HIGGS PARTICLES FROM PURE GAUGE FIELDS

Hagen Kleinert

Physics Department
Free University Berlin
1000 Berlin 33

I Introduction

One of the important outstanding questions in unified gauge theories of weak and electromagnetic interactions is the physical nature of the Higgs particles required by renormalization. Until recently, ideas have been guided by the first historical appearance of such fields in the context of superconductivity. In 1934, Gorter and Casimir[1] first proposed the use of a space time independent order parameter for the description of the temperature behavior of the specific heat in the superconductive phase transition. In 1937, Landau[2] made the order parameter a local order field by introducing gradient terms permitting the study of spatial fluctuations. This theory has the characteristic that as the temperature of the system passes below a certain temperature T_c, the mass term which stabilizes fluctuations changes sign, thereby leading to a non-vanishing expectation value $\langle\varphi\rangle \equiv \varphi_0 \neq 0$ of the order field. There is a second order phase transition to an ordered phase.

If the order field $\varphi(\underset{\sim}{x})$ is complex and the energy depends only on $\varphi^+\varphi$, there are two degenerate fluctuations above T_c. Below T_c there is the phenomenon of spontaneous symmetry breakdown: The size of the order parameter has massive, the phase, however, has long-range massless fluctuations which join smoothly to symmetry transformations in the limit $\underset{\sim}{k} \to 0$. These are called Nambu-Goldstone bosons. In 1950, finally, Ginzburg and Landau[3] took this theory and coupled it to a vector potential in a gauge invariant way. In this way they arrived at the prototype of present models of unified gauge theories. These are characterized by two properties. First, due to the covariant coupling

$$|(\underset{\sim}{\partial} - ie\underset{\sim}{A})\varphi|^2 \tag{1}$$

the ordered phase generates a photon mass term which leads to a finite magnetic penetration depth (Meissner effect[4]). Simultaneously, the gauge invariance of the coupling allows for the removal of the phase fluctuations by a gauge transformation and absorb them completely in the vector field A. There they become part of the massive gauge field fluctuations required by rotational symmetry (Higgs effect)[5]. The Meissner-Higgs effect has turned out to be crucial for generating a short range weak interaction without destroying the renormalizability of long-range gauge theories[6].

The remaining size fluctuations of the order field are presently referred to as Higgs particles. Nine years after Ginzburg and Landau, Gorkov[7] was able to explain the order field microscopically as a field of bound states of pairs of electrons (Cooper pairs). Inspired by this it is widely believed that the Higgs particles in unified gauge theories might be bound states of some more fundamental underlying microscopic constituents (e.g. technicolor quarks).

It is the purpose of this lecture to draw attention to another type of Higgs particles which exist in a variety of many-body systems and which could, in fact, represent the correct explanation for the origin of these fields in the context of unified theories of weak and electromagnetic interactions. These Higgs fields appear naturally in many non-linear problems in which the non-linearity leads to line-like singular, or almost singular, field configurations. They have physical properties opposite to the order field and are therefore called disorder fields. In contrast to order fields, they acquire non-vanishing expectation as the temperature passes above a certain critical value. We are far from being able to describe their specific role in the context of non-abelian gauge theories. In abelian gauge theories, however, they emerge quite naturally as a way of parametrizing global group excitations. Since such excitations are known to be of extreme importance also in non-abelian lattice gauge theories it is suggested that the Higgs fields appearing in unified theories are nothing but a convenient way of parametrizing non-linear line-like singular field configurations of a pure gauge theory. The development presented in this chapter will be completely parallel to recent discussions for crystals[8], pion condensates[9], and magnetic superconductors[10]. In all systems, line-like, almost singular field configurations are known under the name of defect lines. They play an important role in understanding fluctuation properties and phase transitions.

II Lattice Gauge Theory

Let us recall Wilson's definition[11] of a gauge theory on a simple, cubic lattice. If $\underset{\sim}{x}$ denotes the discrete lattice sizes and $\underset{\sim}{i}$ the oriented link going from $\underset{\sim}{x}$ to the next neighbor along the possible $\hat{x}_i$ axes, the partition function is given by

$$Z = \prod_{\underset{\sim}{x}, i} \int dU_i(\underset{\sim}{x})\, e^{\frac{\beta}{2d_c} \operatorname{tr} \sum_{\underset{\sim}{x}, i+j} \left(\operatorname{Re} U_{ij}^{\square} - 1\right)} \tag{2}$$

where dU_i is the invariant group measure and $U_{ij}^{\square}$ a product of four group elements, one for every link on a plaquette formed by $\underset{\sim}{x}, \underset{\sim}{x}+\underset{\sim}{i}, \underset{\sim}{x}+\underset{\sim}{i}+\underset{\sim}{j}, \underset{\sim}{x}+\underset{\sim}{j}$:

$$U_{ij}^{\square} \equiv U_i(\underset{\sim}{x})\, U_j(\underset{\sim}{x}+\underset{\sim}{i})\, U_i^{+}(\underset{\sim}{x}+\underset{\sim}{j})\, U_j^{+}(\underset{\sim}{x}) \tag{3}$$

