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UNIVERSAL ENTROPY OF LARGE SPHERES OF SPONTANEOUS STRINGS*

H. KLEINERT
Institut für Theoretische Physik, Freie Universität Berlin, 1 Berlin 33, Germany

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We show that large spherical surfaces in Euclidean space time made of spontaneous strings (strings with extrinsic curvature plus a spontaneously generated tension) have a universal entropy $P(R^2) \propto (R^2)^{d/6} e^{-\text{const.} R^2}$ in the limit of large dimensionality d. The power of R is universal in the sense that it is independent of the model parameter ν (normality) which characterizes the amount of Nambu-Goto tension with respect to the spontaneously generated tension ($\nu = \infty$ is the pure Nambu-Goto String).

Strings with extrinsic curvature stiffness seem to be an excellent representation of the strings that form in the QCD vacuum and hold quarks together. The curvature action is dimensionless and asymptotically free in the ultraviolet and generates spontaneously a tension.[1,2,3,4,5] This is why we call such strings "spontaneous".[3,4] They are characterized by a parameter $\nu \equiv$ normality.[3,4] It specifies the ratio between Nambu-Goto and spontaneously generated tension ($\nu = \infty$ is the Nambu-Goto, $\nu = 0$ the purely spontaneous, string). At small temperature and large distance, two important finite size properties of spontaneous strings have recently been shown to be universal in the limit of large dimension d: the thermal deconfinement transition[3] and the Coulomb-like $1/R$ term in the quark potential.[5,6] Both are independent of ν. Here we add one more quantity to this list, namely the entropy of spherical surfaces of large R. They may be viewed as the string version of the instantons of QCD.

For the Polyakov string with the action $M^2 \int d^2\xi \sqrt{g}(\partial x^a)^2$, the number of spherical states has been shown[7] to grow with the area by $\Gamma(R^2) \propto (R^2)^{(d-25)/6} e^{\Lambda^2 R^2}$ where Λ^{-1} is the short-distance cutoff. We shall find that for leading order in d, the same result is true for spontaneous strings of any ν.

Consider a background sphere of radius R_{ext} in the parametrization $x_s^a(\xi, \varphi) = R_{\text{ext}}(\cos\varphi \,\text{ch}^{-1}\xi, \sin\varphi\,\text{ch}^{-1}\xi, \text{th}\,\xi, 0, 0, \dots)$ with the metric $g_{ij} = R_{\text{ext}}^2 \text{ch}^{-2}\xi\,\delta_{ij}$.

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Inserting this into the action with tension and extrinsic curvature stiffness

$$A = ((d-2)/2) \int d^2\xi \sqrt{g}\, [\tilde{M}_{\rm NG}^2 + (1/2\tilde{\alpha})[(D^2 x^a)^2 + \lambda^{ij}(\partial_i x^a \partial_j x^a - g_{ij})]] \tag{1}$$

(in which x^a, g_{ij}, λ^{ij}, are independent field variables, $\tilde{M}_{\rm NG}^2$ is the Nambu-Goto tension, and $1/\tilde{\alpha}$ is the bending stiffness, the tilde standing for a factor $((d-2)/2)$) and performing the integral over all fluctuations in $x^a (a = 2, \ldots, d-1)$ [assuming that $g_{ij} = \rho R_{\rm ext}^2 {\rm ch}^{-2}\xi\delta_{ij}$, $\lambda^{ij} = \lambda g^{ij}$, with $\rho, \lambda \equiv$ const.] gives, in the limit $d \to \infty$, the action $A = ((d-2)/2)\,4\pi R^2{}_{\rm ext}\,\rho f_{\rm tot}$ with the free energy density

$$f_{\rm tot} = 4/2\tilde{\alpha}\rho R^2 + \lambda/\tilde{\alpha}\rho + f_{\rm fl} - (1/4\pi R^2)\ln(R^2\mu^2) \tag{2}$$

where $f_{\rm fl} = f_{\rm fl}^\lambda + f_{\rm fl}^0$ are the fluctuation corrections

$$f_{\rm fl}^\lambda = (1/4\pi R^2)\sum_{l=1}^{\infty}(2l+1)\ln[(l(l+1) + \lambda R^2)/\mu^2 R^2] - \lambda/\tilde{\alpha} \tag{3}$$

due to the trace log $(1/4\pi R^2)\,{\rm tr}\ln(-D^2(-D^2+\lambda))$ with the zero mode removed. Its effect is accounted for by the last term in (2) since it produces a volume factor $(LR)^{d-2}$ in the partition function $Z = e^{-A}$. The action has to be evaluated at the extremum with respect to variations in ρ, λ at fixed $R_{\rm ext}^2 = R^2/\rho$. In order to calculate the sums in (3), we add and subtract the regularized flat-space integrals

$$\int \frac{d^2 g}{(2\pi)^2}\ln(q^2+\lambda) - \lambda/\tilde{\alpha} = -(\lambda/4\pi)\ln(\lambda/\bar{\lambda}), \tag{4}$$

with the quantity $\bar{\lambda}$ being the dimensionally transmuted coupling constant of the spontaneous string, as in Ref. 3. The difference between sums and integrals can be performed using the generalized Euler-MacLaurin series

$$\left(\sum_{l=0}^{\infty} - \int_{-q}^{\infty} dl\right)(l+q)^\nu h(l+q) = \sum_{k=0}^{\infty} k!^{-1}\zeta(-\nu-k, q)h^{(k)}(0) \tag{5}$$

