THERMAL SOFTENING OF CURVATURE ELASTICITY IN MEMBRANES

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Recently, there have been three determinations of the reduction of curvature elasticity due to thermal fluctuations. The results obtained by Helfrich, Peliti and Leibler, and Förster are $\kappa_R = \kappa - (T/4\pi)I \log(q_{max}/q_{min})$, with $I = 1$, 3, and 2 respectively. We discuss the differences between the calculations and show that, despite a careless handling of the path integration measure, Peliti and Leibler's result is correct. Förster's paper, on the other hand, has the correct measure but the wrong algebra. As a further result, the gaussian curvature constant is shown to change with temperature as $\bar\kappa_R = \bar\kappa + (T/4\pi) \times 4 \log(q_{max}/q_{min})$.

In order to compare different membrane experiments, Helfrich [1] estimated the softening of the curvature elastic constant due to thermal fluctuations to be

$$\kappa_R = \kappa - (T/4\pi) \log(q_{max}/q_{min}). \tag{1}$$

By looking at the same problem in two different ways, Peliti and Leibler [2], and Förster [3] found an enhancement by a factor three and two, respectively. Naturally, the question arises as to which of these three is correct. All three authors consider the curvature energy

$$E = \tfrac{1}{2}\kappa \int d^2\xi \sqrt{g}\,(c_1 + c_2)^2 + \bar\kappa \int d^2\xi \sqrt{g}\, c_1 c_2, \tag{2}$$

where $c_1 + c_2$ is the mean, $c_1 c_2$ the gaussian curvature, and g is the determinant of the metric of the surface. If $x^a(\xi^i)$ with $a = 1,\dots, 3; i = 1, 2$ is the parametrization of the surface, the metric is

$$g_{ij} = \partial_i x^a \partial_j x^a = D_i x^a D_j x^a. \tag{3}$$

The curvatures are obtained from the second fundamental form

$$D_i D_j x^a \equiv C_{ij} n^a \tag{4}$$

(where $D_i v_j = \partial_i v_j - \Gamma_{ij}^k v_k$ is the covariant derivative and n^a the unit normal vector of the surface) as follows[‡1]

$$c_1 + c_2 = g^{ij} C_{ij} \equiv C_i{}^i \equiv C, \quad c_1 c_2 = \det C_i{}^j = \tfrac{1}{2}(C^2 - C_i{}^j C_j{}^i) \equiv K \equiv \tfrac{1}{2}R. \tag{5}$$

Hence (2) can also be written as

$$E \equiv E_M + E_G = \tfrac{1}{2}\kappa \int d^2\xi \sqrt{g}\, D^2 x^a D^2 x^a + \tfrac{1}{2}\bar\kappa \int d^2\xi \sqrt{g}\,(D^2 x^a D^2 x^a - D_i D^j x^a D_j D^i x^a), \tag{6}$$

which shows that the second term is a pure surface energy, proportional to the Euler characteristic $2 - 2h$ (h = number of handles) of the surface. Helfrich (H) and Peliti and Leibler (PL) go to the special parametrization in which ξ^1, ξ^2 are a euclidean base space and the surface is given by a vertical displacement field, i.e.,

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[‡1] The Ricci tensor is $R_i{}^j = CC_i{}^j - C_i{}^l C_l{}^j$ and $R \equiv R_i{}^i$ is the scalar curvature.

$$x^a(\xi) = \begin{pmatrix} \xi^1 \\ \xi^2 \\ u(\xi^1\xi^2) \end{pmatrix}. \tag{7}$$

Then

$$\partial_i x^a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ u_1 & u_2 \end{pmatrix}, \quad n^a = (1+u_l^2)^{-1/2} \begin{pmatrix} -u_1 \\ -u_2 \\ 1 \end{pmatrix}, \quad g_{ij} = \delta_{ij} + u_i u_j, \quad g^{ij} = \delta^{ij} - u_i u_j/(1+u_l^2), \quad g = 1 + u_l^2, \tag{8}$$

where $u_i \equiv \partial_i u$, and

$$C_{ij} = u_{ij}/(1+u_l^2)^{1/2}, \quad C_i{}^j = -\partial_i n^j, \quad C = -\partial_i n^i = -\partial_i\,[u_i/(1+u_l^2)^{1/2}], \tag{9}$$

and hence

$$E_{\rm M} = \tfrac{1}{2}\kappa \int d^2\xi\, (1+u_l^2)^{1/2}\, \{\partial_i\,[u_i/(1+u_l^2)^{1/2}]\}^2. \tag{10}$$

