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Glueballs from spontaneous strings

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I point out that in Euclidean space spontaneous strings (≡strings with extrinsic curvature stiffness and spontaneously generated tension) can form glueballs, even at the classical level (in contrast with the ordinary string). In the limit of infinite dimensions, the fluctuations can be integrated out and the glueballs are found to have a mass to tension ratio of ≈3.5 in the purely spontaneous case (≡no Nambu-Goto tension term in the action).

Recently, Polyakov[1] and the author[2] have called for the addition of an intrinsic curvature term

$$A_K=(1/2\alpha)\int d^2\xi\sqrt{g}\,(D^2x^a)^2 \tag{1}$$

to the ordinary Nambu-Goto action $A_{\rm NG}=M_{\rm NG}^2\int d^2\xi\sqrt{g}$ [where $x^a(\xi)$ parametrizes the surface in d dimensions, g_{ij} is the metric $\partial_i x^a\partial_i x^a$, and D_i are the covariant derivatives]. The string emerging from the new action has many more features in common with the string formed between quarks in QCD than the Nambu-Goto string: At the classical level it is scale invariant, but fluctuations generate spontaneously a tension.[1-3] This gives an additional linear rise in the quark potential at long distances[4,5] but does not change Lüscher's universal $(d-2)\pi/24R$ correction.[6,7] In a thermal environment, the tension decreases and vanishes at some deconfinement temperature[8] T_d of the order of the tension. The most dramatic advantage over the Nambu-Goto string, however, is the existence of the potential at short distances[7] where it has precisely the same asymptotic-freedom behavior $\propto 1/R$ behavior as the short-distance force due to gluons. Even the quantitative behavior is apparently correct—the prefactor has twice the Lüscher value, in agreement with potential fits to the spectrum of the Ψ/J family.[7,9]

The above properties have been demonstrated only in the limit of infinite dimensions. If they were to hold at $d=4$, the spontaneous string would provide a major step towards understanding the QCD forces in terms of surface physics.

In this paper we would like to draw attention upon another pleasant property of spontaneous strings. They give rise, in a natural way, to glueballs in Euclidean space and, at the classical level, this is obvious. A tube of extrinsic radius $R_{\rm ext}$ and timelike length $\beta_{\rm ext}$ has an action

$$A_{\rm cl}=2\pi R_{\rm ext}\beta_{\rm ext}(1/2\alpha R_{\rm ext}^2+M_{\rm NG}^2)\ . \tag{2}$$

This is minimal at $R_{\rm ext}=1/\sqrt{2\alpha M_{\rm NG}^2}$ and gives a mass to tension ratio for a glueball ground state of $M_G/M_{\rm NG}=2\pi\sqrt{2/\alpha}$.

In Minkowski space, the stiffness leads to the existence of closed static string solutions in addition to the rotating ones of the Nambu-Goto action. These have been investigated by Curtright and co-workers.[10] Braaten and Zachos[11] looked at the quantum instability of the static solutions caused by the presence of higher-derivative terms in the action.

The purpose of this paper is to study the effect of fluctuations upon the Euclidean glueball state exactly in the limit $d\to\infty$. First we make the quantities g_{ij} and x_0 independent fluctuating variables by means of a Lagrange multiplier term

$$(1/2\alpha)\int d^2\xi\sqrt{g}\,\lambda^{ij}(\partial_i x^a\partial_j x^a-g_{ij})\ .$$

Then we expand the action around the background

$$x=(\xi^0,R_{\rm ext}\cos(\xi^1/R_{\rm ext}),R_{\rm ext}\sin(\xi^1/R_{\rm ext})0,\ldots,0)\,,$$

where $\xi^0\in(0,\beta_{\rm ext})$, $\xi^1\in(0,2\pi R_{\rm ext})\equiv(0,L_{\rm ext})$. If g_{ij} has the form $\rho_i\delta_{ij}$, then the curvature action of the background is

$$A_{K0}=(1/2\alpha)\int d^2\xi\sqrt{g}\,(D^2x_0)^2$$
$$=2\pi R_{\rm ext}\beta_{\rm ext}\sqrt{\rho_1\rho_2}\,1/2\alpha\rho_1R_{\rm ext}^2\ . \tag{3}$$

