THERMAL DECONFINEMENT TRANSITION FOR SPONTANEOUS STRINGS

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Within the recently proposed membrane model of quark confinement we test the possibility that all string tension M is generated spontaneously, by calculating the deconfinement temperature T_d. We find $T_d \approx 0.68M$, in reasonable agreement with Monte Carlo data for SU(3) lattice gauge theories.

We want to suggest that strings between quarks follow an action that involves *only* the extrinsic curvature of a two-dimensional surface $x^\mu(\xi)$ and reads

$$\mathscr{A}_K = \frac{1}{2\alpha}\int d^2\xi \sqrt{g}\, K^2, \tag{1}$$

where $K^2 = (D_i x^\mu)^2$ and g_{ij}, D_i are the metric and covariant derivatives, respectively. The possibility of adding this term to the usual string action have previously been realized by Polyakov and by the author [1,2] (independently) and several consequences have been studies [3]. We would like to point out that if the action (1) is taken by itself, as a model of its own, it reveals several desirable physical properties. In contrast to the ordinary string model, the coupling constant α is dimensionless and always contains interactions, in any parametrization. These interactions generate spontaneously a string tension. For this reason we shall call (1) a "spontaneous string model". The full quark potential of the model was calculated in a previous paper [4] (where we have also allowed for an additional Nambu–Goto term, for generality).

As a first test of the idea we calculate the thermal deconfinement transition and find that it lies near the place where it is expected for strings between quarks. Above this temperature, the confining tension is gone and the surface begins ondulating without control. The situation is similar to that in microemulsions [5,6], where the addition of surfactants drives the surface tension negative and leads to a proliferation of surfaces.

The calculation is easiest in the Gauss parametrization

$$x^\mu(\xi) = (\xi^0, \xi^1, x^a), \quad a = 2, \ldots, d-1,$$

with an action in which the internal and external fluctuations are treated separately [1,3]

$$\mathscr{A}_K = \frac{1}{2\alpha}\int d^2\xi \sqrt{g}\left[(D^2x^a)^2 + \lambda^{ij}\left(\partial_i x^a \partial_j x^a - g_{ij} + S_{ij}\right)\right], \tag{2}$$

with fluctuating x^a, g_{ij}, λ^{ij}. The fluctuations in x^a can be integrated out giving a one-loop contribution

$$\mathscr{A}_{fl} = \tfrac{1}{2}(d-2)\,\mathrm{Tr}\log\left((D^2)^2 - D_i\lambda^{ij}D_j\right) - \frac{1}{2\alpha}\int d^2\xi \sqrt{g}\left(\lambda^{ij}g_{ij} - \lambda^{ii}\right), \tag{3}$$

which for $d \to \infty$ can be evaluated at the extremum.

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For a surface spanned between two static quarks at a large extrinsic distance $R_{\rm ext}$ in a thermal environment of inverse temperature $\beta_{\rm ext}=1/T_{\rm ext}$, we can take $g_{ij}=\rho_i\delta_{ij}=$ const., $\lambda^{ij}=\lambda^i g^{ij}=$ const. and have the effective action

$$\mathscr{A}_{\rm fl}=\tfrac{1}{2}(d-2)R_{\rm ext}\beta_{\rm ext}\sqrt{\rho_0\rho_1}\left[f_0^{T=0}+\Delta f^T+\Delta f^{\rm an}+(1/2\alpha)(\lambda_0/\rho_0+\lambda_1/\rho_1)\right], \tag{4}$$

with

$$f^{T=0}=\int\frac{d^2q}{(2\pi)^2}\log(q^4+\tilde\lambda q^2)-\frac{\tilde\lambda}{\tilde\alpha},$$

$$\Delta f^T=\left(T\sum_{m=-\infty}^{\infty}-\int_{-\infty}^{\infty}\frac{d\omega_m}{2\pi}\right)\int_0^\infty\frac{dq}{\pi}\left[\log(\omega_m^2+q^2+\tilde\lambda)+(\tilde\lambda=0)\right],$$

$$\Delta f^{\rm an}=T\sum_{m=-\infty}^{\infty}\int_0^\infty\frac{dq}{\pi}\Big\{\log\left[(\omega_m^1+q^2)^2+\tilde\lambda(\omega_m^2+q^2)-\tilde\lambda\,\delta(\omega_m^2-q^2)\right]$$
$$-\log\left[(\omega_m^2+q^2)^2+\tilde\lambda(\omega_m^2+q^2)\right]\Big\},$$