Expanding this up to the quartic power in u

$$E_{\rm M} = \tfrac{1}{2}\kappa \int d^2\xi\, (u_{ii}^2 - \tfrac{1}{2}u_{ii}^2 u_l^2 - 2u_{ii}u_k u_l u_{kl} + \ldots) \tag{11}$$

and inserting $u = U + \epsilon$ it is easy to calculate the quartic fluctuation energy up to second order in U as follows

$$E_{\rm M} = \tfrac{1}{2}\kappa \int d^2\xi\, [(U_{ii}^2 + 2U_{ii}\epsilon_{jj} + \epsilon_{ii}^2) - \tfrac{1}{2}(U_{ii}^2\epsilon_l^2 + U_l^2\epsilon_{ii}^2 + 4U_{ii}U_l\epsilon_{jj}\epsilon_l)$$
$$- 2(U_{ii}U_k\epsilon_l\epsilon_{kl} + U_{ii}U_l\epsilon_k\epsilon_{kl} + U_{ii}U_{kl}\epsilon_k\epsilon_l + U_kU_l\epsilon_{ii}\epsilon_{kl} + U_kU_{kl}\epsilon_{ii}\epsilon_{kl} + U_lU_{kl}\epsilon_{ii}\epsilon_k) + \ldots]. \tag{12}$$

The authors H and PL differ in their treatment of the fluctuations. PL assumes a measure for the path integration $\int \mathcal{D}u(\xi)$ which leads, in lowest order, to the equipartition theorem

$$\langle \epsilon(\boldsymbol{x})\,\epsilon(\boldsymbol{0})\rangle = \frac{T}{\kappa}\int \frac{d^2k}{(2\pi)^D} \exp(i\boldsymbol{k}\cdot\boldsymbol{x})\,k^{-4},$$

and thus, in our notation, to a renormalized energy

$$E_{\rm M}^{\rm R} = \tfrac{1}{2}\int d^2\xi\, [\kappa U_{ii}^2 - \tfrac{1}{2}T(U_{ii}^2 L + U_l^2 Q) - 2T(U_{ii}U_{kl}L_{kl} + U_kU_lQ_{kl})], \tag{13}$$

where

$$L = \int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2} = \frac{1}{2\pi}\log(q_{\rm max}/q_{\rm min}), \quad L_{kl} = \int \frac{d^2k}{(2\pi)^2}\frac{k_kk_l}{k^4} = \tfrac{1}{2}\delta_{kl}L,$$

$$Q = \int \frac{d^2k}{(2\pi)^2}, \quad Q_{kl} = \int \frac{d^2k}{(2\pi)^2}\frac{k_kk_l}{k^2} = \tfrac{1}{2}\delta_{kl}Q$$

are one-loop integrals with logarithmic and quadratic divergences. Hence κ is renormalized to

$$\kappa_{\rm R} = \kappa - T \times \tfrac{3}{2}L = \kappa - (T/4\pi)\times 3\log(q_{\rm max}/q_{\rm min}). \tag{14}$$

Apart from this, there is a thermal generation of *negative* surface energy

$$E_{\rm S}^{\rm R} = -\tfrac{3}{4}TQ\int d^2\xi\, U_l^2 = -\tfrac{3}{2}TQ\int d^2\xi\,(1+U_l^2)^{1/2} + \tfrac{3}{2}TQ, \tag{15}$$

which PL do not mention.

Helfrich also takes the equipartition theorem $\langle\epsilon_l^2\rangle = (T/\kappa)\,L$, but neglects the contribution from the last parenthesis in (12). So, the origin of the discrepancy between his result and PL's is clear.

Let us now look at Förster's calculation. He uses a covariant approach and remains within the general $x^a(\xi)$ description of the surface. Setting $x^a \to x^a + \delta x^a$ and expanding in δx^a gives $D^2 \to D^2 - \delta g^{ij} D_i D_j - \delta \Gamma_i{}^{ik} D_k + \delta g^{ik} \delta g_k{}^j D_i D_j$ and thus, due to $D_i D_j x^a \cdot D_k x^a = 0$, the energy