Since x appears only quadratically in the path integral, the $d-2$ transverse degrees of freedom can be integrated out and we find

$$A=A_{K0}+[(d-2)/2]\left[\operatorname{tr}\ln(D^4-D_i\lambda^{ij}D_j)-\int d^2\xi\sqrt{g}\,\lambda^{ij}(g_{ij}-\delta_{ij})\right]\ . \tag{4}$$

In the limit $d\to\infty$, the total action is determined by a space-independent saddle point at which it reads

$$A=[(d-2)/2]2\pi R_{\rm ext}\beta_{\rm ext}\sqrt{\rho_0\rho_1}g\ , \tag{5a}$$

where, at infinite circumference of the cylinder $L_{\rm ext}$, ρ_0 and ρ_1 are equal, $\rho_1\equiv\rho_2=\rho$, and[5]

$$g=\tilde M_{\rm NG}^2+\int\frac{d^2k}{(2\pi)^2}\ln(k^4/\rho^2+\tilde\lambda k^2/\rho)$$
$$-\frac{\tilde\lambda}{\tilde\alpha}+\frac{1}{2\tilde\alpha}(\lambda_0/\rho_0+\lambda_1/\rho_1) \tag{5b}$$

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with $\tilde{\alpha} \equiv [(d-2)/2]\alpha$ and $\tilde{M}^2_{\rm NG} \equiv [2/(d-2)]M^2_{\rm NG}$. For symmetry reasons we have set $\lambda_{ij} = \lambda_i g_{ij}$ and defined $\tilde{\lambda}$ as $(\lambda_1+\lambda_2)/2$. Renormalization with an intrinsic cutoff mass μ gives

$$g = \left[\frac{\tilde{\lambda}_\nu}{4\pi}\nu - \frac{\tilde{\lambda}}{4\pi}\ln(\tilde{\lambda}/\bar{\lambda}) + \frac{1}{2\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0} + \frac{\lambda_1}{\rho_1}\right]\right], \quad (6)$$

where $\bar{\lambda} \equiv \mu^2 4\pi e^{-\gamma}\exp(-4\pi/\tilde{\alpha}+1)$. We also introduce $\delta \equiv (\lambda_1 - \lambda_2)/2\tilde{\lambda}$. At $L_{\rm ext} = \infty$, λ_{ij} is isotropic, i.e., $\delta = 0$. For finite $L_{\rm ext}$, g receives a finite-size correction $\Delta g^L + \Delta g^\delta$ where $L \equiv L_{\rm ext}\sqrt{\rho_1}$ is the intrinsic circumference and

$$\begin{aligned}\Delta g^L &= -\pi/3L^2 - 2(\sqrt{\tilde{\lambda}}/\pi L)\sum_{\bar{n}=1}^{\infty} K_1(L\sqrt{\tilde{\lambda}}\,\bar{n})/\bar{n} \\ &= -2\pi/3L^2 + \sqrt{\tilde{\lambda}}/L \\ &\quad + (2/L)\sum_{n=1}^{\infty}\left(\sqrt{q_n^2+\tilde{\lambda}} - q_n - \tilde{\lambda}/2q_n\right) \\ &\quad + (\tilde{\lambda}/4\pi)\ln(L^2/\bar{L}^2) \end{aligned} \quad (7)$$

[with $q_n \equiv 2\pi n/L$, $\bar{L} \equiv 4\pi^2 e^{-\gamma}/\sqrt{\bar{\lambda}}$, $K_1(z) \equiv$ Bessel function] and

$$\begin{aligned}\Delta g^\delta &= (1/L)\sqrt{\tilde{\lambda}}(\sqrt{1-\delta}-1) \\ &\quad + (2/L)\sum_{n=1}^{\infty}\left(A_n^+ + A_n^- - \sqrt{q_n^2+\tilde{\lambda}} - q_n\right) + \tilde{\lambda}\delta/4\pi \end{aligned} \quad (8)$$

with

$$A_n \equiv (q_n^2 + \tilde{\lambda}[(1-\delta)/2]\{1 \pm [1 - 8\delta\, q_n^2/\tilde{\lambda}(1-\delta)^2]^{1/2}\})^{1/2}$$

The action has to be maximized in $\tilde{\lambda},\delta$ and minimized in $\rho_{0,1}$. For $L=\infty$, the extremum is given by $4\pi/\tilde{\alpha}\rho = 1+\nu$, $\tilde{\lambda} = \tilde{\lambda}_\nu \equiv \bar{\lambda}e^\nu$, where ν is a number, called "normality," which characterizes the content of the ordinary Nambu-Goto tension in the action. Its value is fixed by $\tilde{M}^2_{\rm NG} = \tilde{\lambda}_\nu \nu/4\pi$. The total tension is $(d-2)/2$ times $\tilde{M}^2_{\rm tot} = \tilde{\lambda}_\nu(1+\nu)/4\pi$.

