LIMITATIONS TO THE COLEMAN–WEINBERG MECHANISM OF SPONTANEOUS MASS GENERATION

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We point out that the second order phase transition of compact U(1) lattice gauge theory implies that a certain range of abelian Higgs models escapes the Coleman–Weinberg mechanism of spontaneous mass generation and suggest ways for a Monte Carlo investigation of the full range of its validity.

A decade ago, Coleman and Weinberg [1] argued that the abelian Higgs model in four dimensions

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 + \tfrac{1}{2}|(\partial_\mu - ieA_\mu)\varphi|^2 + \tfrac{1}{2}m^2|\varphi|^2 - \tfrac{1}{4}\lambda|\varphi|^4 \qquad (1)$$

should, for $m^2 = 0$, generate spontaneously masses for both the scalar field φ as well as the photon field A_μ, with a ratio

$$m_\varphi^2/m_A^2 = (3/\pi)\,e^2/4\pi\,. \qquad (2)$$

An equivalent statement is that the model has a first order phase transition at $m^2 > 0$.

The claim was derived from perturbation theory, thus requiring sufficiently small e^2 *and* λ.

For small $\kappa^2 \equiv \lambda/e^2 \ll 1$, the conclusion follows directly from the observation, that the "black body" radiation of the photons in the presence of a constant φ field leads to an additional effective potential

$$\Delta V = (3e^4/64\pi^2)|\varphi|^4\,[\log(|\varphi|^2/M^2) - \tfrac{25}{6}]\,, \qquad (3)$$

where M is an arbitrary mass scale. The corrected potential has its minimum away from the field origin at $|\varphi| = M$ where the curvature of ΔV is $m_\varphi^2 = (3e^4/8\pi^2) \times |\langle\varphi\rangle|^2$. This minimum spontaneously violates the U(1) symmetry $\varphi \to e^{i\gamma}\varphi$. Due to the minimal coupling, the photon acquires a mass $m_A^2 = e^2|\langle\varphi\rangle|^2$, thus leading to the ratio (2).

The initial restriction of small K could apparently be removed by a renormalization group argument [‡1] by which the coupling constant λ could be changed by a large amount with almost no change in e^2 such that any range of K could be covered [‡1].

Unfortunately, this argument considered by the authors as a virtue, is actually a source of a severe difficulty: When starting at a certain mass scale in the perturbative regime and changing this scale towards the infrared, the coupling λ rapidly becomes larger and thus escapes into a regime where perturbation theory is no longer valid and non-expandable terms of the type $e^{-1/\lambda}$ become important. These can easily generate an infrared stable fixed point in which case the model would certainly have a second phase transition such that there would be no spontaneous mass generation.

That something like this can, indeed, happen is seen in three dimensions where (1) coincides with the Ginzburg–Landau theory of the superconductive phase transition.

In the neighborhood of the critical temperature T_c, the mass term behaves as $m^2 = (T/T_c - 1)$ and the couplings λ, e are of the order $\lambda \sim \rho^{-3/2}(T_c/T_F)^2$, $e \sim \rho^{1/4} \times (\alpha v_F/c)^{1/2}$ where T_F is the Fermi temperature, ($T_F = p_F^2/2m_{el} \sim 10^4$ K), v_F the Fermi velocity ($v_F = p_F/m_{el} \sim 10^8$ cm/s), c the light velocity, α the fine-structure constant, and ρ is roughly the ratio of elec-

[‡1] See the last sentences in section V of ref. [1].

tron mean free path over the clean superconductor's coherence length $\xi_0 \sim v_F/T_c$. In dirty samples, ρ and thus e can be made arbitrarily small. In clean superconductors we have to take $\rho = 1$ and $e \sim 10^{-2}$. The parameter $\kappa = (\lambda/e^2)^{1/2} \sim \rho^{-1}(c/\alpha v_F)^{1/2} T_c/T_F$ is the ratio of magnetic penetration depth versus coherence length times $1/\sqrt{2}$ and it is customary to distinguish type I and type II superconductors according to $\kappa \lesseqgtr 1/\sqrt{2}$. For the superconductor, the Coleman–Weinberg argument [2] leads to an additional cubic potential $\Delta V = -(1/6\pi)\, e^3 |\varphi|^3$ and this suggests a first order phase transition with a latent heat $\Delta S \sim e^6/\lambda^2$ at a precocious temperature $T_{\rm prec}$ slightly above T_c:

$$T_{\rm prec}/T_c - 1 \sim e^6/\lambda \sim \rho^3(\alpha v_F/c)^3(T_F/T_c)^2 . \qquad (4)$$

Unfortunately, these quantities are too small to be detectable, ($T_{\rm prec} - T_c \sim 10^{-6}$ K, typically), so the conclusion was rather academic.