$$E_M = \tfrac{1}{2}\kappa \int d^2\xi \sqrt{g}\, \{(D^2x^a)^2 + \tfrac{1}{2}\delta g^k{}_k (D^2x^a)^2 + 2D^2x^a(D^2x^a - \delta g^{ij} D_i D_j x^a) + [-\tfrac{1}{4}\delta g^{ij}\delta g_{ij} + \tfrac{1}{8}(\delta g^i{}_i)^2](D^2x^a)^2$$
$$+ \delta g^k{}_k D^2x^a(D^2\delta x^a - \delta g^{ij} D_i D_j x^a) + (D^2\delta x^a)^2 + (\delta g^{ij} D_i D_j x^a)^2 + (\delta\Gamma_i{}^{ik} D_k x^a)^2$$
$$- 2(D^2\delta x^a D_k x^a + D^2x^a D_k \delta x^a)\delta\Gamma_i{}^{ik} - 2(D^2\delta x^a \delta g^{ij} D_i D_j x^a + D^2 x^a \delta g^{ij} D_i D_j \delta x^a)$$
$$+ 2\delta g^{ik}\delta g_k{}^i D_i D_j x^a D^2 x^a + \ldots\}. \qquad (16)$$

The third term from the end can be combined to $-(\delta\Gamma_i{}^{ik})^2$ (since $\delta\Gamma_i{}^{ik} = D^2\delta x^a D_k x^a + D^2 x^a D_k \delta x^a + \ldots$).

In order to isolate the physical contribution to the fluctuations he notes correctly that only the part of δx^a which points along the *normal* vector is physical. The transverse part corresponds merely to a reparametrization of the surface. Thus he sets

$$\delta x^a = \nu n^a. \qquad (17)$$

He further notes that as long as one wants to calculate the influence of the short-wavelength fluctuations upon the long-wavelength background field, it is sufficient to consider only terms of the form $\nu D^2 D^2 \nu$, $\nu D_i D_j \nu$. Then he drops all linear terms in $\delta g^i{}_i$ and remains with

$$\delta^2 E_M = \tfrac{1}{2}\kappa \int d^2\xi \sqrt{g}\, [(D^2\delta x^a)^2 - (\delta\Gamma_i{}^{ik})^2 - 2(D^2\delta x^a \delta g^{ij} D_i D_j x^a + D^2 x^a \delta g^{ij} D_i D_j \delta x^a)$$
$$+ \delta g^k{}_k D^2 x^a D^2 \delta x^a + \ldots]. \qquad (18)$$

Using $\delta g_{ij} = D_i \delta x^a D_j x^a + (ij) \approx -2\nu C_{ij}$ and $n^a D_i D_j n^a = -C_i{}^l C_{lj}$, he arrives at [‡2]

$$\delta^2 E_M = \tfrac{1}{2}\kappa \int d^2\xi \sqrt{g}\, \{\nu[(D^2D^2 - 2C_i{}^j C_j{}^i D^2 - 4C^i{}_l C^{lj} D_i D_j) - (-C^2D^2 + 4(C C^{ij} - C^i{}_l C^{lj}) D_i D_j)$$
$$+ 4(C_i{}^j C_j{}^i D^2 + C C^{ij} D_i D_j) - 2C^2 D^2]\,\nu + \ldots\}$$
$$= \tfrac{1}{2}\kappa \int d^2\xi \sqrt{g}\, \{\nu[D^2D^2 - C^2D^2 + 2C_i{}^j C_j{}^i D^2]\,\nu + \ldots\}, \qquad (19)$$

which after integrating out ν and inserting $C_i{}^j C_j{}^i = C^2 - 2K$ amounts to the additional curvature energy

$$E_M^R - E_M = \tfrac{1}{2} T\, \mathrm{Tr}\log[D^2D^2 + (C^2 - 4K)D^2 \ldots] = T\,\mathrm{Tr}\log D^2 + \tfrac{1}{2} T \int d^2\xi \sqrt{g}\, \mathrm{Tr}[D^{-2}(C^2 - 4K) + \ldots]. \qquad (20)$$

If only short wavelength fluctuations are integrated and the background field is sufficiently smooth, this amounts to

$$\kappa^R = \kappa - (T/4\pi) \times 2\log(q_{max}/q_{min}), \qquad (21)$$

$$\bar\kappa^R = \bar\kappa + (T/4\pi) \times 4\log(q_{max}/q_{min}). \qquad (22)$$

The first equation gives 2/3 PL's result. It is easy to find the reason for this discrepancy. The linear terms δg_{ij} in (16) are sensitive to the quadratic variations in ν and it is necessary to keep the *full*

$$\delta g_{ij} = D_i \delta x^a D_j x^a + (ij) + D_i \delta x^a D_j \delta x^a = -2\nu C_{ij} + D_i \nu D_j \nu + \nu^2 C_{il} C_j{}^l. \qquad (23)$$

