# DYNAMICAL GENERATION OF STRING TENSION *AND* STIFFNESS IN STRINGS AND MEMBRANES ☆

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Simple field theoretic models are presented in which a surface acquires an extrinsic curvature stiffness dynamically. This, in turn, leads to a spontaneous generation of tension with all the attractive features of spontaneous strings (glueballs, thermal deconfinement, quark potential with good long- and short-distance behaviour). The simplest model mimics the generation of curvature stiffness in oil–water interfaces by soap molecules and is therefore of potential use in the theory of microemulsions. Another accounts for the emission and reabsorption of thin color electric flux lines, into extrinsic space, from a world-sheet of a string thereby producing bending stiffness.

Surfaces with curvature stiffness play an important role in the physics of living cells, bilipid vesicles, microemulsions, and liquid crystals [1]. Recently [2,3] they have been proposed to be a good approximation to the color-electric flux sheets spanned by the quark loops of quantum chromodynamics. At a fixed time, such a sheet forms a new type of string in which part of the tension is generated by surface fluctuations [4] (entropic or spontaneous tension due to undulation). It is even possible to generate *all* the tension from fluctuations alone, in which case we speak of a "spontaneous string". Such strings have a realistic quark potential [5–7], even at shorter distances [7]. In particular, the short distance $1/R$ term comes out to be about twice as big as the Lüscher term at large distance [7,8], just as required by analyses of heavy meson spectra [9].

If $x^a(\xi)$ $[\xi=(\xi^1, \xi^2), a=1, 2, ..., D]$ is a parametrization of the surface, the curvature action may be written as

$$\mathcal{A}=\frac{1}{2\alpha}\int d^2\sqrt{g}\,(D^2x^a)\,, \tag{1}$$

where the $D_i$ are the covariant derivatives in the induced metric $g_{ij}=\partial_i x^a\partial_j x^a$. While the statistical mechanics of such an action is well-defined, the quantum theory of a string in spacetime has immediate trouble with unitarity, as first pointed out in ref. [3] and stressed further by others. The presence of four derivatives in $\mathcal{A}$ induces ghosts (states with negative norm). Also the ordinary Nambu–Goto string, with tension only, has ghosts, but these can at least be avoided by working in $D=26$ dimensions, while those coming from $D^4$ cannot. It is the purpose of this note to present the simplest possible action that possesses all the virtues of the spontaneous string but does not share the most obvious unitarity problems of the higher derivatives. At the simplest level, the action involves an antisymmetric tensor field $\varphi_{ab}$ $(a=1, ..., D)$ coupled linearly to the normals of a random surface. For simplicity, let us look only at $D=3$ where there is a single normal vector $N^a=\epsilon^{abc}(1/2\sqrt{g})\epsilon^{ij}\partial_i x^b\partial_j x^c$ and we can work with a vector field $\varphi_a=\epsilon_{abc}\varphi_{bc}$. Then the action reads

$$\mathcal{A}_0=\int d^3x\,\tfrac{1}{2}[(\partial\varphi_a)^2+m^2\varphi_a^2]+M^2\int D^2\xi\,\sqrt{g}-\gamma\int d^2\zeta\sqrt{g}\,N^a\varphi_a\,. \tag{2}$$

It simulates the effect of soap molecules on an oil–water interface which, by itself, would have an initial surface

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tension $M^2$. Away from the interface, $\varphi_a$ fluctuates around zero. Along the interface, $\varphi_a$ has a non-zero average parallel to $N^a$. It describes the accumulation of the soap molecules on the interface and their preference of the normal orientation. We can integrate out the soap field $\varphi_a$ and find the interaction between surface elements

$$\mathscr{A}_{\rm int} = -\tfrac{1}{2}\gamma^2 \int d^2\xi \sqrt{g} \int d^2\xi' \sqrt{g'}\, N^a(\xi) G_m[x(\xi) - (\xi')] N^a(\xi') , \tag{3}$$

where $G_m(x)$ is the Yukawa potential

$$G_m(x) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{ikx}}{k^2+m^2} = \frac{e^{-mr}}{4\pi r} . \tag{4}$$