The number d_c is a conventional factor denoting the dimensionality of the U_i matrices. For small field fluctuations, $U_i \sim 1$ can be parametrized in terms of $d_c \times d_c$ matrices A_i as

$$U_i(\underset{\sim}{x}) = e^{iagA_i(\underset{\sim}{x})} = 1 + iagA_i(\underset{\sim}{x}) + \tfrac{1}{2}(iag)^2 A_i(\underset{\sim}{x})^2 + \ldots$$

and one finds the limit $A_i \to 0$

$$U_i(\underset{\sim}{x})U_j(\underset{\sim}{x}+\underset{\sim}{i}) = 1 + iag\,(A_i + A_j) + \tfrac{1}{2}(iag)^2(A_i^2 + A_j^2) + ia^2g\,\partial_i A_j + (iag)^2 A_i A_j + \ldots$$

such that

$$\begin{aligned} \mathrm{Re}\, U_{ij}^{\square} - 1 &= -\tfrac{1}{2}\big(U_i(\underset{\sim}{x})U_j(\underset{\sim}{x}+\underset{\sim}{i}) - U_j(\underset{\sim}{x})U_i(\underset{\sim}{x}+\underset{\sim}{j})\big) \\ &\quad \cdot \big(U_j^+(\underset{\sim}{x}+\underset{\sim}{i})U_i^+(\underset{\sim}{x}) - U_i^+(\underset{\sim}{x}+\underset{\sim}{j})U_j^+(\underset{\sim}{x})\big) \\ &\to -\tfrac{1}{2}a^4g^2F_{ij}^2 \equiv -\tfrac{1}{2}a^4g^2\{\partial_i A_j - \partial_j A_i + ig[A_i,A_j]\}^2 \end{aligned} \tag{4}$$

thus arriving at the conventional continuum theory in D dimensions

$$Z = \prod_{\underset{\sim}{x},i} \int_{-\infty}^{\infty} dA_i(\underset{\sim}{x})\, e^{-\frac{1}{4}\int d^Dx\, F_{ij}^2} \tag{5}$$

if one identifies

$$\beta \equiv \frac{d_c}{a^{4-D}}\,\frac{1}{g^2} \tag{6}$$

and a as the lattice spacing. In other words, the temperature $T \equiv \beta^{-1}$ in such a description is proportional to the square of the coupling constant. In the case of an abelian theory $U_i = e^{iagA_i}$

with A_i=real and the partition function becomes

$$Z = \prod_{\underset{\sim}{x},i} \int_{-\pi}^{\pi} \frac{dA_i(\underset{\sim}{x})}{2\pi} e^{-\beta \sum_{\underset{\sim}{x},i<j} (1-\cos(\nabla_i A_j - \nabla_j A_i))} \tag{7}$$

where

$$\nabla_i A_j(\underset{\sim}{x}) \equiv A_j(\underset{\sim}{x}+\underset{\sim}{i}) - A_j(\underset{\sim}{x}) \tag{8}$$

is defined as the lattice derivative.

The important feature which distinguishes this lattice gauge theory from the small field limit (5) is the physical spectrum. In (5), there are only non-interacting photon fields A_i. In (7) there are, in addition, global group excitations which in the following will be called defects. They arise from the possibility that if the difference of two adjacent A_i fields $\nabla_i A_j(\underset{\sim}{x}) = A_j(\underset{\sim}{x}+\underset{\sim}{i}) - A_j(\underset{\sim}{x})$ changes by an integer multiple of 2π, the cosine function is invariant and the energy is unchanged.

Defects have an important impact upon the statistical mechanics of the system due to their high entropy. This is why they lead to a drastic change of thermodynamic properties at a certain critical value β_c. For small β, i.e. in the hot phase, they proliferate and cause a phase transition due to the well-known relation

$$F = E - TS$$

which shows the importance of configurations of high entropy at high temperature even if the energy is large.

The point we would now like to make is that starting from a pure gauge theory, the proliferation of these defects can most easily be classified by rewriting the theory as another gauge theory involving the <u>dual</u> field strength coupled minimally to complex

Higgs fields. The Higgs field accounts for the random ensemble of closed defect lines. For $T > T_c$, it takes a non-vanishing expectation. Thus it is properly a disorder field in the sense defined above.

Since the field energy is dual to the original one, the ensemble of defect lines is nothing but the world lines of gas of magnetic monopoles. It is the magnetic analogue of Debye screening that in this gas color electric flux lines are compressed into small tubes thereby leading to a linear potential between color electric quarks. In order to develop this Higgs field theory, it is most convenient to approximate the periodic energy $1-\cos(\nabla_i A_j - \nabla_j A_i)$ by another expression which has the same periodicity and shape around the minima of the energy, where the configurations are most probable, and introduces an error only where the energy is large, i.e. for improbable configurations. This approximation was invented by Villain and is excellent for not too small values of β [12]. He observed that for any configuration $A_i(\underset{\sim}{x})$ near the minimum of the energy there is always a configuration of integer numbers $n_{ij}(\underset{\sim}{x})$ such that

$$1-\cos\left(\nabla_i A_j(\underset{\sim}{x}) - \nabla_j A_i(\underset{\sim}{x})\right) \sim \frac{1}{2}\left(\nabla_i A_j(\underset{\sim}{x}) - \nabla_j A_i(\underset{\sim}{x}) - 2\pi n_{ij}(\underset{\sim}{x})\right)^2 \tag{9}$$

Thus, if we perform this replacement in the partition function and sum over all $n_{ij}(\underset{\sim}{x})$ we introduce little error since the wrong values of $n_{ij}(\underset{\sim}{x})$ are strongly suppressed by a high energy. In this way, Villain arrived at a partition function of the form

$$Z \sim Z_V = \prod_{\underset{\sim}{x}, i} \int_{-\pi}^{\pi} \frac{dA_i(\underset{\sim}{x})}{2\pi} \sum_{\{n_{ij}(\underset{\sim}{x})\}} e^{-\frac{\beta}{2}\sum_{\underset{\sim}{x}, i<j} (\nabla_i A_j - \nabla_j A_i - 2\pi n_{ij}(\underset{\sim}{x}))^2} \tag{10}$$