This gives a further term

$$\nu[-\tfrac{1}{2} C^2 D^2 + 2C C^{ij} D_i D_j]\,\nu \qquad (24)$$

[‡2] We have listed each term in (18) separately to facilitate checking the calculation.

in the curly brackets of (19) and changes eq. (20) to

$$T\,\mathrm{Tr}\log D^2 + \tfrac{1}{2}T\int d^2\xi\sqrt{g}\,\mathrm{Tr}[D^{-2}(\tfrac{1}{2}C^2 - 4K) + 2D^{-4}CC^{ij}D_iD_j]. \tag{25}$$

When doing the momentum integrals, the new term raises the prefactor 2 in eq. (21) to 3.

Notice that (25) has precisely the same tensor structure in C_{ij} as (13) has in U_{ij} [as it should, since $C_{ij} = 1/(1+u_l^2)^{1/2}\,U_{ij}$].

Even though the correct result agrees with PL it must be realized that this is somewhat accidental. The measure of integration should be

$$\int \mathcal{D}\nu\, g^{1/4} \equiv \prod_{\xi}\int d\nu(\xi)\,[g(\xi)]^{1/4}. \tag{26}$$

This can also be expressed in a parametrization independent manner as

$$\prod_{a,\xi}\left(\int dx^a\, g^{1/4}\right)\exp\left(-\frac{1}{2\epsilon}\int d^2\xi\sqrt{g}\,g^{ij}\delta x^a D_i x^a \delta x^k D_j x^k\right). \tag{27}$$

It is useful to find out why the error in using the measure $\int\mathcal{D}u$ happens to cancel. For this we notice that the normal variation

$$x^a + \delta x^a = x^a + \nu n^a = \begin{pmatrix} \xi^1 - \nu(1+U_l^2)^{-1/2}U_1 \\ \xi^2 - \nu(1+U_l^2)^{-1/2}U_2 \\ U(\xi) + \nu(1+U_l^2)^{-1/2} \end{pmatrix} \approx \begin{pmatrix} \xi^1 - \nu U_1 \\ \xi^2 - \nu U_2 \\ u(\xi) + \nu - \tfrac{1}{2}\nu U_l^2 \end{pmatrix} \tag{28}$$

has the parametrization (6) only after going to the new coordinates

$$\xi'^i = \xi^i - \nu(\xi)[1 + U_l^2(\xi)]^{-1/2}\,U_i(\xi) \approx \xi^i - \nu(\xi)\,U_i(\xi). \tag{29}$$

Hence the energy associated with (28) has the form (10) only in these new coordinates

$$E'_{\mathrm{M}} = \tfrac{1}{2}\kappa\int d^2\xi'(1+u_{l'}'^2)^{1/2}(\xi')\,\{\partial'_{i'}[u_{i'}(1+u_{l'}'^2)^{-1/2}]\}^2, \tag{30}$$

where the primed subscripts denote derivatives with respect to ξ'^i and $u'(\xi') = U(\xi) + \nu(\xi)/(1+U_k^2)^{1/2}$. With (29) we have

$$\partial\xi'^i/\partial\xi^j \approx \delta^{ij} - \nu_j U_i - \nu U_{ij}, \qquad d^2\xi' \approx d^2\xi(1 - \nu_i U_i - \nu U_{ii}), \qquad \partial_{l'} \approx \partial_l + \nu_l U_k\partial_k + \nu U_{lk}\partial_k, \tag{31}$$

such that, with respect to the unprimed energy E_{M}, there is the change (12), with ϵ replaced by ν, plus the following additional contributions, all from $\int d^2\xi'\,u_{i'j'}'^2$:

$$\int d^2\xi\,(1-\nu_iU_i-\nu U_{ij})\,(2U_{ii}\nu_{jj} + 2U_{ii}U_k\nu_{ij}\nu_k + 4U_{ii}U_k\nu_i\nu_{kj} + 4U_{ii}U_{kl}\nu_k\nu_l + 4U_{ii}U_{kl}\nu\nu_{kl} + 2U_{ii}U_{jjk}\nu\nu_k$$
$$+ U_l^2\nu_{ii}\nu_{jj} + 4U_kU_{kl}\nu_k\nu_{ii} + 2U_{kl}U_{kl}\nu\nu_{ii}) + \ldots\,. \tag{32}$$

