Tricritical Ratio of Length Scales in the $D=4$ Abelian Higgs Model

H. Kleinert[a]
Department of Physics, University of California, San Diego, La Jolla, California 92093
(Received 16 December 1985)

We use the duality between the Abelian Higgs model and pure U(1) lattice gauge theory to estimate the ratio $\sqrt{2}\kappa \equiv \lambda/\xi$ of penetration depth and coherence length at the tricritical point to be $\sqrt{2}\kappa \approx 0.93$, thus placing the tricritical point slightly on the type-I side of the borderline between type-I and type-II superconductivity in four dimensions.

PACS numbers: 11.15.Ha

About four years ago it was pointed out[1] that the Monte Carlo observation by Lautrup and Nauenberg[2] of a continuous phase transition in the U(1) lattice gauge theory (LGT) in two dimensions, if correct, was a counterexample to the Coleman-Weinberg mechanism[3] in the Abelian Higgs model. The agreement was based on the fact that after a couple of Villain approximations[4] and a duality transformation, the U(1) LGT

$$Z=\prod_x \int dA_\mu(x) \exp(\beta \sum_{x,\mu<\nu} \cos(\nabla_\mu A_\nu - \nabla_\nu A_\mu) \tag{1}$$

could directly be transformed into the following Abelian Higgs model[5]:

$$Z=\prod_{x,\mu}\int \frac{d\tilde{A}_\mu(x)}{(2\pi\beta)^{1/2}} S(\nabla \tilde{A}_\mu(x)) \prod_x \int \frac{d\phi(x)\, d\phi^\dagger(x)}{16\pi/\tilde{t}} \times \exp\left\{-\frac{1}{2\beta}\sum_{x,\mu<\nu} \tilde{F}'^2_{\mu\nu} - \sum_x \left[\frac{1}{32}|\tilde{D}_\mu\psi|^2 + \frac{\tilde{t}-4}{16}|\psi|^2 + \frac{1}{64}|\psi|^4 + \dots\right]\right\}. \tag{2}$$

Here x denotes the sites of the hypercubic lattice, μ the oriented links, ∇_μ, $\overline{\nabla}_\mu$ the lattice derivatives

$$\nabla_\mu \psi(x) = \psi(x+\mu) - \psi(x), \quad \overline{\nabla}_\mu \psi(t) = \psi(x) - \psi(x-\mu),$$

and $\tilde{D}_\mu$ the covariant derivatives

$$\tilde{D}_\mu \psi(x) = e^{-2\pi i A_\mu(x)} \psi(x+\mu) - \psi(x).$$

The field $\psi(x)$ is a *disorder field*[6] whose Feynman diagrams respresent the world lines of the magnetic monopoles in the U(1) LGT, and $\tilde{A}_\mu(x)$ is the *dual vector potential* by which these monopoles interact. The prime on the field strength $\tilde{F}'^2_{\mu\nu}(x)$ indicates that the propagation of $\tilde{A}(x)$ has received a subtraction to make it vanish at the origin. In the Landau gauge we have

$$\langle A_\mu(x) A_\nu(x')\rangle = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} (\delta_{\mu\nu} - K_\mu K_\nu^*/K^*\cdot K)[1/\mathbf{K}^*\cdot\mathbf{K} - v(0)], \tag{3}$$

where $K_\mu = e^{ik_\mu} - 1$ and

$$v(0) = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{1}{\mathbf{K}^*\cdot\mathbf{K}} \approx 0.155.$$

The quantity $\tilde{t}$ is equal to

$$\tilde{t} = 4\pi^2 v(0)\beta. \tag{4}$$

If the Coleman-Weinberg[3] claim is correct, that the Abelian Higgs model should have a first-order phase transition, then so should the U(1) LGT, in apparent contradiction with Lautrup and Nauenberg's result.

In the past three years, this observation has spurred renewed interest in the U(1) LGT on the one hand and the Abelian Higgs model on the other, with some surprising results.

First, it now appears as though the U(1) LGT itself really does undergo a first-order phase transition,[7] although with a very small entropy jump per site, $\Delta s \simeq 0.02$. Hence there is no longer any conflict with the Coleman-Weinberg claim. Instead, there now seems to exist another counterexample, namely the Villain approximation to the U(1) LGT. This indeed seems to have a second-order transition.[8,9]

Second, the Abelian Higgs model itself has been investigated by Monte Carlo techniques and been found to possess a tricritical point[10] where the order of the transition changes from first to second, just as expected after the discussion in Ref. 1.