This has the great advantage that the exponent is quadratic in all fluctuating variables. Therefore, we can introduce an antisymmetric auxiliary field tensor F_{ij} and rewrite the partition function in the form[13,14]

$$Z_V = \prod_{\underset{\sim}{x},i} \int_{-\pi}^{\pi} \frac{dA_i(\underset{\sim}{x})}{2\pi} \sum_{\{n_{ij}(\underset{\sim}{x})\}} \prod_{\underset{\sim}{x},i<j} \int_{-\infty}^{\infty} \frac{dF_{ij}(\underset{\sim}{x})}{\sqrt{2\pi\beta}} e^{-\frac{1}{2\beta}\sum_{\underset{\sim}{x},i<j} F_{ij}^{2} + i\sum_{\underset{\sim}{x},i<j} F_{ij}(\nabla_i A_j - \nabla_j A_i - 2\pi n_{ij})} \tag{11}$$

The sum over $n_{ij}(\underset{\sim}{x})$ can be performed using Poisson's formula

$$\sum_{n=-\infty}^{\infty} e^{-2\pi i F n} = \sum_{f=-\infty}^{\infty} \delta(F-f) \tag{12}$$

which squeezes the field F_{ij} onto the integers f_{ij}. If we now execute the A_i integrals, the integer numbers f_{ij} are seen to have no lattice divergence. Hence

$$\begin{aligned}
&\prod_{\underset{\sim}{x},i} \int_{-\pi}^{\pi} \frac{dA_i(\underset{\sim}{x})}{2\pi} \sum_{\{n_{ij}(\underset{\sim}{x})\}} e^{i\sum_{\underset{\sim}{x},i<j} F_{ij}(\nabla_i A_j - \nabla_j A_i - 2\pi n_{ij})} \\
&= \sum_{\{f_{ij}(\underset{\sim}{x})\}} \delta_{\overset{*}{\nabla}_i f_{ij},0} \; \delta(F_{ij} - f_{ij})
\end{aligned} \tag{13}$$

In order to arrive at this result we have used the lattice analogue of partial integration, $\sum_{\underset{\sim}{x}} g^{*}(\underset{\sim}{x}) \nabla_i h(\underset{\sim}{x}) = -\sum_{\underset{\sim}{x}} (\overset{*}{\nabla}_i g(\underset{\sim}{x}))^{*} h(\underset{\sim}{x})$, where

$$\overset{*}{\nabla}_i g(\underset{\sim}{x}) = g(\underset{\sim}{x}) - g(\underset{\sim}{x} - \underset{\sim}{i}) \tag{14}$$

Because of the lattice divergence, we can rewrite f_{ij} as a curl of an integer valued lattice vector potential which is dual to A_i

$$f_{ij}(\underset{\sim}{x}) = \varepsilon_{ijkl} \overset{*}{\nabla}_k \hat{a}_l(\underset{\sim}{x} - \underset{\sim}{l}) \tag{15}$$

a decomposition which is gauge invariant under $\hat{a}_l(\underset{\sim}{x}) \to \hat{a}_l(\underset{\sim}{x}) + \nabla_l \hat{\Lambda}(\underset{\sim}{x})$ In this way we arrive at

$$Z = (2\pi\beta)^{-3N} \sum_{\{\hat{a}_l(\underset{\sim}{x})\}} \delta_{a_1,0}\, e^{-\frac{1}{2\beta}\sum_{\underset{\sim}{x}, i<j} \tilde{f}_{ij}^2} \tag{16}$$

where $\delta_{a_1,0}$ is some gauge fixing which is compatible with the integer valuedness of $\hat{a}_l$, N is the total number of lattice sites, and

$$\tilde{f}_{ij} \equiv \nabla_i \hat{a}_j - \nabla_j \hat{a}_i = \varepsilon_{ijkl} f_{kl} \tag{17}$$

is the dual field strength of f_{ij} .

Since we are not used to thinking in terms of integer fields it is advantageous to allow $\hat{a}(\underset{\sim}{x})$ to be continuous variables but enforce their integer values via one more application of Poisson's formula (12). Then Z_1 can be written as

$$Z_1 = (2\pi\beta)^{-N} \prod_{\underset{\sim}{x},i} \int_{-\infty}^{\infty} \frac{d\hat{A}_i(\underset{\sim}{x})}{\sqrt{2\pi\beta}} \, \delta(\tilde{A}_1) \, e^{-\frac{1}{2\beta}\sum_{\underset{\sim}{x},i<j} \hat{\tilde{F}}_{ij}^2} \tag{18}$$

$$\cdot \sum_{\{l_i(\underset{\sim}{x})\}} \delta_{\overset{*}{\nabla}_i l_i, 0} \; e^{2\pi i \sum_{\underset{\sim}{x},i} l_i \hat{A}_i}$$

The $\delta_{\overset{*}{\nabla}_i l_i, 0}$ restriction has to be inserted to account for the gauge invariance of $\hat{F}_{ij}$. This restriction implies that the sum over integer $l_i(\underset{\sim}{x})$ configurations

$$\sum_{\{l_i(\underset{\sim}{x})\}} \delta_{\overset{*}{\nabla}_i l_i, 0} \; e^{2\pi i \sum_{\underset{\sim}{x},i} l_i \hat{A}_i} \tag{19}$$

can be decomposed into the sum over closed loops in which each line carries the number $l_i(\underset{\sim}{x}) = 1$. It is this sum which represents the additional degree of freedom due to the line-like defects in the lattice gauge theory. The coupling (19) is the same as the minimal coupling of an electric current. Here, however, the field A_i is dual such that the currents are the world lines of magnetic monopoles with respect to the A_i fields.