Performing the contractions $\langle\nu\nu\rangle$ we find the additional energy

$$TL(U_{ii}U_{kk} - U_{kl}U_{kl}),$$

which, in this approximation, amounts to $TL \times 2K$ and therefore renders the correct softening of the gaussian curvature, eq. (22). Besides, there is one more contribution to the surface energy

$$Q\int d^2\xi\,U_l^2, \tag{33}$$

which changes the previously obtained value (15) into $-\tfrac{1}{2}TQ\int d^2\xi\,(1+U_l^2)^{1/2} + \tfrac{1}{2}TQ$. A further term of this type comes from the measure $\int \mathcal{D}v\, g^{1/4} = \int \mathcal{D}v\, (1+U_l^2)^{1/4}$. In the exponent this gives $\frac{1}{4} Q \int d^2\xi \log(1+U_l^2)$, thus bringing the surface energy to

$$E_s = -TQ \int d^2\xi\, (1+U_l^2)^{1/2} + TQ. \tag{34}$$

In the covariant formulation, the corresponding energy is contained in the first term of (20) which was due to the fluctuation energy $\frac{1}{2}\kappa \int d^2\xi \sqrt{g}\,(D^2 v)^2$ and reads, in the present parametrization,

$$\tfrac{1}{2}\kappa \int d^2\xi \sqrt{g}\,(D^2v)^2 \approx \tfrac{1}{2}\kappa \int d^2\xi\, (1+U_l^2)^{1/2} (v_{ii} - U_{ii}U_k v_k - U_i U_j v_{ij} + \ldots)^2$$

$$\approx \tfrac{1}{2}\kappa \int d^2\xi\, (v_{ii}^2 + \tfrac{1}{2} U_l^2 v_{ii}^2 - 2U_iU_j v_{ij} v_{ll} + \ldots)$$

and is seen to generate, via the measure

$$\int \mathcal{D}v\, g^{1/4} = \int \mathcal{D}v\, \exp\!\left(\tfrac{1}{4} Q \int d^2\xi \log(1+U_l^2)\right),$$

exactly the same surface energy (34).

It was noticed by Polyakov [5] in another context that it is possible to calculate the fluctuation energy

$$E_F = T\,\mathrm{Tr}\log D^2 \tag{35}$$

exactly by rewriting it as [6]

$$E_F = -T \int d^2\xi \int_0^\infty \frac{dt}{t} \langle \xi | e^{tD^2} | \xi \rangle \tag{36}$$

and going to an orthogonal coordinate frame in which $g_{ij}(\xi) = \rho(\xi)\,\delta_{ij}$ and the covariant derivative $D^2 = (1/\sqrt{g}) \times (\partial_i \sqrt{g}\, g^{ij} \partial_j)$ becomes simply $\rho^{-1}\partial_i^2$. Then, under changes $\delta\rho(\xi)$,

$$\delta E_F = T \int d^2\xi \frac{\delta\rho}{\rho} \int_0^\infty dt\, \langle \xi | D^2 e^{tD^2} | \xi \rangle = T \int d^2\xi \frac{\delta\rho}{\rho} \int_0^\infty dt\, \langle \xi | \frac{d}{dt} e^{tD^2} | \xi \rangle = -T \int d^2\xi \frac{\delta\rho}{\rho} \langle \xi | e^{(\epsilon\partial^2)/\rho} | \xi \rangle \tag{37}$$

and using the formula in D dimensions [7]

$$(\sqrt{g})^{-D/2} \langle \xi | e^{\epsilon D^2} | \xi \rangle = (4\pi\epsilon)^{-D/2} (1 + \epsilon R/6 + \ldots). \tag{38}$$

Polyakov found

$$\delta E_F = -T \int d\xi\, \delta\rho (1/4\pi\epsilon + R/24\pi + \ldots), \tag{39}$$

where the curvature in orthogonal coordinates is simpy $R = -\rho^{-1}\partial_i^2 \log \rho$. The divergent term $1/4\pi\epsilon$ is equal to $\langle \xi | \xi \rangle = \delta^{(2)}(0) = \int d^2k/(2\pi)^2$ such that we can replace it by Q. Thus, integrating (39) in ρ gives

$$E_F = -T \int d^2\xi\, [Q\rho + (1/48\pi)(\partial_i \log\rho)^2] + T \times \text{const.} \tag{40}$$

The first term agrees with (34) (since $-TQ \int d^2\xi\, \rho = -TQ \int d^2\xi \sqrt{g}$ corresponds to $-TQ \int d^2\xi (1+U_l^2)^{1/2}$) and the second term gives a further *finite* reduction in membrane energy which is missed in the naive approach via (12).