To evaluate this further we use the Gauss parametrization of the surface $x^1=\xi^1$, $x^2=\xi^2$, $x^3=u(\xi)$, and write

$$G_m = \int \frac{d^2k}{(2\pi)^2} \int \frac{dk_3}{(2\pi)} \exp\{ik_3[u(\xi) - u(\xi')]\} \frac{e^{ik\xi}}{k_3^2+k^2+m^2} . \tag{5}$$

The first exponential plays a similar role in undulation physics as the Debye–Walker factor does in phonon physics. The undulations soften the correlations. The general evaluation of $G_m$ does not give much insight. It is more instructive to imagine $m$ to be large and $\Delta u = u(\xi) - u(\xi')$ to be small and smooth. Then we can find an expansion of $\mathscr{A}_{\rm int}$ into local higher derivative terms:

$$G_m = \left[\frac{1}{2m}\left(1 - \frac{\partial^2}{2m^2}\right) + (\Delta u)^2 \tfrac{1}{4} m \left(1 - \frac{\partial^2}{2m^2} - \frac{\partial^4}{8m^4}\right)\right.$$
$$\left. + (\Delta u)^4 \tfrac{1}{48} m^3 \left(1 - \frac{3\partial^2}{2m^2} + \frac{3\partial^4}{8m^4} + \frac{\partial^6}{6m^6}\right) + \dots\right] \delta^{(2)}(\xi - \xi') . \tag{6}$$

Choosing $\xi'=0$ and expanding $\Delta u = u_i\xi_i + \tfrac{1}{2}u_{ij}\xi_i\xi_j + \dots$ and $\sqrt{g} = 1 + \tfrac{1}{2}u_l^2 + u_l u_{lk}\xi_k + \tfrac{1}{2}u_{lk}u_{li}\xi_k\xi_i - \tfrac{1}{8}u_l^2 u_m^2 - \tfrac{1}{2}u_l^2 u_i u_{jk}\xi_k + \dots$, we can rewrite $\mathscr{A}_{\rm int}$ after some tedious algebra in the following simple covariant form:

$$\mathscr{A}_{\rm int} = \tfrac{1}{2}\gamma^2 \int d^2\xi \sqrt{g} \left( -\frac{1}{2m} N^2 - \frac{1}{4m^3} N D^2 N - \frac{1}{16m^3} (C_i^j)^2 \right) + \dots , \tag{7}$$

where the $C_i^j$ is the extrinsic curvature matrix, defined by $D_i N^a = -C_i^j \partial_j x^a$. In the gaussian map it becomes $C_i^j \equiv -\partial_i N^j = \partial_i (u_j/\sqrt{1+u_l^2})$. The second term in (7) can be partially integrated to $(\gamma^2/8m^3)\int d^2\xi \sqrt{g}\, (C_i^j)^2$. Using, furthermore, the fact that the scalar gaussian curvature $R = C^2 - C_i^j C_j^i$ ($C \equiv C_i^i$) is a pure surface term, the action reads up to the order $1/m^2$

$$\mathscr{A}_{\rm int} = -\frac{\gamma^2}{4m} \int d^2\xi \sqrt{g} + \frac{3\gamma^2}{32m^3} \int d^2\xi \sqrt{g}\, (C)^2 + \dots . \tag{8}$$

The first term gives a negative contribution to the surface tension. It is a well-known experimental property of soap layers, exploited in the ordinary washing process, to reduce tension.

The second term in (8) coincides with the curvature action (1) of the spontaneous string. The curvature stiffness is now dynamically generated and given by action (1) with the inverse stiffness constant

$$\alpha = \frac{16m^3}{3\gamma^2} . \tag{9}$$

If we would take the limit of large $m$ and $\gamma$, at fixed $\alpha$, the string physics following from the new action (2) would be precisely the same as that of the spontaneous strings [2–8]. For finite $m^2$, $\gamma$, however, the new action no longer possesses the obvious $D^4$ ghosts of the action (1).