If the A_i fields are integrated out, the partition function becomes[13,14]

$$Z_1 = (2\pi\beta)^{-3N/2} \sum_{\{l_i(\underset{\sim}{x})\}} \delta_{\overset{*}{\nabla}_i l_i, 0} \; e^{-\frac{\beta}{2} 4\pi^2 \sum_{\underset{\sim}{x},\underset{\sim}{x}'} l_i(\underset{\sim}{x}) \, v(\underset{\sim}{x}-\underset{\sim}{x}') \, l_i(\underset{\sim}{x}')} \tag{20}$$

where

$$v(\underset{\sim}{x}) \equiv \sum_{\underset{\sim}{k}} e^{i\underset{\sim}{k}\underset{\sim}{x}} \frac{1}{2\sum_{i=1}^{4}(1-\cos k_i a)} \tag{21}$$

$$\equiv \sum_{\underset{\sim}{k}} e^{i\underset{\sim}{k}\underset{\sim}{x}} \frac{1}{|K|^2}$$

is the lattice version of the Coulomb potential. Eq. (20) is recognized as the sum over Biot-Savart type of energies of the magnetic current loops. It is convenient to remove the large value at $\underset{\sim}{x}=o$ and rewrite

$$v(\underset{\sim}{x}) \equiv v'(\underset{\sim}{x}) + v(\underset{\sim}{o})\,\delta_{\underset{\sim}{x},\underset{\sim}{o}} \tag{22}$$

where $v(\underset{\sim}{o}) \simeq .155$ and

$$v'(\underset{\sim}{x}) \equiv \sum_{\underset{\sim}{k}} e^{i\underset{\sim}{k}\underset{\sim}{x}} \left(\frac{1}{|K|^2} - \frac{1}{N}\, v(\underset{\sim}{o})\right) \tag{23}$$

is a potential which vanishes at $\underset{\sim}{x}=o$. Then (20) becomes

$$Z_1 = (2\pi\beta)^{-3N/2} \sum_{\{l_i(\underset{\sim}{x})\}} \delta_{\bar{\nabla}_i l_i, 0} \tag{24}$$

$$\cdot\, e^{-\frac{\beta}{2} 4\pi^2 \sum_{\underset{\sim}{x},\underset{\sim}{x}'} l_i(\underset{\sim}{x})\, v'(\underset{\sim}{x}-\underset{\sim}{x}')\, l_i(\underset{\sim}{x}') - \frac{\beta}{2} 4\pi^2 v(\underset{\sim}{o}) \sum_{\underset{\sim}{x},i} l_i^2(\underset{\sim}{x})}$$

The first term can again be rewritten in the form (18) and becomes

$$Z_1 = (2\pi\beta)^{-N} \prod_{\underset{\sim}{x},i} \int_{-\infty}^{\infty} \frac{d\hat{A}_i(\underset{\sim}{x})}{\sqrt{2\pi\beta}}\, \delta(\overset{*}{\nabla}_i \hat{A}_i)\, e^{-\frac{1}{2\beta}\sum_{\underset{\sim}{x},i<j} F'^{2}_{ij}}$$

$$\cdot \sum_{\{l_i(\underset{\sim}{x})\}} \delta_{\overset{*}{\nabla}_i l_i,0}\, e^{-\frac{\beta}{2}4\pi^2 v(0)\sum_{\underset{\sim}{x},i} l_i^2 + 2\pi i \sum_{\underset{\sim}{x},i} l_i \tilde{A}_i} \tag{25}$$

The prime of the field strength denotes the fact that F_{ij}^2 in momentum space is multiplied by $\left(1-\frac{v(0)}{N}|k|^2\right)^{-1}$ corresponding to the replacement $v(\underset{\sim}{x}) \to v'(\underset{\sim}{x})$. This expression has the advantage that it displays the suppression of magnetic current loops of higher current strength due to the self-interaction. Since the vector potential $\hat{A}(\underset{\sim}{x})$ propagates with (23), the residual interactions have no self-energy due to

$$\left\langle \tilde{A}_i(\underset{\sim}{x})^2 \right\rangle = 0 \tag{26}$$

We are now ready to show that (25) can be rewritten as a Higgs like disorder field theory representing the grand-canonical ensemble in the original pure gauge theory!

III Higgs Disorder Fields

The derivation proceeds in two steps. First, we follow Peskin[14] and observe that the sum over closed l_i loops in (25) is equivalent to an XY model in an external vector potential A_i, by which we mean the following partition function[14]

$$Z_{1\,\text{loops}}^{\hat{A}_i} = \prod_{\underset{\sim}{x}} \int_{-\pi}^{\pi} \frac{d\gamma(\underset{\sim}{x})}{2\pi}\, e^{\frac{1}{t}\sum_{\underset{\sim}{x},i} \cos(\nabla_i \gamma - 2\pi \hat{A}_i)} \tag{27}$$

The auxiliary temperature t of this model is to be identified with the factor of the $\frac{1}{2}\sum_{\underset{\sim}{x},i} \ell_i^2$ term as follows

$$t = \beta\, 4\pi^2 v(0) \tag{28}$$

The proof is quite simple: Using Villain's approximation, (27) becomes

$$Z_{\text{loops}}^{\hat{A}_i} \sim \prod_{\underset{\sim}{x}} \int_{-\pi}^{\pi} \frac{d\gamma(\underset{\sim}{x})}{2\pi} \sum_{\{m_i(\underset{\sim}{x})\}} e^{-\frac{1}{2t}\sum_{\underset{\sim}{x},i}\left(\nabla_i\gamma - 2\pi\hat{A}_i - 2\pi m_i\right)^2} \tag{29}$$

Following the same arguments as after (10) we can rewrite this as

$$Z_{\text{loops}}^{\hat{A}_i} \sim \prod_{\underset{\sim}{x},i} \int_{-\infty}^{\infty} \frac{d\,L_i(\underset{\sim}{x})}{\sqrt{2\pi/t}} \prod_{\underset{\sim}{x}} \int_{-\pi}^{\pi} \frac{d\gamma(\underset{\sim}{x})}{2\pi} \sum_{\{m_i(\underset{\sim}{x})\}} \tag{30}$$

$$\exp\left\{-\frac{t}{2}\sum_{\underset{\sim}{x},i} L_i^2 - i\sum_{\underset{\sim}{x},i} L_i\left(\nabla_i\gamma - 2\pi\hat{A}_i - 2\pi m_i\right)\right\}$$