While superconductors cannot be used to test the mechanism, there exists yet another physical transition with a completely different looking energy density which nevertheless is structurally isomorphic to the superconductive case, as far as the Coleman–Weinberg mechanism is concerned [2]. This is the smectic A-nematic phase transition. Here the precocity $T_{\rm prec} - T_c$ is of the order of 10^{-2} K and therefore detectable. Nevertheless, the transition is found, experimentally, to be of second order, with a temperature resolution of 10^{-3} K [3].

As we understand it now, the Coleman–Weinberg mechanism in three dimensions breaks down if there are strong fluctuations due to vortex loops. These invalidate, on the one hand, the $\varphi \approx$ const. assumption underlying the calculation of ΔV since inside each core of a vortex line $\varphi = 0$, on the other hand, the perturbative renormalization group argument, due to non-expandable $e^{-1/\lambda}$ type terms in the β function.

The vortex fluctuations increase with increasing κ. The temperature interval in which they are important was estimated by Ginzburg [4]. It is given by the condition that condensation energy density times coherence volume be of the order of the temperature, which here amounts to

$$T_G/T_c - 1 \sim \lambda^2 \sim \rho^{-3}(T_c/T_F)^4 . \qquad (5)$$

Compared with (4) we see that the ratio is κ^6. Hence, for large κ (deep type II), $T_{\rm prec}$ lies within the Ginzburg interval and the $\varphi \sim$ const. assumption cannot possibly be trustworthy. By constructing a dual field theory of fluctuating vortex loops [5] it was possible to estimate a tricritical value [6] $\kappa_{tc} \sim 0.8/\sqrt{2}$ where the transition changes from first to second order. This vortex field theory turns out to be a $g|\varphi|^4$ theory, due to the short-range interaction between the vortex lines, with the coupling g changing sign at the tricritical value of κ_{tc}.

Naturally, the question arises as to the range of validity of the Coleman–Weinberg argument in *four dimensions*. It is well known that close to the critical point, the abelian Higgs model forms the dual representation of the abelian U(1) lattice gauge model [7] whose partition function reads

$$Z = \prod_{x,\mu} \int_{-\pi}^{\pi} \frac{dA_\mu(x)}{2\pi} \times \exp\Big(\beta_2 \sum_{x,\mu\neq\nu} [\cos(\nabla_\mu A_\nu - \nabla_\nu A_\mu) - 1]\Big) , \qquad (6)$$

where μ, ν are oriented link labels ($\mu, \nu = 1, 2, 3, 4$) and x runs over all lattice points. This partition function has been thoroughly investigated by Monte Carlo methods [8] with the result that at $\beta \sim 0.995$ there is a second order phase transition. The dual representation is found as follows: A Villain approximation [9] brings Z to the periodic gaussian form

$$Z \sim \prod_{x,\mu} \int_{-\pi}^{\pi} \frac{dA_\mu(x)}{2\pi} \times \sum_{n_{\mu\nu}(x)} \exp\Big(-\beta_2 \sum_{x,\mu>\nu} (\nabla_\mu A_\nu - \nabla_\nu A_\mu - 2\pi n_{\mu\nu})^2\Big). \qquad (7)$$

The sum over $n_{\mu\nu}(x)$ can be rewritten as

$$\sum_{n_{\mu\nu}(x)} \prod_x \int_{-\infty}^{\infty} df_{\mu\nu}(x) \exp\Big(\sum_{x,\mu>\nu} [-f_{\mu\nu}^2/2\beta + i f_{\mu\nu}(\nabla_\mu A_\nu - \nabla_\nu A_\mu - 2\pi n_{\mu\nu})]\Big)$$

$$= \sum_{f_{\mu\nu}(x)=\text{integer}} \delta_{\bar\nabla_\mu f_{\mu\nu},0} \exp\Big(-\frac{1}{2\beta} \sum_{x,\mu>\nu} f_{\mu\nu}^2\Big) .$$