The dynamically generated curvature decreases rapidly for increasing bending. It is easy to evaluate $\mathscr{A}_{\rm int}$ of (3) exactly for a sphere or a cylinder of radius $a$. The result is $\mathscr{A}_{\rm int} = \tfrac{1}{2}\gamma^2 \cdot \text{Area} \cdot h$, where $h$ are the integrals

$$h_s = -a^2 \int_0^{\pi} d\varphi \, d\theta \sin\theta \cos\theta \frac{e^{-mr}}{4\pi r}, \quad h_c = -2a \int_0^{\pi} d\theta \cos\theta \frac{k_0}{2\pi}, \tag{10}$$

with $r=2ma\sin(\frac{1}{2}\theta)$, which give

$$h_s = -\frac{1}{2m}\left(1-\frac{1}{m^2 a}\right)(1-e^{-2ma}), \quad h_c = -aI_1(ma)K_1(ma), \tag{11}$$

where $I_1$, $K_1$ are modified Bessel functions. Up to order $1/a^2$ we expand $h_s \approx -1/2m + C^2/8m^3$, $h_c \equiv -1/2m + 3C^2/16m^3$. These are in accordance with (8) and determine, in addition, the pure surface term $\propto R$ of the action

$$\mathscr{A}_{\text{int,sf}} = \frac{\gamma^2}{16m^3} \int d^2\xi \sqrt{g} R. \tag{12}$$

When plotted as a function of $1/R$, both bending energies show rapid softening, most extremely in the cylindrical case (see fig. 1). This is the origin of microemulsions with cylindrical surface structures.

In microemulsions, the energy per area element of cylinders of small radii is usually larger than that of small spheres. In order to account for this we need another parameter which allows assigning the gaussian and mean curvature energies in (8) and (12) independently. This is possible by taking into account, in (2), the existence of a second independent gradient term for the $\varphi_a$ field, $\frac{1}{2}\lambda(\partial_a\varphi_a)^2$. Then there is a second interaction energy in (3),

$$\mathscr{A}'_{\text{int}} = -\frac{\lambda\gamma^2}{2(1+\lambda)m^2} \int d^2\xi \sqrt{g} \int d^2\xi' \sqrt{g'}\, \partial_a N^a(\xi) \{G_m[x(\xi)-x(\xi')] - (m=0)\} \partial_a N^a(\xi'). \tag{13}$$

Since $(\partial_a N^a)^2 = C^2$, this contributes only to the mean curvature energy so that the parameter $\lambda$ indeed provides the desired degree of freedom.

Also a scalar field $\varphi(x)$ coupled locally to the surface generates curvature stiffness, Albeit much less effectively. Dropping in the action (2) the vector index $a$ and in the coupling term the normal vector $N^a$, we can

