The New Aspects of Subnuclear Physics

Edited by
Antonino Zichichi

European Physical Society
Geneva, Switzerland

Erice School 1978

PLENUM PRESS • NEW YORK AND LONDON

SUPERSYMMETRIC THEORIES OF FUNDAMENTAL INTERACTIONS

S. Ferrara

CERN

Geneva, Switzerland

1. INTRODUCTION

Supersymmetry[1] is a new symmetry of local quantum field theory which extends in a non-trivial way the relativity group of space-time. The new symmetry operations are carried by spinorial charges which, according to the spin-statistic theorem, obey anticommutation relations. Particles are classified, in any supersymmetric field theory, according to representations of the symmetry group, and irreducible representations contain particle states of different spin, both integer and half-integer. Supersymmetry is therefore the first example of a genuine relativistic spin containing symmetry.

Previous no-go theorems which prevented possible relativistic generalizations of $SU_w(6)$ are now circumvented.

The key point is that the algebraic structure related to supersymmetry is not an ordinary Lie algebra, to which the above-mentioned theorems applied, but rather a graded Lie algebra (GLA).

Graded Lie algebras contain generators obeying both commutation and anticommutation relations. The spinorial charges obeying anticommutation relations are called the odd elements of the GLA, while

D I S C U S S I O N

CHAIRMAN: S. Ferrara

Scientific Secretaries: K.L. Giboni, A.D. Kennedy, S. Templeton

DISCUSSION No. 1

- THIRRING:

A cancellation between boson and fermion contributions is needed in gravitation for the vacuum expectation value of the energy-momentum tensor. There is always a positive zero point energy for bosons and a negative zero point energy for fermions, and one wants these to cancel because otherwise one would get a large cosmological constant - in fact they cancel to a fantastically good accuracy. Is this something that would be guaranteed by supersymmetry?

- FERRARA:

If one has exact supersymmetry, it immediately follows that the vacuum expectation value of the energy-momentum tensor is exactly zero.

- THIRRING:

This is for an exact symmetry; but if it's broken, things are not so good.

- FERRARA:

You don't know whether it is broken. The stress tensor is obtained through a supersymmetry rotation of the spinor current and if the spinor charges annihilate the vacuum, its vacuum expectation value is zero. Consequently, there is no induced cosmological term in any exact supersymmetric theory.

- HA:

Can one extend the idea of supersymmetry to the domain of

nuclei and construct a phenomenological theory of nuclear spectra?

- FERRARA:

In principle, supersymmetry can apply to any system, there is no reason why it should be a symmetry particularly relevant to particle physics. In any system with bosons and fermions which are almost degenerate in mass, and which seem to fit into a multiplet structure, one could think of applying supersymmetry.

- HA:

That means that fermions and bosons don't have to be fundamental particles, they could be composite ones.

- ZICHICHI:

So the conclusion was that you don't need fundamental particles and fundamental interactions in order to create supersymmetries? This disturbs me very much.

- FARRAR:

There is no reason to expect it to be supersymmetric, but if you look it might be.

- KLEINERT:

If you take a nucleus with a single degenerate unfilled shell, and you proceed to fill that shell, then indeed there is a slightly broken supersymmetry which describes the levels of this nucleus. A supermultiplet consists of the system with 1, 2, ... nucleons in the shell up to a full shell. Pickup and stripping reactions proceed via a fermionic charge just as weak interactions proceed via I^+ or I^-. It has to be decided by comparison with data to what extent supersymmetry can be found also in the interactions.

- ZICHICHI:

It is like asking whether chemistry would obey SU(3) symmetry: it could do it, but why?

- FARRAR:

I think the thing making supersymmetry interesting for particle physics is that the actual Lagrangian is invariant under the symmetry, not just that one can organize the states. I would be astonished if the interaction would have that symmetry in the case of a nucleus.

- HA:

If gauge invariance is consistent with supersymmetry, both spin ½ fermions as well as vector bosons can be gauge particles. What are these spin ½ gauge particles for the strong, weak and electromagnetic interactions?

- FERRARA:

In a sense they are gauge particles because they are the same superparticle - they are two different states of the same supermultiplet. In some sense one can say that one is unifying matter and radiation. For example in supersymmetric Q.C.D. one has an octet of coloured gluons and an octet of massless spin ½ Majorana particles, which are called gluinos. These are only massless if supersymmetry is unbroken; however, if you want to apply supersymmetry to the real world in a realistic way, it must be spontaneously broken. In this case, provided the colour symmetry remains unbroken, the gluons will remain massless but the gluinos may acquire a mass.