Notice that in the field representation $\rho = e^{\phi}$, the energy

$$E_c = \int d^2\xi\, [Qe^{\phi} + (1/48\pi)(\partial_i\phi)^2] \tag{41}$$

is the field energy whose partition function would give the Mayer expansion of a Coulomb gas. The overall sign in (40), however, is negative.

For completeness, let us include here also the simplest application [8] of the corrected version of (21) together with the new result (22) [9]. For this we perform the $\mathrm{Tr}(1/D^2)$ for spherical membranes by replacing the momentum sum by a sum over angular momenta

$$L = \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2} = \frac{1}{2\pi} \log(q_{max}/q_{min}) \to \frac{1}{4\pi} \sum_l (2l+1) \frac{1}{l(l+1)} \sim \frac{1}{2\pi} \log l_{max} \sim \frac{1}{4\pi} \log M, \qquad (42)$$

where

$$M = \sum_{l=0}^{l_{max}} (2l+1) \sim l_{max}^2$$

is the total number of modes, which is proportional to the number N of molecules in the membrane. As a consequence, the energy of spherical membranes with no spontaneous curvature has a temperature dependence

$$E = 8\pi\kappa\,[1 - (T/\kappa) \times \tfrac{3}{2} L] - 4\pi\bar{\kappa}\,[1 - (T/\kappa) \times 2L] = 8\pi\kappa - 4\pi\bar{\kappa} - 4\pi TL = E_0 - T \log N. \qquad (43)$$

Thus the size distribution

$$P(N) \propto e^{-(E-\mu N)/T} \propto N\, e^{\mu N/T} \qquad (44)$$

happens to be the same as obtained recently by Helfrich [8] on the basis of an approximate treatment of spherical membranes, similar to his first. There is an accidental cancellation of the renormalization of the gaussian curvature, which he ignores, with the factor 3 in the softening of κ.

It is gratifying to note that such distributions (of the Shultz type $\propto N^a e^{-(\mathrm{const} \times N)}$ have indeed been used to fit data from quasi-elastic light scattering [10,11]. For a proper derivation of such distributions, which must include the spontaneous curvature, see ref. [9].

Let us finally mention that it was not really necessary to keep track of all surface energy terms since membrane fluctuations take place at constant area [12] which, as we have learned from the work of Brochard et al. [13] make surface energies disappear.

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References

[1] W. Helfrich, J. Phys. (Paris) 46 (1985) 1263.
[2] L. Peliti and S. Leibler, Phys. Rev. Lett. 54 (1985) 1690.
[3] D. Förster, Berlin preprint (August 1985).
[4] W. Helfrich, Z. Naturforsch. 30c (1975) 84.
[5] A.M. Polyakov, Phys. Lett. 103B (1981) 207.
[6] E. Onofri, M.A. Virasoro, Nucl. Phys. B201 (1982) 159;
E. Onofri, Comm. Math. Phys. 86 (1982) 321;
D. Friedan, Les Houches Lectures (1982);
O. Alvarez, Nucl. Phys. B216 (1983) 125
[7] N.V. Romanov and A.S. Schwarz, Teor. Mat. Fiz. 41 (1979) 190; [Theor. Math. Phys. 41 (1979) 967].
[8] W. Helfrich, Berlin preprint (July 1985).
[9] H. Kleinert, UCSD preprint (Sept/Oct 1985).
[10] R. Pecora, in: Measurement of suspended particles by quasi-elastic light scattering, ed. B. Dahneke (Wiley, New York, 1983);
M. Bertero, A. Brianzi, E.R. Pike, G. de Villiers, K.H. Lahn and N. Ostrowsky, J. Chem. Phys. 82 (1985) 1551;
A.L. Larrabee, Biochemistry 18 (1979) 3321;
G. Duckwitz-Peterlein, W. Frisch and J.F. Holzwarth, Biochim. Biophys. Acta 469 (1981) 311.
[11] J.F. Holzwarth and R. Groll, unpublished notes.
[12] E.A. Evans, J. Biophys. 13 (1973) 926;
R. Skalak, A. Tozeren, R.P. Zorda and S. Chien, J. Biophys. 13 (1973) 245;
A.E. Evans and E. la Celle, Blood 45 (1975) 29;
Hochmuth et al., J. Biomed. 5 (1972) 501.
[13] F. Brochard, P.G. de Gennes and D.P. Pfeuty, J. Phys. (Paris) 37 (1976) 1099.