Summing over $m_i(\underset{\sim}{x})$ forces L_i to be integers ℓ_i, due to (12), and integrating out $\gamma(\underset{\sim}{x})$ we find $\delta_{\overset{*}{\nabla}_i \ell_i,0}$, just as in (13).
Thus we arrive at

$$Z_{\text{loops}}^{\hat{A}_i} \sim \left(\frac{2\pi}{t}\right)^{-3N/2} \sum_{\{\ell_i(\underset{\sim}{x})\}} \delta_{\overset{*}{\nabla}_i \ell_i,0}\, e^{-\frac{t}{2}\sum_{\underset{\sim}{x},i}\ell_i^2 + 2\pi i\sum_{\underset{\sim}{x},i}\ell_i\hat{A}_i} \tag{31}$$

which is indeed the loop sum in (25), after identifying (28). But there is no problem in turning (27) into a complex Higgs like field theory. For $\tilde{A}_i = o$ we can write

$$\sum_{\underset{\sim}{x}, i} \cos \nabla_i \gamma = \sum_{\underset{\sim}{x}, i} \cos \left(\gamma(\underset{\sim}{x}+\underset{\sim}{i}) - \gamma(\underset{\sim}{x}) \right)$$

$$= \sum_{x, i} \left[\cos \gamma(\underset{\sim}{x}+\underset{\sim}{i}) \cos \gamma(\underset{\sim}{x}) + \sin \gamma(\underset{\sim}{x}+\underset{\sim}{i}) \sin \gamma(\underset{\sim}{x}) \right] \tag{32}$$

$$= \sum_{\underset{\sim}{x}, i, a} S_a(\underset{\sim}{x}) \left(1 + \nabla_i \right) S_a(\underset{\sim}{x})$$

where

$$\underset{\sim}{S}(\underset{\sim}{x}) = \left(\cos \gamma(\underset{\sim}{x}), \sin \gamma(\underset{\sim}{x}) \right)$$

Moreover, using

$$\sum_{\underset{\sim}{x}, i} S_a(\underset{\sim}{x}) \nabla_i S_a(\underset{\sim}{x}) = \frac{1}{2} \sum_{\underset{\sim}{x}, i} S_a(\underset{\sim}{x}) \nabla_i S_a(\underset{\sim}{x}) - \overset{*}{\nabla}_i S_a(\underset{\sim}{x}) S_a(\underset{\sim}{x})$$

$$= \frac{1}{2} \sum_{\underset{\sim}{x}, i} S_a(\underset{\sim}{x}) (\nabla_i - \overset{*}{\nabla}_i) S_a(\underset{\sim}{x}) = \frac{1}{2} \sum_{\underset{\sim}{x}, i} S_a(\underset{\sim}{x}) \left(S_a(\underset{\sim}{x}+\underset{\sim}{i}) - S_a(\underset{\sim}{x}) + S_a(\underset{\sim}{x}-\underset{\sim}{i}) - S_a(\underset{\sim}{x}) \right) \tag{33}$$

$$= \frac{1}{2} \sum_{\underset{\sim}{x}, i} S_a(\underset{\sim}{x}) \overset{*}{\nabla}_i \nabla_i S_a(\underset{\sim}{x})$$

we can rewrite (27) as

$$Z_{1\,\text{loops}}^{\tilde{A}_i = 0} \sim \prod_{\underset{\sim}{x}} \int_{-\pi}^{\pi} \frac{d\gamma(\underset{\sim}{x})}{2\pi} e^{\frac{D}{T} \sum_{\underset{\sim}{x}, a} S_a(\underset{\sim}{x}) \left(1 + \sum_i \overset{*}{\nabla}_i \nabla_i / 2D \right) S_a(\underset{\sim}{x})} \tag{34}$$

where D=4 is the space dimensionality.

Now we introduce a two component real auxiliary field φ_a and rewrite

$$Z^{\hat{A}_i=0}_{\text{loops}} \sim \det\left(1+\sum_i \overset{*}{\nabla}_i\nabla_i/2D\right)^{-1} \prod_x \int_{-\infty}^{\infty} \frac{d\varphi_1 d\varphi_2}{4\pi D/t} \prod_x \int_{-\pi}^{\pi} \frac{d\gamma(\underset{\sim}{x})}{2\pi} \tag{35}$$

$$\exp\left\{-\frac{t}{4D}\sum_{\underset{\sim}{x},\underset{\sim}{x}',a} \varphi_a(\underset{\sim}{x})\left(1+\sum_i \overset{*}{\nabla}_i\nabla_i/2D\right)^{-1}(\underset{\sim}{x},\underset{\sim}{x}')\varphi_a(\underset{\sim}{x}') + \sum_{\underset{\sim}{x},a} S_a\varphi_a\right\}$$

The integration over $\gamma(\underset{\sim}{x})$ gives $I_0(\sqrt{|\varphi|^2})$. In order to remove the determinantal factor it is useful to go to fields

$$\psi_a(\underset{\sim}{x}) \equiv \sum_{\underset{\sim}{x}'} \left(1+\sum_i \overset{*}{\nabla}_i\nabla_i/2D\right)^{-\frac{1}{2}}(\underset{\sim}{x},\underset{\sim}{x}')\,\varphi(\underset{\sim}{x}') \tag{36}$$

such that

$$Z^{\hat{A}_i=0}_{\text{loops}} \sim \int \frac{d\psi\, d\psi^+}{4\pi D/t} \tag{37}$$

$$\cdot \exp\left\{-\frac{t}{4D}\sum_{\underset{\sim}{x}}|\psi(\underset{\sim}{x})|^2 + \sum_{\underset{\sim}{x}} \log I_0\left(\left|\left(1+\sum_i \overset{*}{\nabla}_i\nabla_i/2D\right)^{\frac{1}{2}}\psi\right|\right)\right\}$$

where we have gone to complex field notation. The field $\psi(\underset{\sim}{x})$ may be considered as the second quantized field version of the random loops represented in (31) (at $\tilde{A}_i$=o)(in the euclidean version).