- ZICHICHI:

As the gluinos are also coloured, why don't they likewise remain massless due to SU(3) colour symmetry?

(The gluons must remain massless since they are the gauge bosons of the colour symmetry, whereas the gluinos are not. They transform as separate multiplets under colour transformations, but into each other under the supersymmetry transformations which are spontaneously broken.)

- FARRAR:

Another similar example is that for every quark there must be a scalar quark, which must become massive because they are not seen. However, I will be discussing these points in my lectures.

- GROSSE:

In a Yang-type Lagrangian in which you unify spin 0 with spin 2 you get higher than second derivatives. There is the Velo-Zwanziger phenomenon implying that sometimes higher order equations of motion admit wave solutions travelling faster than light. Does this happen in supersymmetric theories?

- FERRARA:

In normal supergravity one finds the usual kinetic energy term having no higher than second derivatives. However, in Weyl

invariant (Superconformal) gravity one finds that the spin two particle propagates with a quartic derivative and the spin 3/2 one with a cubic derivative.

- GROSSE:

Why do we always use Majorana spinors?

- FERRARA:

The essential reason for the basic building block being the Majorana spinor is because of the hermiticity property of the anti-commutator. On the right hand side the momentum is hermitian, and you can play around with the γ-matrices to show that the spinors Q on the left hand side must be self-conjugate. Of course this is no restriction, since one can have as many spinors as one wishes.

- BHANOT:

Can you give an intuitive argument why the Coleman-Mandula and O'Raifeartaigh no-go theorems do not apply to Graded Lie Algebras?

- FERRARA:

For a long time people tried to combine internal symmetries with Poincaré invariance, and eventually these no-go theorems were proved as purely mathematical theorems, and their physical content is unclear.

- BLASI:

The no-go theorems state that one can only combine internal symmetries with Poincaré invariance trivially (as a direct product) if one tries to do this at one space-time point. This is not the case in supersymmetry as it involves neighbouring points.

- FERRARA:

This is so because one has a derivative in the supersymmetric transformation; it is easy to see that this comes about because the product of two supersymmetry transformations is a translation, and the generator of translations is a derivative.

- KENNEDY:

Why can't you introduce this convenient derivative for a Lie algebra?

- FERRARA:

Many people have tried this, but none have succeeded; the basic problem is that one cannot do this with a finite number of fields, and this is one of the assumptions of the Coleman theorem.

- KLEINERT:

Maybe it is because we never tried to put a derivative on the right hand side; and let us not forget that the no-go theorem was really derived for the purpose of trying to get a multiplet with different masses into a combined representation of Poincaré and internal group, and the proof showed that the masses would have to be equal so that relativistic SU(6) would never work out. In supersymmetric theories the masses are still degenerate.

- FERRARA:

Yes, but the spins are different.

- ROTH:

If certain infinities can be eliminated by generalizing the Lie algebras to Graded Lie Algebras, why not look at more generalized algebras to try to eliminate the remaining infinities?

- FERRARA:

Supersymmetry is a good example of the fact that when you go to a richer algebraic structure you can get unexpected benefits. In principle there is no reason to suppose that there shouldn't be richer structures, but no one has found them yet.

- SCHELLEKENS:

Do supersymmetry cancellations also occur in supersymmetric φ^3 theory in such a way that it is finite, and if so to what order has this been checked?

- FERRARA:

It is not finite, but it is renormalizable. It also preserves the supersymmetry to all orders.

- THIRRING:

Is it finite in one space one time dimension?

- FERRARA:

Some people in Dublin have proved it to be.

- BENGTSSON:

Would you care to elaborate somewhat on the differences between particles under SO(7) and SO(8), since the number of particles with a given helicity seems to be the same in the two representations.

- FERRARA:

Several multiplets are identical, such as those with N = 7 and 8 and $\lambda_{max} = 2$, or with N = 3 and 4 and $\lambda_{max} = 1$. They are exactly the same; if you start with N = 3 you automatically build up a multiplet with higher symmetry.

- NILSSON:

Why is the largest symmetry you consider SO(8), why don't you extend this to SO(10)?

- FERRARA:

First of all one gets 10 gravitons, which we don't like because they are gauge particles and we would have to find an invariance for each of them, and we only have one momentum to gauge. Thus this would be very difficult, unless you believe in the strong gravity of Salam and Strathdee. No one has been able to construct a consistent theory containing particles of spin greater than 2, and we would need a spin 5/2 particle for SO(10). Spin 1 particles are usually associated with gauging internal symmetries, spin 3/2 with gauging spin ½ supersymmetry generators, and spin 2 with gauging the Poincaré group. A spin 5/2 particle would have to be associated with gauging a spin 3/2 supersymmetric generator, and this has been ruled out in a paper by Haag, Lopuszanski and Sohnius.