In none of the recent works on the Abelian Higgs model has it been possible to measure the only model-independent physical parameter of the tricritical point, which is the ratio of the two length scales

$$\sqrt{2}\kappa = \frac{\text{penetration depth } \lambda}{\text{coherence length } \xi} = \frac{m_H}{m_W}. \tag{5}$$

This is due to the large number of fluctuating variables (four gauge-field components plus one complex field, all noncompact). In this note we would like to point out that on the basis of the technique developed in Ref. 5, it is possible to find a simple estimate for this ratio. This estimate can then be made more precise by use of the data of a Monte Carlo experiment in the pure U(1) LGT performed by Berg and Panagiotakopoulos[11] in 1984. These authors determine the energies E_H, E_W of scalar and vector combinations of Wilson loops at momentum $k=\pi/2$ and find the results shown in Fig. 1. According to the duality transformation given above, these lengths are a direct measure of the two length parameters ξ, λ in the Abelian Higgs model $[E_H=(1/\xi^2+2)^{1/2}, E_W=(1/\lambda^2+2)^{1/2}]$.

Let us first give the analytic estimate for $\sqrt{2}\kappa$. A simple mean-field analysis of the action (2) gives the following results: For low $\tilde{t} < \tilde{t}_c = 4\pi^2 v(0)\beta_c = 4$ the disorder field destabilizes and takes a nonzero vacuum expectation value

$$|\psi|^2 = 8(1-\tilde{t}/\tilde{t}_c) = 8(1-\beta/\beta_c). \tag{6}$$

This is the signal for the condensation of monopoles in the U(1) LGT. Via the covariant derivative $|\tilde{D}_\mu\psi|^2$, there is a Meissner effect on the dual photon which acquires a finite mass m_W (equal to the inverse penetration depth $1/\lambda$)

$$m_W^2 = \lambda^{-2} = \tfrac{1}{4}\pi^2\beta|\psi|^2 = 2\pi^2\beta(1-\beta/\beta_c). \tag{7}$$

The disorder field itself has a mass m_H (equal to the inverse coherence length $1/\xi$) given by

$$m_H^2 = 1/\xi^2 = 16(1-\beta/\beta_c), \quad t < t_c. \tag{8}$$

Thus we obtain for the ratio of the two length scales the mean-field result

$$\sqrt{2}\kappa = m_H/m_W = (8/\pi^2\beta)^{1/2}. \tag{9}$$

The actual value of β_c is renormalized to be around $\beta_c \approx 1$. Since the U(1) LGT is practically tricritical, this leads to a first crude estimate of the tricritical value,

$$\sqrt{2}\kappa_{\rm tr} \approx 0.90. \tag{10}$$

For $\beta > \beta_c$, the disorder field has no expectation value and its mass is equal to

$$m_H^2 = 8(\beta/\beta_c - 1). \tag{11}$$

In addition there is a massless Goldstone mode. In this phase, there is no Meissner effect so that also the photon mass vanishes:

$$m_W^2 = 0. \tag{12}$$

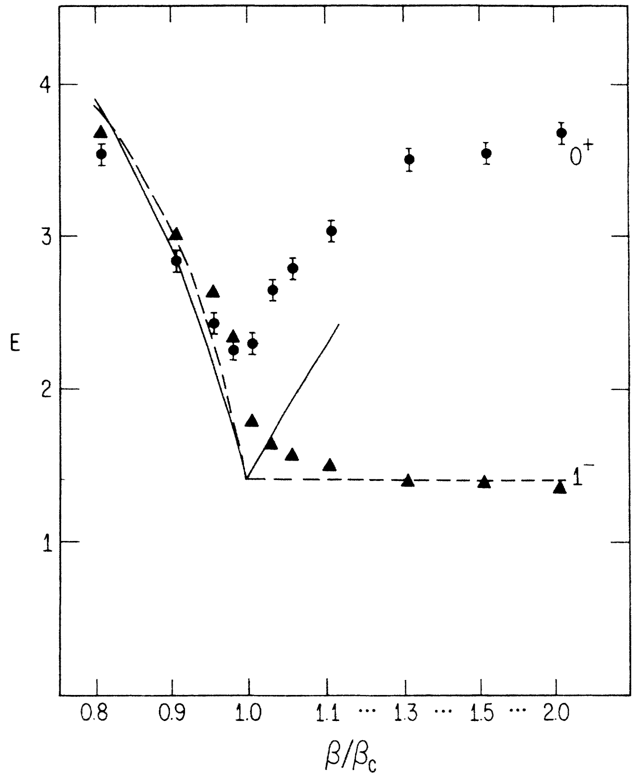

FIG. 1. The scalar and vector energies $(m^2+\mathbf{K}\cdot\mathbf{K}^*)^{1/2}$ at lattice momentum $k=\pi/2$ (i.e., $\mathbf{K}\cdot\mathbf{K}^*=2$) as obtained in Ref. 11 via Monte Carlo simulation. The curves show our mean-field results, based on formulas (7) and (8) with the masses renormalized by a factor of 2 to bring the absolute scale to the correct size.