For finite L we have to solve the equations

$$-1/2\tilde{\alpha}\rho_1\tilde{\lambda}_L + \tilde{M}^2_{\rm NG}/\tilde{\lambda} - (1/4\pi)\ln(\tilde{\lambda}/\bar{\lambda}) + (\partial/\partial\tilde{\lambda})(\Delta g^L + \Delta g^\delta) = 0\,, \quad (9)$$

$$\lambda_0/\tilde{\alpha}\rho_0 = g^L_{\rm tot}\,, \quad (10a)$$

$$\lambda_1/\tilde{\alpha}\rho_1 = [(1+\delta)/(1/\tilde{\lambda}_L + 1 + \delta)](\tilde{\lambda}/2\pi + 2\tilde{M}^2_{\rm NG} - g^L_{\rm tot})\,, \quad (10b)$$

$$\partial\Delta g^\delta/\partial\delta = [\tilde{\lambda}/4\pi(1-\delta^2)][\delta(1+4\pi\tilde{M}^2_{\rm NG}/\tilde{\lambda}) + (1-\delta)4\pi/2\tilde{\alpha}\rho_1\tilde{\lambda}_L + (4\pi/\tilde{\lambda})(g^L_{\rm tot} - \tilde{M}^2_{\rm NG}) - 1]\,, \quad (11)$$

where $\tilde{\lambda}_L \equiv \tilde{\lambda}L^2/4\pi^2$ and

$$g^L_{\rm tot} \equiv 2\tilde{M}^2_{\rm NG} - (\tilde{\lambda}/4\pi)[\ln(\tilde{\lambda}/\bar{\lambda}) - 1] + \Delta g^L + \Delta g^\delta \quad (12)$$

is the total g at the extremum. The solution is found by iteration and yields an action which rises linearly in $L_{\rm ext}$ with a minimum at very small $L_{\rm ext}$ but finite intrinsic L. For $\nu=0$ (purely spontaneous string) we find a ratio

$$M_G/M_{\rm tot} \approx \sqrt{(d-2)/2} \times 3.5\,. \quad (13)$$

An analytic estimate of the order of magnitude of the ratio is possible using the large-L expansion of $v = L_{\rm ext}g^L_{\rm tot}$ in which

$$\begin{aligned} &\delta \approx -2(1+\nu)/(1+2\nu/3)\tilde{\lambda}_L, \quad \tilde{\lambda}/\tilde{\lambda}_\nu \approx 1 - 1/2\tilde{\lambda}_L\,, \\ &\rho_0/\bar{\rho} \approx 1 + (1+4\nu/9)/2\tilde{\lambda}_L(1+\nu)(1+2\nu/3)\,, \\ &\rho_1/\bar{\rho} \approx 1 + (1+8\nu/9)/2\tilde{\lambda}_L(1+\nu)(1+2\nu/3)\,; \end{aligned} \quad (14)$$

$$v \approx L_{\rm ext}[\tilde{\lambda}_\nu(1+\nu)\bar{\rho}/4\pi][1 + (1+3\nu)/6\tilde{\lambda}_L(1+\nu)]\,, \quad (15)$$

where $\bar{\rho} \equiv 4\pi/\tilde{\alpha}(1+\nu)$, so that the minimum of v gives

$$\begin{aligned} M_G/M_{\rm tot} &\approx \sqrt{(d-2)/2}\sqrt{\pi(1+\nu)} \\ &\quad \times \sqrt{\tilde{\lambda}_L}[1 + (1+3\nu)/6\tilde{\lambda}_L(1+\nu)]\,\Big|_{\rm min} \\ &\approx \sqrt{(d-2)/2}\sqrt{2\pi(1+3\nu)/3}\,. \end{aligned} \quad (16)$$

The true value at $\nu \approx 0$ in Eq. (13) is about twice as large.

Computer experiments of lattice QCD without fermions[12] give a value of around 3. In principle, a determination of this ratio could be used to find out which is the value of ν chosen by the string of QCD.

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