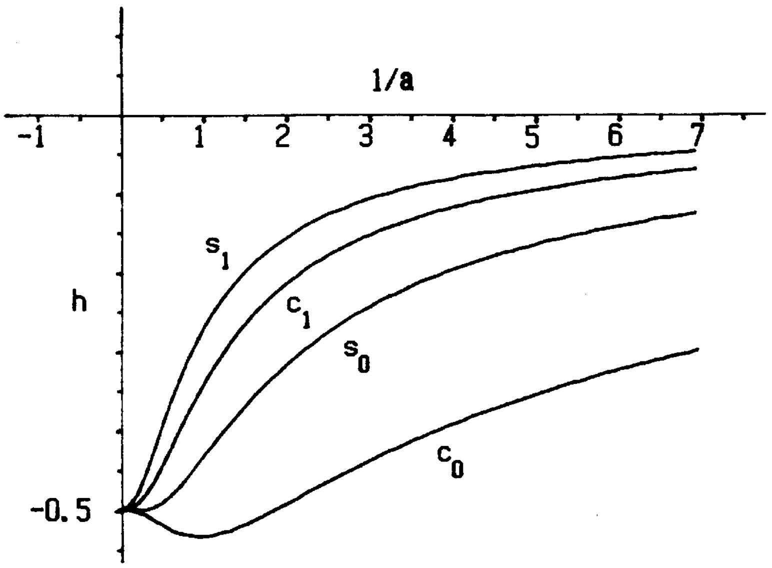


Fig. 1. The dynamically generated curvature energy as a function of the inverse radius $1/a$ for a sphere (s) where $C=2/a$, $R=2a^2$ and a cylinder (c) where $C=1/a$, $R=0$. The curves $s_1$, $c_1$ and $s_0$, $c_0$ are generated by a vector and a scalar field, respectively.

calculate, for spheres and cylinders, the energy density from integrals like (10), but with cos $\theta$ absent. The result is

$$h_{s0} = -\frac{1}{2m}(1-e^{-2ma}), \quad h_{c0} = -aI_0(ma)K_0(ma) . \tag{14}$$

Fig. 1 shows these curves, which indicate a resistance to bending only at larger curvature. The spherical energy does not have any term quadratic in $1/a$. Moreover, the cylinder has initially a negative curvature stiffness, $h_{c0} \approx -(1/2m)(+\frac{1}{8}C^2 + \frac{27}{128}C^4 + ...)$. This implies that the general curvature energy has an expansion $h_{c0} \approx -(1/2m)[1+\frac{1}{8}(C^2-2R^2)+...)$. An interesting feature of this model is that the cylindrical energy has a minimum at a non-zero curvature $1/a \approx 1$. the surface will therefore prefer to be crumpled, similar to the QCD string in the weak coupling regime, behind the roughening transition.

The physics can be enriched by allowing for a self-interaction of the $\varphi$ or $\varphi_a$ fields, say of the $\varphi^4$ type, $\mathcal{A}_{\varphi^4} = \frac{1}{4}g\int d^3x\,(\varphi_a^2)^2$. It parametrizes the local repulsion between soap molecules. There could also be an extra $\varphi^4$ term localized at the interface. In the presence of such an interaction, we are able to describe the process of condensation of soap molecules on the interface by assuming $m^2$ to become negative at the interface, with an extra action $\mathcal{A}_\varphi^2 = -\frac{1}{2}\int d^2\xi\,\sqrt{g}\,m_0^2\phi_a^2$. Then the $O(D)$ symmetry is spontaneously broken at the mean field level, and the model reduces effectively to a non-linear $O(N)$ $\sigma$-model on the surface

$$\mathcal{A} \equiv \int d^2\xi\,\sqrt{g}\,\{(Dn^a)^2 + \lambda[(n^a)^2-1] + M^2 - \gamma N^a n^a\} , \tag{15}$$

where $n^a \equiv \varphi^a/|\varphi|$. This model shows a dynamic generation of curvature stiffness with $\langle\lambda\rangle \neq 0$ playing a similar role as previously $m^2$.

The question remains which of these models could apply to QCD strings. For this we observe that color-electric flux is not completely confined to the string but reaches out into the vacuum up to a distance $1/\Lambda \sim 1/300$ MeV. This is relevant in nuclear matter of ultra-high densities, where strings move so close to each other that they almost merge. Infinitely thin color flux tubes will be emitted from the surface and reabsorbed after travelling an average distance $1/\Lambda$. Under the approximation of a random path between creation and annihilation, we can represent the ensemble of such paths by a fluctuating scalar *disorder field* [10], whose Feynman diagrams are the direct topological pictures of the random lines. Such a field would have precisely the properties of the above scalar $\phi(x)$.

Hopefully, we have convinced the reader that a more detailed investigation of the actions (2), (2) with $\phi_a$ replaced by a scalar field and $N^a$ by 1, and (15) will be a useful enterprise with a wide range of applications.

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