The data of Ref. 11 reflect this qualitative structure. For $\beta < \beta_c \simeq 1$ there is a massive vector particle (W) and a massive scalar particle (H), the first being slightly more massive than the second just as in Eqs. (8) and (9). For $\beta > \beta_c$, the vector particle is massless while scalar particle is again more massive [just as in Eqs. (12) and (13)]. It is curious to see that the simple mean-field results (7) and (8) provide an excellent fit to the data if both masses are renormalized by a factor of 2.

The data also show that just as in the mean-field estimate (9), the mass of the disorder field is slightly smaller than the photon mass, but not quite as small as in the mean-field estimate (11). Giving the largest weight to the data point at $\beta \sim 0.9$, we estimate

$$\sqrt{2}\kappa_{\rm tr} \sim m_H/m_A \sim 0.93. \tag{13}$$

This value lies slightly on the side of a type-I superconductor in four dimensions.

There is obvious need for an improvement of the length measurements in the U(1) LGT. In particular, they should be performed for the mixed action

$$\sum_{x,\mu<0}[\beta\cos(\nabla_\mu A_\nu-\nabla_\nu A_\mu)+\gamma\cos2(\nabla_\mu A_\nu-\nabla_\nu A_\mu)] \tag{14}$$

at various values of γ, since this parameter allows one to vary the strength of the first-order jump and to move the system right through the tricritical point ($\gamma\approx-0.15$). Certainly, the tricritical length scales obtained in this way will have to be confirmed in direct simulations of the Abelian Higgs model, but until these become available, the U(1) LGT will be a valuable substitute.

For a similar estimate of the three-dimensional Abelian Higgs model, compare earlier work.[12]

The author is grateful to Professor N. Kroll and J. Kuti for their hospitality at University of California, San Diego, and to D. Toussaint for valuable discussions. This work was supported in part by Deutsche Forschungsgemeinschaft under Grant No. Kl 256 and by University of California, San Diego/U.S. Department of Energy Contract No. DEAT-03-81ER40029.

[(a)]Permanent address: Institut für Theorie der Elementarteilchen, FU Berlin, Arnimallee 14, 1 Berlin 33, Germany.

[1]H. Kleinert, Phys. Lett. **93A**, 86 (1982).

[2]B. Lautrup and M. Nauenberg, Phys. Lett. **95B**, 63 (1980).

[3]S. Coleman and E. W. Weinberg, Phys. Rev. D **7**, 1880 (1973).

[4]J. Villain, J. Phys. **36**, 581 (1975). For a recent discussion of the quality of the approximation, see W. Janke and H. Kleinert, Nucl. Phys. B (to be published).

[5]H. Kleinert, in *Gauge Interactions,* edited by A. Zichichi (Plenum, New York, 1982).

[6]See H. Kleinert, *Gauge Theory of Stress and Defects* (Gordon and Breach, New York, 1986) (to be published).

[7]J. Jersák, T. Neuhaus, and P. Zerwas, Phys. Lett. **133B**, 103 (1983); H. G. Everts, J. Jersák, and T. Neuhaus, Nucl. Phys. **B251 [FS13]**, 279 (1985).

[8]T. A. DeGrand and D. Toussaint, Phys. Rev. D **24**, 466 (1981); J. Jersák, T. Neuhaus, and P. M. Zerwas, Nucl. Phys. **B251 [FS13]**, 299 (1985).

[9]The subtle differences between the two models have been analyzed recently by Janke and Kleinert in Ref. 4.

[10]Y. Munehisa, Phys. Rev. D **30**, 1310 (1984), and **31**, 1522 (1985) [also Phys. Lett. **155B**, 159 (1985), for the three-dimensional case]. D. Toussaint and R. Sugar, to be published; G. Koutsoumbas, Phys. Lett. **B140**, 379 (1984). There may also be a tricritical point in the non-Abelian version. H. Kühnelt, C. B. Lang, and G. Vones, Nucl. Phys. **B230 [FS70]**, 16 (1981); K. Jansen, J. Jersák, C. B. Lang, T. Neuhaus, and G. Vones, Phys. Lett. **155B**, 268 (1985), and to be published.

[11]B. Berg and C. Panagiotakopoulos, Phys. Rev. Lett. **52**, 94 (1984).

[12]H. Kleinert, Lett. Nuovo Cimento Soc. Ital. Fis. **35**, 405 (1982).