Close to a phase transition, we can expand the exponent in powers of the field ψ and include only the lowest gradient terms

$$\sum_{\underset{\sim}{x}} \left\{ -\frac{t}{4D} |\psi|^2 + \frac{1}{4} |\psi|^2 - \frac{1}{8D} |\nabla_i \psi|^2 - \frac{1}{64} |\psi|^4 \right\} + \ldots \tag{38}$$

We can now include the coupling to the gauge field $\hat{\tilde{A}}_i$ by simply replacing the derivatives by their covariant versions

$$\begin{aligned} \nabla_i \psi(\underset{\sim}{x}) &\to D_i \psi(\underset{\sim}{x}) \equiv e^{-2\pi i A_i(\underset{\sim}{x})} \psi(\underset{\sim}{x}+\underset{\sim}{i}) - \psi(\underset{\sim}{x}) \\ \overset{*}{\nabla}_i \psi(\underset{\sim}{x}) &\to \overset{*}{D}_i \psi(\underset{\sim}{x}) \equiv \psi(\underset{\sim}{x}) - e^{2\pi i A_i(\underset{\sim}{x}-\underset{\sim}{i})} \psi(\underset{\sim}{x}-\underset{\sim}{i}) \end{aligned} \tag{39}$$

As a result we find for the original <u>pure electric</u> $U(1)$ lattice gauge theory a <u>magnetic</u> $U(1)$ lattice gauge theory <u>with Higgs fields.</u>

$$Z \sim \prod_{\underset{\sim}{x},i} \int_{-\infty}^{\infty} \frac{d\hat{A}_i}{\sqrt{2\pi\beta}} \prod_{\underset{\sim}{x}} \int \frac{d\psi \, d\psi^+}{4\pi D/t} \tag{40}$$

$$\exp\left\{ -\frac{1}{4\beta} \sum_{\underset{\sim}{x},i,j} \tilde{F}_{ij}'^2 - \sum_{\underset{\sim}{x}} \left(\frac{1}{8D} |D_i \psi|^2 + \frac{t-D}{4D} |\psi|^2 + \frac{1}{64} |\psi|^4 \right) \right\}$$

It is now straightforward to see that there is a phase transition at

$$t = D \tag{41}$$

For $t < D$, i.e. $T > T_c \equiv \frac{D}{4\pi^2 v(0)}$, which corresponds to strong coupling, the monopole disorder proliferates leading to a non-vanishing $\psi_0 \neq 0$. As a consequence of the minimal coupling, which in the long wavelength limit reduces to the form (1), the dual vector potential becomes massive and the color electric flux is screened, implying linear potentials and confinement. For $t > D$ i.e. $T < T_c$ (weak coupling) there are no monopoles and the forces are Coulomb like. It is gratifying to note that this temperature will not be shifted by seagull diagrams

$$\langle \tilde{A}_i^2 \rangle^{\psi^+\psi} = \tag{42}$$

since the propagator $\omega'(\underline{x})$ ensures the absence of closed photon loops (recall (26)).

Up to now, all operations have been performed on a D dimensional lattice whose spacing a was normalized to unity. In order to extract the consequences for a field theory on a four dimensional continuous space-time we have to reintroduce the parameter a at the appropriate places and take the limit $a \to 0$. Then lattice derivatives go over into proper derivatives as $\nabla_i \psi \to a \partial_i \psi$. In order to arrive at the conventional gradient term $\int d^4x |\partial_i \psi|^2$ we have to renormalize the field by a factor a i.e. we have to replace $\psi \to \sqrt{24}\, a \psi$ Then $\nabla_i \psi \to \sqrt{24}\, a^2 \partial_i \psi$ and the expansion (38) becomes in four dimensions

$$-\int d^4x \left\{ \frac{2}{a^2}(t-D)|\psi|^2 + |\underline{\partial}\psi|^2 + 9|\psi|^4 \right\} + \dots \tag{43}$$

If the fields are to have a finite mass, say M^2, we have to require that $t-D$ approaches zero with a as

$$t - D = \frac{a^2}{2} M^2 \to 0 \tag{44}$$

Thus the continuum limit lies automatically in the critical regime.

Physically, this situation may be described as follows: Critical fluctuations are characterized by a very large coherence length as compared with the lattice spacing. But then, in the limit of zero lattice spacing a, any finite wavelength M^{-1} is extremely long as compared with respect to a such that it is critical in the underlying infinitesimally fine lattice. We can convince ourselves that indeed all higher terms which are present in the exponent of (37) disappear in this continuum limit: Every higher power of ψ carries an extra factor a and so does every further gradient ∂_i.

Thus as announced in the introduction, starting from a pure gauge theory we have indeed arrived, by purely formal manipulation, at another dual gauge theory coupled to Higgs fields. The conclusions of this observation may be far reaching:

IV Conclusion

The discussion in this work was based on abelian gauge theory on the lattice. The question arises as to what we can learn from this as far as non-abelian theories are concerned. The basic lesson to be extracted from the many Monte Carlo computer calculations is that abelian and non-abelian theories are both characterized by the proliferation of monopoles above a certain value of the coupling. This is a consequence of global group excitations which are present in the compact abelian theory as well as the non-abelian theory. The difference between the two arises only in the weak coupling limit. While in the $U(1)$ theory for $D > 3$, there is a value β_c above which there are no more monopoles, and therefore no confinement, the non-abelian version has no phase transition and remains confining for arbitrarily large β. We therefore are led to conjecture that whatever we derive in the abelian theory for strong coupling should be valid in the non-abelian theory also for weak coupling. In fact, this is necessary if we want to use information from lattice calculations and extract from them statements about the continuum limit of the theory. Assuming that this can be done we arrive at the following exciting hypothesis[15] concerning the structure of strong and weak and electromagnetic interactions. The strong interactions are a pure gauge theory involving 8 colored gluons with a Lagrangian $\mathcal{L} = \frac{1}{4g_s^2} \sum_{i,j,a} F_{ij}^{a\,2}$. The weak interactions are also a pure gauge theory with 3 flavored vector mesons W_i : $\mathcal{L} = \frac{1}{4g_w^2} \sum_{i,j,\alpha} \tilde{F}_{ij}^{\alpha\,2}$. Contrary to the strong interactions, the action involves the dual tensor $\tilde{F}_{ij}^{\alpha}$ of the fields W_i . The coupling g_W is much larger than g_s , opposite to what one would naively think. Due to the weakness of g_s , there are very few color magnetic monopoles in the ground state which lead to color electric confinement but allow for a large energy range where perturbation theory is applicable.

The weak interactions, on the other hand, are characterized by a strong g_W . Therefore there exists a high "monopole" density of the dual theory which are flavor electric particles. They are most conveniently described by Higgs fields. These have a non-zero vacuum expectation such that the W_i fields are massive. Due to the largeness of g_W , the world lies deeply in the Higgs phase and perturbation theory with massless W's is impossible up to very high energies. This is observed as the short range property of weak interactions. At the same time, the flavor electric interaction between Higgs particles is very weak. This picture is quite different from current hypotheses of unifying weak, electromagnetic, and strong interactions via a single, multicomponent non-abelian gauge theory. In our case the unified theory has the generic form

$$\mathcal{L} = \frac{1}{4g_s^2} \sum_{i,j,a} F_{ij}^{a\,2} + \frac{1}{4g_w^2} \sum_{i,j,\alpha} \tilde{F}_{ij}^{\alpha\,2}$$

with small g_s and large g_W. The quarks are electric sources for A_i^a and magnetic ones for $\tilde{W}_i^\alpha$. Certainly, many details remain to be investigated.

It goes without saying that the field theory on the lattice with the partition function (11) can also form the basis for studying real magnetic monopoles, if the single event discovered recently by Cabrera finds successors. In this case the field A_i is the usual electromagnetic one which is coupled immediately to electrons. The dual version (40) can be seen as a possible generalization of quantum electrodynamics which naturally gives rise to Dirac monopoles.

Let us recall that Dirac[17] derived his quantization condition by postulating the invisibility of the backflow of magnetic field lines with respect to electrons. In the present formulation, this is automatically ensured if the electron is coupled minimally via the covariant derivative $D_i \Psi(\mathbf{x}) = e^{-2\pi i A_i} \Psi(\mathbf{x}+\mathbf{i}) - \Psi(\mathbf{x})$, since this derivative is indifferent to jumps of A_i by 2π. There was, however, one problem which Dirac was not able to solve within continuum electrodynamics: Even though he had succeeded in making the string invisible to electrons, it still carried an infinite density. In order to circumvent this problem, many tricky procedures were invented and you have heard about them in Prof. Yang's lecture. It is gratifying to note that the present modification (7) of the electromagnetic action completely avoids these energetic problems without involving additional mathematical structures. The scalar fields, on the other hand, describe spin zero particles which couple locally to the dual electromagnetic potential $\tilde{A}_i$. Thus, with respect to the original potential A_i, they correspond to magnetic monopoles. Their magnetic charge is $4\pi\sqrt{\beta} = \frac{4\pi}{g}$ as we can read off equ. (40). This is precisely Dirac's value for magnetic monopoles which has magnetic charge= 4π /electric charge (in our units). Thus the partition function (7), (10) has a chance of being the correct extension of quantum electrodynamics into the short-distance regime and the monopoles would be scalar particles with well-defined complicated repulsive self-interactions.

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D I S C U S S I O N

CHAIRMAN: H. KLEINERT

Scientific Secretaries: T. Hofsäss and P. van Baal

DISCUSSION

- *GOCKSCH:*

How is your action related to the standard Wilson action? In particular, can one explain the appearance of the $n_{ij}(x)$ through this action?

- *KLEINERT:*

I did not have time to explain this during the lecture, but you will find it in the lecture notes, paragraph II up to equation (9).

- *KREMER:*

I am afraid that the monopoles in your theory might form dipoles or some condensate. How can you be sure that the monopoles exist individually?

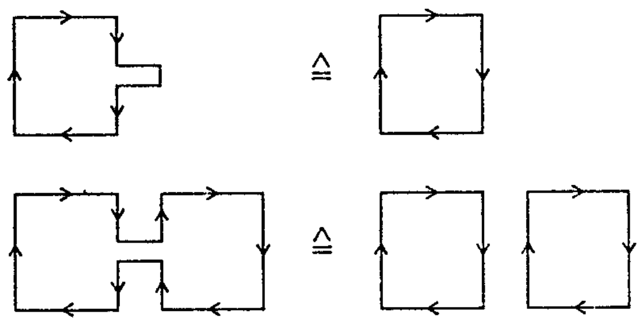
- *KLEINERT:*

Indeed the monopoles do form a condensate if the coupling is so strong such that the system is inside the Higgs phase. In the weak coupling phase, monopoles and antimonopoles can form dipoles through the electromagnetic interaction. These dipoles can be calculated from my final action. All forces are known: the long range ones from the electromagnetic coupling $g = 2\pi/e$, the short range ones from the interaction terms in the Higgs field. Notice that these terms are unknown in Prof. Yang's theory of monopoles while here they arise from the fact that the sum:

$$\sum_{\ell_i(x)\in Z} \delta_{\nabla_i \ell_i(x),0}$$

(cf. equation (25)) does not involve all random loops but that backtracking is forbidden.