- ZICHICHI:

There is only one charge that we really understand in nature and that is the gravitational charge, which corresponds to the curvature of space-time. In these supersymmetric theories how are we to understand the role of the other charges which are important in nature, can one obtain these charges as some kind of generalized curvature in the space on which supersymmetry is based? What should our physical intuition be? Einstein died without understanding the connection between electric charge and mass because he could not believe there to be something absolutely irreconcilable with the concept of space-time.

- FERRARA:

If one takes the view advocated by Gell-Mann that there is only one fundamental multiplet in nature which contains all fundamental particles, then the maximum symmetry one can have is an SO(8) gauge symmetry. In this one has a collection of particles, one has the graviton and 8 spin 3/2 gravitinos. The graviton is the gauge particle related to the Poincaré group. The gravitinos are gauging the 8 supersymmetric charges. You have 28 spin 1 bosons which are the gauge particles of the 28 dimensional SO(8) internal symmetry group. This theory has only 2 coupling constants, the gravitational charge and the dimensionless gauge coupling constant

which only arises in curved space with cosmological term.

- ZICHICHI:

What if I want to work out the strong, weak and other coupling constants?

- FERRARA:

These come from the 2 coupling constants through spontaneous symmetry breaking. This theory is not sufficient for this unification because SO(8) does not contain $SU(3)_{colour} \otimes SU(2)_{weak} \otimes U(1)_{em}$. If you do a reduction of SO(8) into SU(3) as Gell-Mann did, you find that you can predict the charges of the quarks; this is the first model in which you can theoretically predict that the quarks possess fractional charges of the correct values.

- ZICHICHI:

Without cheating? Genuine?

- FERRARA:

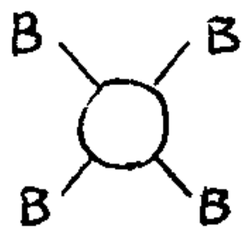
Yes.

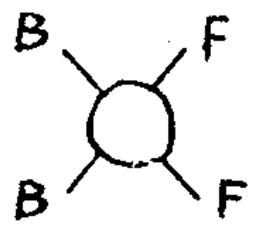
- KENNEDY:

I want to ask a question about the connection between Supergroups and ordinary Lie groups. I find it hard to visualize a manifold in which some of the variables correspond to anticommuting elements of a Grassmann algebra; how do the group elements correspond to finite transformations when one can only expand to some finite order in the anticommuting parameters? To what extent do Lie's theorems and their converses hold for supergroups?

- FERRARA:

One cannot visualize any physical transformations which realize the supersymmetry transformations because of the fermionic nature of the charges. Nevertheless, a supergroup is a mathematically well defined object. Although the fermionic charges are not observable due to their spinorial nature, they give rise to physically observable effects in amplitudes for scattering processes such as





Almost all the theorems that hold for Lie algebras can be extended to Graded Lie algebras; there has been an extensive investigation

of the structure of Graded Lie groups by Sternberg and Konstant at M.I.T. and Harvard. There is an analogue of the Cartan classification of all compact semi-simple Lie algebras for Graded Lie algebras.

- FELTESSE:

Do you violate the principle of causality with this space-time symmetry which transforms fields at different points into each other?

- FERRARA:

No, because the transformation involves only a finite number of derivatives.

The following result holds in any supersymmetric theory and has not been proved for any other type of theory: by taking the relation

$$\{Q_\alpha , \bar{Q}_\beta\} = -2\, \gamma^{\mu}_{\alpha\beta}\, P_\mu$$

and multiplying both sides by γ and taking the trace one can also show that the energy is positive definite as

$$H = P^0 = \sum_\alpha Q_\alpha^2 \geq 0$$

or at least non-negative. Therefore the supersymmetric theory is a local theory which looks better than the usual one.

- NILSSON:

There is a supergravity theory based on the conformal group, can you get the Einstein theory out of this?

- FERRARA:

Conformal supergravity is completely invariant under local conformal transformations and it has a dimensionless gravitational constant. Thus the theory is completely different from Einstein gravity which has a dimensional coupling constant. There is one connection which is that any solution of the vacuum Einstein equations $R_{\mu\nu} = 0$ is also a solution of the conformally invariant Weyl theory. The action for the Weyl theory is

$$\int d^4x \left(R_{\mu\nu}^2 - \tfrac{1}{2} R^2\right) \sqrt{|g|}$$

- KENNEDY:

That looks rather like the Yang-Stephenson theory which was introduced to make gravitation look more like the Yang-Mills theory.