where $\tilde\alpha\equiv\frac{1}{2}(d-2)\alpha$, $\tilde\lambda\equiv\frac{1}{2}(\lambda_0+\lambda_1)$, $\delta=(\lambda_1-\lambda_0)/2\tilde\lambda$, $\omega_m=2\pi Tm$ with $1/T=(1/T_{\rm ext})\rho_0$ being the intrinsic temporal size of the surface. The $T=0$ piece $f^{T=0}$ is renormalized dimensionally so that it takes the form

$$f^{T=0}\equiv f_0(\tilde\lambda)-\tilde\lambda/4\pi\equiv-(\tilde\lambda/4\pi)\left[\log(\tilde\lambda/\bar\lambda)-1\right]-\tilde\lambda/4\pi, \tag{5}$$

where $\bar\lambda\equiv\mu^2\exp\{-[2/(d-2)]4\sigma/\alpha(\mu^2)+1\}$ is the dimensionally transmuted coupling constant, being defined as the place where $f^{T=0}$ has its saddle point (= maximum for real $\tilde\lambda$) with a value $\bar\lambda$. All quantities λ_i, ρ_i, $\tilde\lambda$ are now finite and we may, moreover, renormalize ρ_0, ρ_1 to be equal to unity at $T=0$. The quantity $\mu^2=\frac{1}{2}(d-2)\bar\lambda/4\pi$ is the spontaneously generated string tension of the system.

The finite temperature correction of $f^{T=\infty}$ is

$$\Delta f^T=2T\int_0^\infty\frac{dq}{\pi}\left[\log\left\{1-\exp\left[-(k^2+\lambda)^{1/2}/T\right]\right\}+(\lambda=0)\right]$$
$$=-\frac{2T\sqrt{\tilde\lambda}}{\pi}\sum_{\tilde m=1}^{\infty}K_1\left(\frac{\tilde m}{T}\sqrt{\tilde\lambda}\right)/\tilde m-\tfrac{1}{3}\pi T^2, \tag{6}$$

where $K_1(z)$ is the modified Bessel function $\int_0^\infty ds\,\exp(-\sqrt{s^2+1}\,z)$. The term $-\frac{1}{3}\pi T^2$ is, of course, just the free energy of two-dimensional massless "black body radiation". Alternatively, we can expand

$$f_0+\Delta f^T=\frac{\tilde\lambda_0}{4\pi}\left(-\log(T/\bar T)^2+\frac{2}{\sqrt{\tilde\lambda_T}}-\frac{2}{3\tilde\lambda_T}+\frac{4}{\tilde\lambda_T}\sum_{m=1}^{\infty}\left[(m^2+\tilde\lambda_T)^{1/2}-m-\tilde\lambda_T/2m\right]\right), \tag{7}$$

where $\bar T\equiv\sqrt{\bar\lambda}\,e^\gamma/4\pi$, $\tilde\lambda_T\equiv\tilde\lambda/4\pi^2T^2$. Finally, we have to take into account the possible anisotropy of the gap δ via

$$\Delta f^{\rm an}=2T\sum_{m=1}^{\infty}\left[A_m^++A_m^--(\omega_m^2+\tilde\lambda)^{1/2}-m\right]+T\left(\sqrt{\lambda_1}-\sqrt{\tilde\lambda}\right)-\frac{\tilde\lambda}{4\pi}\delta, \tag{8}$$

with

$$A_m^\pm\equiv\left(\omega_m^2+\tfrac{1}{2}\lambda_1\left\{1\pm\left[1+(4\Delta\lambda/\lambda_1^2)\omega_m^2\right]^{1/2}\right\}\right)^{1/2},\quad \Delta\lambda\equiv\lambda_1-\lambda_0.$$

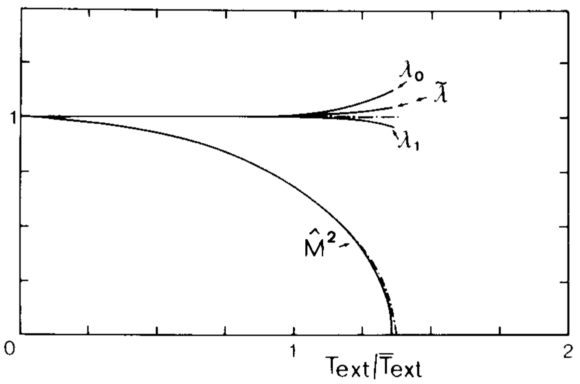

Fig. 1. The string tension as a function of the reduced temperature $T_{\rm ext}/\bar T_{\rm ext}$ where $\bar T_{\rm ext} = \{e^{\gamma}/[2\pi(d-2)]^{1/2}\}M \approx 0.502M$ and M is the string tension. The curves are for a purely spontaneous string. The deconfinement temperature is seen to lie at $T_{\rm ext} \approx 0.68M$. The curve $-\cdot-\cdot-$ represents the simple analytic approximation (13) explained in the text. The upper three curves show the corresponding gaps λ_0, λ_1, $\tilde\lambda = \frac{1}{2}(\lambda_1 + \lambda_0)$ as a function of temperature and $-\cdot-\cdot-$ is our simple approximation.