Examples:



The loops on the left-hand side are absent, since they would double-count the equivalent loops on the right-hand side.

These restrictions are automatically ensured by the short range self-interactions of the ψ field shown in equation (37).

- *SCHÄFER:*

What will happen to your construction of the Higgs field if you make the same analysis for a non-Abelian theory?

- *KLEINERT:*

In detail, I cannot say. But it appears as if Abelian and non-Abelian theories are quite similar in the strong-coupling phase. The principal difference between the two seems to lie in the fact that in four dimensions the physics in the Abelian theory changes abruptly at a critical coupling, where the spaghetti of monopoles disappears, while in non-Abelian theories the strong-coupling physics can be extrapolated without discontinuity into the weak coupling regime (with only a roughening transition wiping out the memory of the lattice). By transitivity, we might expect that the strong-coupling Abelian theory can be used to learn something about the weak-coupling non-Abelian theory.

- *LYKKEN:*

Can you determine the parameters of the Higgs potential in your scheme?

- *KLEINERT:*

Yes, because the coefficients of the $|\psi|^{2n}$ expansion are completely determined by the theory, compare equation (37).

- *VAN DER BIJ:*

Can you tell us what the masses of the Higgs and the gauge particles are?

- *KLEINERT:*

Yes, from equation (40) you see that the Higgs mass is given by ($t<D$):

$$m_H^2 = 4D(D-t)$$

and the mass of the neutral vector meson is:

$$m_Z^2 = \frac{(2\pi)^2\beta}{4D}\,|\langle\psi\rangle|^2$$

inserting $|\langle\psi\rangle|^2 = 8(D-t)$ and $\beta = e^2/(2\pi)^2$ we find:

$$m_H/m_Z = \sqrt{2}\,D/e \simeq 20$$

which brings m_H into the TeV range. Before we compare with experiment, however, we should not forget this is not the full Georgi-Weinberg-Salam theory but just a model.

- *TUROK:*

But how do we obtain in this lattice theory the weak interaction scale of approximately 100 GeV?

- *KLEINERT:*

In every renormalizable theory you have an observable mass, which is given by:

$$m = a^{-1}f(g) \quad (*)$$

It is characteristic of renormalizability that m (and any other observable parameter) is invariant under a change of the scale of a as long as the coupling is appropriately adjusted as a function of a. Thus the relation (*) does not really determine the mass, but everytime you choose a new a you have to adjust g such that m is the mass you want it to be. After all, the mass is the proper dimensionally transmuted parameter to characterize the theory (which replaces the pair (a,g(a)) to be used otherwise).

- *MUKHI:*

Can you distinguish your Higgs particle from those of other theories?

- *KLEINERT:*

By more detailed dynamical consequences of the theory. Since all parameters are fixed, these are now calculable. For example, my Higgs particles will never split into smaller fermionic sub-structures (such as technicolor quarks).

- *DOBREV:*

You were stressing that you do not need to have sections. In the continuum they are needed to have a mathematically well defined description of monopoles. Of course, on the lattice you do not need sections, for topological reasons.

- *KLEINERT:*

It is not the topology which makes the strings invisible in my lattice formulation but the specific form of the energy. Had I added a term $-n_{ij}^2$ in the exponent, strings would have shown up.

My reservation against the section way of avoiding a string energy derives from the impossibility of making a quantum field theory in which a quantum field $\psi(x)$ is supposed to know which is the upper and which the lower section of the many-monopole states it can describe. Maybe Prof. Yang can explain that.

- *YANG:*

How to go from a finite number of particles to a field theory is a similar problem as in usual QED, and there is no difficulty. The difficulty that there is no Feynman rule derives from the fact that there is no interaction representation.

- *KLEINERT:*

Exactly, and this in turn is due to the sections.

- *CASTELLANI:*

Are there other ways to generate Higgs fields without using the lattice formulation?

- *KLEINERT:*

Most non-linear systems manufacture their own defects which have finite energy and therefore can appear in the laboratory (solitons, vortex lines in superfluid ^{4}He, and disclinations in liquid crystals). All these can in principle be represented by a Higgs field through a lattice formulation, and you can find this in recent Phys. Lett.A and Lettere Nuovo Cimento of mine.

What I presented can be seen as an idealization of this type of non-linear systems, in which smooth defects are sharpened to be local, which can be described by a local field theory, and in which the residual interactions among the defects are completely linearized. In this idealization, the lattice is convenient in preventing energetic infinities to appear. Also, it simplifies the construction of the Higgs field.

- *CASTELLANI:*

I was thinking more of recent appearances of Higgs fields in dimensional reduction schemes.

- *KLEINERT:*

How those Higgs fields are related to my defect Higgs fields, I do not know, but would be interested in finding out.

- *HOFSÄSS:*

I want to point out, that if one uses your procedure to generate the Higgs field which is needed to unify electromagnetic and weak interactions, one has a very restrictive scheme: as all the parameters for the Higgs field are fixed, the scheme could predict the mass of the electron.

- *PERNICI:*

Do you have an analogue of the Dirac quantization condition?

- *KLEINERT:*

Yes. My monopole satisfies eg = 2π which, by the way is the same as Dirac's 2eg = 1, since he used the convention:
$\mathcal{L} = \frac{1}{16\pi}F_{\mu\nu}^2$ instead of $\frac{1}{4}F_{\mu\nu}^2$

- *TUROK:*

What is the spin of your monopole? Don't you need spin 1/2 for the full electromagnetic symmetry?

- *KLEINERT:*

My monopole's spin is zero.

- *ZICHICHI:*

Spin zero is just garbage.

- *KAPLUNOVSKY:*

Why for heaven's sake should it have spin 1/2?

- *KLEINERT:*

Don't you know. If Prof. Zichichi wants it to have spin 1/2 it must have spin 1/2?