- FERRARA:

Since $\int (R_{\mu\nu\rho\lambda})^2$ can be written as a linear combination of $\int R^2$ and $\int R_{\mu\nu}^2$, all of these theories are similar.

- ROTH:

Is supergravity two loop finite or just two loop renormalizable?

- FERRARA:

It is two loop finite.

- ROTH:

Is there hope that it's going to be finite to all orders or renormalizable to all orders?

- FERRARA:

The hope is that it will prove to be finite to all orders - it is not renormalizable in the sense that the Green's functions are infinite. What happens is that when you go onto mass shell all the infinities cancel, it is finite in the sense of S-matrix elements just like the ordinary Einstein theory without matter. This comes about, because the coefficients of the counter-terms vanish when you use the field equations.

- ROTH

I don't understand. If you have counter terms then you are doing renormalization, aren't you?

- FERRARA:

The theory is not renormalizable as you get an infinite number of counter terms, but as these all vanish on mass shell (at least to the two loop level) all this is unimportant.

- KENNEDY:

The meaning of renormalizability is unclear for such Lagrangians, in any case, as we have a non-polynomial interaction due to the presence of the $\sqrt{|g|}$ in the tensor density.

- FERRARA:

One only has a finite number of terms up to the two loop level, but the theory certainly is not renormalizable.

- ANASTAZE:

If we decompose the 56-plet of Majorana spinors into Dirac spinors you miss some particles such as the muon, the muon neutrino and the tau. Is it possible to explain this, and is this consistent with what we know about elementary particles?

- FERRARA:

If you insist on a scheme in which all particles sit in a single SO(8) multiplet and you believe in Q.C.D. then there are missing particles. This scheme is unsatisfactory from this point of view. At present I don't know of any way out other than putting them into more than one multiplet.

DISCUSSION No. 2 (Scientific Secretaries: K.L. Giboni, A.D. Kennedy, S. Templeton)

- BHANOT:

I would like to ask you about your model with gluons and gluinos. Could you please comment on the independent degrees of freedom and on the ghosts that are needed?

- FERRARA:

All the Lagrangians we have written are classical Lagrangians, if you want to quantize them you must add Fadeev-Popov ghosts; and then for supersymmetric gauge theories follow the usual renormalization procedure.

- BHANOT:

I thought these were not renormalizable but finite.

- FERRARA:

No, the supersymmetric Yang-Mills theory is renormalizable but not finite; only supergravity is one and two loop finite but not renormalizable. It is worth noticing that for example in the supersymmetric Yukawa theory supersymmetry leads us to expect that the wave function, mass and coupling constant renormalizations are each common to all members of the multiplet. Furthermore, the dynamics leads to further relations between the renormalization constants $M_R = Z\, M_0$, $g_R = Z^{3/2} g_0$, where Z is the wave function renormalization. We see that there is only one basic divergence in the theory.

- BHANOT:

Is this a consequence of the generalized Ward identities?

- FERRARA:

No, they are a consequence of the dynamical structure of the theory and the topology of the Feynman diagrams corresponding to the particular form of the Lagrangian chosen.

- JACOB:

You gave a beautiful review of supersymmetry as it now stands after several years of development. Could you please itemize those topics which are particularly "hot" at present and give us an idea of where the field is presently progressing?

- FERRARA:

The prospects for supersymmetry are of two different kinds, firstly technical developments and secondly there are developments which are endeavouring to construct realistic models. In the latter case I refer you to Glennys Farrar's lectures. Putting aside supergravity for the moment, one would like to construct super-symmetric versions of Q.C.D., and of the weak and electromagnetic interactions: in doing this the problem is having a proliferation of particles. All the common particles have partners of different spin, for example the quarks have scalar partners and the gluons have fermionic partners, the gluinos. The problem is to find definite predictions which can be tested experimentally.

The most ambitious aim is to make a finite theory of gravitation, which requires putting matter particles all into the same multiplet as the graviton. There is a programme at present trying to get around the problem that the largest internal symmetry allowable is SO(8), as discussed before.

- BLASI:

What is the meaning of "on the mass shell" when applied to the counter terms of supergravity?

- FERRARA:

The situation is the same as in the ordinary Einstein theory of gravity, where in the absence of matter the theory is one loop renormalizable due to some miraculous cancellations (as was first shown by 't Hooft and Veltman). At one loop order the graviton interaction of pure gravity is given by the following diagram