Setting $f_{\mu\nu} = \epsilon_{\mu\nu\lambda\kappa} D_\lambda \tilde{A}_\kappa$, integrating over $\tilde{A}_\kappa$, and enforcing the integerness of $f_{\mu\nu}$ by a sum over closed integer valued monopole world lines, l_μ, this becomes

$$Z = \prod_{x,\kappa} \int_{-\infty}^{\infty} \mathrm{d}\tilde{A}_\kappa(x)\, \delta(\nabla_\mu \tilde{A}_\mu)$$

$$\times \exp\Big(-\frac{1}{2\beta} \sum_{x,\mu>\nu} f_{\mu\nu}^2(x)\Big)$$

$$\times \sum_{l_\mu(x)} \delta_{\bar{\nabla}_\mu l_\mu, 0} \exp\Big(2\pi i \sum_{x,\mu} l_\mu(x)\, \tilde{A}_\mu(x)\Big). \qquad (8)$$

The $l_\mu(x)$ sum can be turned into a disorder field theory [5] which close to the critical point becomes

$$\sum_{l_\mu(x)} \delta_{\bar{\nabla}_\mu l_\mu, 0} \exp\Big(2\pi i \sum_{x,\mu} l_\mu \tilde{A}_\mu\Big)$$

$$\xrightarrow[\text{critical limit}]{} \int \mathcal{D}\varphi\, \mathcal{D}\varphi^+$$

$$\times \exp\{-\tfrac{1}{32}|(\partial - \mathrm{i}eA)\varphi|^2 - \tfrac{1}{16}[e^2 v(0) - 4]|\varphi|^2$$

$$- \tfrac{1}{64}|\varphi|^4 + \mathrm{O}(|\varphi|^6, |\partial\varphi|^2|\varphi|^2, |\partial^2\varphi|^2)\}\,, \qquad (9)$$

where $v(0)$ is the lattice Coulomb propagator at the origin (~0.155) and the charge e is an abbreviation for $e = 2\pi\sqrt{\beta}$. At the mean field level, there is a second order phase transition at $\beta \sim 0.65$ which is moved up to $\beta_c \sim 0.995$ by fluctuations.

Now, the partition function (8) with the disorder fields (9) certainly constitutes an example for an abelian Higgs model with a second order phase transition. The charge e is, however, too large (~2π) to give us information on the range of coupling constants for which the Coleman–Weinberg mechanism was derived. Thus we have to modify the model such as to decrease e^2. This is possible by adding, in (8), a core energy for the monopoles, say $-\frac{1}{2}t l_\mu^2$. In the disorder field theory (9) this modifies the mass term to $-\frac{1}{16}(t + e^2 v(0) - 4)|\varphi|^2$ such that, at the mean field level, there is a line of transitions ending at $e^2 = 0$, $t = 4$. In the original action (7) the core energy is equivalent to adding, in the exponent

$$-\tfrac{1}{2}t(\epsilon_{\mu\nu\lambda\kappa} \nabla_\nu n_{\lambda\kappa})^2\,. \qquad (10)$$

But from the experience with the three-dimensional version [10], as well as other models [11], we know that an increase in defect core energy can at most *soften* the phase transition in which these proliferate. Thus we expect the transition to remain second order along the whole line $t + e^2 v(0) - 4 \sim 0$. Indeed, at $e^2 = 0$ the Higgs model reduces to a pure complex $|\varphi|^4$ theory which is well known to undergo a continuous transition.

In this way, we have generated a whole family of abelian Higgs models with small e^2 which do not show spontaneous mass generation. Thus we know that if, by a change of scale towards the infrared, the coupling λ escapes to the large values implied by (9), there will certainly be a second order phase transition, implying the existence of an infrared stable fixed point.

At present, no information is available for small e *and* λ. Should the three-dimensional situation carry over to four dimensions, this could endanger many of the present applications to particle physics and cosmology (e.g. inflationary universe [12]). Monte Carlo calculations can give valuable information on this regime by taking the core energy as a gaussian random variable fluctuating around some given mean value of t_0. The integration

$$\int \frac{\mathrm{d}t}{(2\pi\epsilon)^{1/2}} \exp[-(t - t_0)^2/2\epsilon]$$

$$\times \exp\{\{-\tfrac{1}{16}[t + e^2 v(0) - 4]\}|\varphi|^2\}$$

$$= \exp\{\{-\tfrac{1}{16}[t_0 + e^2 v(0) - 4]\}|\varphi|^2$$

$$+ [\epsilon/(2\cdot 16^2)]|\varphi|^4\}\,, \qquad (11)$$

generates an additional quartic term which can be chosen to make the $-\frac{1}{64}|\varphi|^4$ term in (9) arbitrarily small. Hence, a Monte Carlo study of the model (7), modified by (10) *and* (11), would make it possible to determine the domain in the coupling constant plane over which the Coleman–Weinberg mechanism is valid.

The absence of the mechanism in actual superconductors at small e *and* λ certainly demonstrates the need for such an investigation.

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