(with Riemann's zeta function $\zeta(-z,q) = \sum_0^\infty (n+q)^z = -B_{z+1}(q)/(z+1)$ where $B_n(q) \equiv$ Bernoulli polynomials). At very large R, $f_{\rm tot} = \tilde{M}_{\rm NG}^2 - (\lambda/4\pi)\ln(\lambda/\bar{\lambda})$ and the action is extremal at $\lambda = \bar{\lambda}_\nu \equiv \bar{\lambda}e^\nu$, where ν is obtained from $\tilde{M}_{\rm NG}^2 \equiv \bar{\lambda}_\nu \nu/4\pi$. The total tension is $\tilde{M}_{\rm tot}^2 = \bar{\lambda}_\nu(1+\nu)/4\pi$. At finite R, the action is $A = ((d-2)/2)g_{\rm tot}$ with

$$g_{\text{tot}} = 4\pi R^2 f_{\text{tot}} = R^2 \bar{\lambda}_v + (4\pi/\tilde{\alpha}\rho)(2 + \lambda R^2) + \Delta g_0 - \lambda R^2 \ln(\lambda/\bar{\lambda})$$
$$+ \Delta g_\lambda - (1/3)\ln(\bar{\lambda}/\mu^2) \tag{6}$$

where

$$\Delta g_0 = a_0 - (1/3)\ln(\bar{\lambda}R^2)\,,$$
$$\Delta g_\lambda = a_\lambda - (1/3)\ln(\bar{\lambda}R^2) + \lambda R^2[\ln(\lambda R^2) - 1] + 2\lambda R^2 \gamma + S_1(\lambda R^2)$$

with

$$a_0 = -4\zeta(-1) + 1/2,$$
$$a_\lambda = -\gamma/2 + 2\zeta(-1) - 1/4 + (1/3)\ln\sqrt{2}\,\zeta(-1) \approx -0.16542114,$$
$$\gamma \approx 0.57721566$$

and

$$S_1(x^2) = \sum_{l=0}^{\infty} \{(2l+1)[\ln[(l+1/2)^2 + x^2] - \ln(l+1/2)^2] - 2x^2/(l+1)\}\,. \tag{7}$$

The ultraviolet cutoff term $-1/3\ln[\bar{\lambda}/\mu^2]$ removed by a counter term of the Gaussian curvature type $(1/2\bar{\alpha})\int d^2\xi\sqrt{g}R$ so that $a_0 - (1/3)\ln(\bar{\lambda}/\mu^2)$ can be considered as a finite constant, to be set to zero since it is irrelevant to our discussion.

Notice that g_{tot} has two terms $-(1/3)\ln(R^2\mu^2)$, one from $\operatorname{tr}\ln(-D^2)$ and one from $\operatorname{tr}\ln(-D^2 + \lambda)$. If both were to survive the extremization process of large R, this would lead to a doubling of the power of R^2 in the entropy. This is, however, not so, the reason being that λ approaches its $R = \infty$ limit $\bar{\lambda}_v$ with corrections of order $1/\bar{\lambda}R^2$ only. In order to see this, we note that in the limit of large R, Δg_λ has the asymptotic behaviour $(1/3)\log(\lambda/\mu^2)$ and g_{tot} becomes

$$g_{\text{tot}} \to R^2\bar{\lambda}_v v + (4\pi/\tilde{\alpha}\rho)(2 + \lambda R^2) - (1/3)\ln(R^2\bar{\lambda})$$
$$- \lambda R^2 \ln(\lambda/\bar{\lambda}) + (1/3)\ln(\lambda/\mu^2)\,. \tag{8}$$

The extremality conditions

$$4\pi/\tilde{\alpha}\rho = (\lambda R^2/(2 + \lambda R^2))[\bar{\lambda}_\nu \nu/\lambda + 1 - 2/3\lambda R^2] \tag{9}$$

$$\lambda R^2(\bar{\lambda}_\nu \nu/\lambda + 1) - 2/3 + 2/\lambda R^2 + (2 + \lambda R^2)[-\ln(\lambda/\bar{\lambda}) - 1$$
$$+ \ln(\lambda R^2) + 2\gamma + S_1'(x^2)] \tag{10}$$

give, in this limit, $4\pi/\tilde{\alpha}\rho \to 1 + \nu - (6 + 5\nu)/3(1 + \nu)\lambda R^2$, $\lambda/\bar{\lambda}_\nu - 1 \to 1 - (2 + 1/3(1 + \nu))/\lambda R^2$. This shows that the large R behaviour of λ has only $1/\lambda R^2$ corrections. The leading R behaviour in (8) comes entirely from the tension plus the fluctuation correction $-(1/3)\ln(\bar{\lambda}R^2)$ due to the $\lambda = 0$ part of the trace log. The $-(1/3)\ln(R^2/\mu^2)$ term of the massive part Δg_λ disappears at large R just as it would if λ was an R-independent mass. The normality ν is irrelevant to this result. The entropy factor $(R^2)^{d/6}$ has therefore the same universal power for all spontaneous strings, with any admixture of Nambu-Goto tension.

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