Extremizing the action (4) in ρ_0, ρ_1, $\tilde\lambda$, δ we obtain $\mathscr{A}_{\rm fl} = \frac{1}{2}(d-2)R_{\rm ext}\beta_{\rm ext}M^2\hat M^2(T)$ where $\hat M^2(T) \equiv \sqrt{\rho_0\rho_1}(4\pi/\tilde\lambda)f_{\rm tot} \equiv \sqrt{\rho_0\rho_1}(4\pi/\tilde\lambda)[f_0 + \Delta f^T + \Delta f^{\rm an}]$ (normalized to unity at $T=0$) and $\tilde\lambda$, δ are determined by the equations

$$(\partial/\partial\tilde\lambda)[f_0 + \Delta f^T + \Delta f^{\rm an}] = 0, \quad (\partial/\partial\delta)\Delta f^{\rm an} = (1/4\pi)[1/(1-\delta^2)][\delta - (4\pi/\tilde\lambda)f_{\rm tot}]. \tag{9,10}$$

The quantities ρ_0, ρ_1 are given by

$$\rho_0 = (1-\delta)[2-(4\pi/\tilde\lambda)f_{\rm tot}]^{-1}, \quad \rho_1 = (1+\delta)[(4\pi/\tilde\lambda)f_{\rm tot}]^{-1}, \tag{11,12}$$

As a lowest approximation we ignore δ and the Bessel functions in (6) and solve the gap equation (9) by $\tilde\lambda \approx \bar\lambda$ such that $f_{\rm tot} \approx \bar\lambda/4\pi - \frac{1}{3}\pi T^2$, $\rho_{\substack{0\\1}} \approx (1 \pm \frac{4}{3}\pi^2T^2/\bar\lambda)^{-1}$ and we obtain

$$\hat M^2(T) \approx [1-(4\pi^2/3\bar\lambda)T^2]^{1/2}/[1+(4\pi^2/3\bar\lambda)T^2]^{1/2}. \tag{13}$$

This gives the estimate for the intrinsic deconfinement temperature

$$T_{\rm d}^0 \approx (3/4\pi^2)^{1/2}\sqrt{\bar\lambda} \approx 1.95\bar T = [6/\pi(d-2)]^{1/2}M \approx 0.98M. \tag{14}$$

The physical, extrinsic, temperature associated with this is

$$T_{\rm ext,d}^0 = T_{\rm d}^0/\sqrt{\rho_1} \approx T_{\rm d}^0/\sqrt{2} \approx 0.69M. \tag{15}$$

In fig. 1, we have plotted $\hat M^2$ of (13) as a function of $T_{\rm ext}/\bar T$, as well as the gap $\bar\lambda$.

A full numerical evaluation of the eqs. (9)–(12) shows that, contrary to the finite R case [4], the gap distortion remains up to the deconfinement transition extremely small, so that there are almost no corrections to the approximate curve (see fig. 1). In fact, if δ is ignored but the formulas evaluated accurately, the curve for $\hat M^2$ does not differ from the one drawn in fig. 1 and the curve for $\tilde\lambda$ coincides with the central full $\tilde\lambda$ curve.

The complete calculation has $T_{\rm ext,d}^0$ lowered to $T_{\rm ext,d} \approx 0.68\ M$. Our deconfinement temperature is in reasonable agreement with Monte Carlo data on lattice gauge theories which find $0.45M$ for SU(2) [7,8] and $0.5M$–M for SU(3) [8] [#1]. The hypothesis that strings arise spontaneously can therefore well be true.

[#1] The large discrepancy is due to various estimates for the relation between the dimensionally transmuted coupling constant and the string tension ($\Lambda^{\rm latt}/M \times 10^3$) found in the literature. Ref. [9] gives a value of 6 ± 1, ref. [10] gives 11 ± 3, ref. [11] gives 9.4 ± 3, ref. [12] gives 11.9 ± 6 and ref. [13] gives 9.6. For an overview see also ref. [14].

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