TWO-LOOP STRING TENSION OF STIFF STRINGS AT FINITE TEMPERATURE IN ANY DIMENSION ☆

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We give a perturbative calculation of the string tension at finite temperature for a string with extrinsic curvature stiffness. This is a two-loop result valid for any dimension of the embedding space. We also discuss the possible existence of a singularity in the string tension as suggested by a previous large-d calculation.

Recently, there has been a proposal to study strings with curvature stiffness [1,2], i.e. to consider the effects of an extrinsic curvature energy in the world sheet of the string embedded in a higher d-dimensional space–time. Among the most interesting properties of this model is its capability of explaining some important features of QCD such as the quark potential. This has recently been studied up to the two-loop level in any dimension [3], thus generalizing previous results obtained in a large-d approximation [4].

A closely related problem is the string tension at finite temperature. The motivation for studying strings at finite temperature is again provided by QCD. In a thermal environment, a phase transition occurs from the confined hadronic phase to a deconfined quark–gluon plasma when the temperature exceeds a certain value T_d. In string models this phenomenon was studied for the Nambu–Goto case a few years ago [5]. In the more realistic model of strings with extrinsic curvature the full nonperturbative problem has been solved by Kleinert in a large-d calculation [6]. There exists also a later two-loop calculation in the large-d limit [7], but this paper contains several mistakes as has been already commented upon [8].

The purpose of this note is to give a derivation of the two-loop string tension at finite temperature for any dimension. This work relies heavily on the previous paper by the authors on the quark potential [3] which the reader will frequently be referred to. The main difference with respect to ref. [3] is a simpler boundary condition. In a thermal environment, there are periodic boundary conditions in the "time" direction

$$x^\mu(r,t)=x^\mu(r,t+1/T)\ . \tag{1}$$

Working at infinite size in the space direction its boundary conditions are irrelevant.

The action for a string with rigidity is [1,2]

$$A=M_0^2\int d^2\xi\sqrt{g}+\frac{1}{2\alpha_0}\int d^2\xi\sqrt{g}\,(D^2x^\mu)^2\ , \tag{2}$$

where we use the Gauss map to parametrize the surface

$$x^\mu(\xi)=(\xi^0,\xi^1,x^2,\dots,x^{d-1})=(\xi^k,\boldsymbol{u})\ , \tag{3}$$

where $k=0$, 1 and $\boldsymbol{u}$ is a vector with $d-2$ transverse components. In terms of the $\boldsymbol{u}$-fields our action has the expansion ($\bar{M}_0^2\equiv\alpha_0 M_0^2$)

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$$A=\int d^2\xi\{M_0^2+(1/2\alpha_0)(\boldsymbol{u}_{ii}^2+\bar{M}_0^2\boldsymbol{u}_i^2)$$
$$+(M_0^2/8\bar{M}_0^2)[\bar{M}_0^2\boldsymbol{u}_i^4-2\bar{M}_0^2(\boldsymbol{u}_i\cdot\boldsymbol{u}_j)^2+2\boldsymbol{u}_{ii}^2\boldsymbol{u}_j^2-4(\boldsymbol{u}_i\cdot\boldsymbol{u}_{jj})^2-8(\boldsymbol{u}_i\cdot\boldsymbol{u}_j)(\boldsymbol{u}_{ij}\cdot\boldsymbol{u}_{kk})+O(u^6)+\ldots]\}\,. \quad (4)$$

The correlation functions are

$$\langle u^a(\boldsymbol{\xi})u^b(\boldsymbol{\xi}')\rangle_F=\delta^{ab}\frac{\alpha_0}{\beta}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dk_1}{2\pi}\frac{\exp[i\omega(t-t')+ik_1(r-r')]}{k^2(k^2+\bar{M}_0^2)}\,, \quad (5)$$

where $\omega\equiv k_0$ is summed over the discrete values $\omega_m=2\pi m/\beta$ with $\beta=1/T$ is the inverse temperature. Using the same methods as in ref. [3], we find

$$\langle u_iu_i\rangle_F^2=(\alpha_0/16\pi)^2(8S_2+4\bar{L}_0+4/\sqrt{\lambda_{0T}}^2)\,, \quad (6a)$$
$$\langle u_iu_j\rangle_F^2=(\alpha_0/16\pi)^2[(-8S_1+8S_2+2+2L_0)^2+(8S_1-2+2L_0+4/\sqrt{\lambda_{0T}})]\,, \quad (6b)$$
$$\langle u_{ii}u_{jj}\rangle_F=-\bar{M}_0^2\langle u_iu_i\rangle_F\,, \quad (6c)$$
$$\langle u_{ij}u_{kk}\rangle_F\langle u_iu_j\rangle_F=-\bar{M}_0^2\langle u_iu_j\rangle_F^2-\bar{M}_0^2(\alpha_0/16\pi)^2[-\tfrac{16}{3}(-\tfrac{1}{2}+2S_1-S_2)/\lambda_{0T}-8/3\lambda_{0T}^{3/2}]\,, \quad (6d)$$

where

$$\lambda_{0T}\equiv\bar{M}_0^2/4\pi^2T^2\,,$$
$$\bar{L}_0\equiv4\pi L_0+\ln[\lambda_{0T}/4\exp(-2\gamma)]\,,$$
$$L_0\equiv\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2+\bar{M}_0^2}=(1/4\pi)\ln(\Lambda^2/\bar{M}_0^2)\,,$$

Λ being an ultraviolet cutoff and S_1, S_2 are the following sums:

$$S_1=\frac{1}{\lambda_{0T}}\sum_{m=1}^{\infty}(\sqrt{m^2+\lambda_{0T}}-m-\lambda_{0T}/2m)\,, \quad (7a)$$

$$S_2=\sum_{m=1}^{\infty}(1/\sqrt{m^2+\lambda_{0T}}-1/m)\,. \quad (7b)$$

Inserting this into (4) we find the string tension at finite temperature up to two loops

$$\beta M^2(\beta)=M_0^2\beta+\bar{\alpha}M_0^2\beta(8S_1+2\bar{L}_0+4/\sqrt{\lambda_{0T}}-4/3\lambda_{0T})$$
$$+\bar{\alpha}^2M_0^2\beta\{-\tfrac{16}{3}(1+2S_1-S_2)/\lambda_{0T}+6-48S_1+24S_2+96S_1^2-96S_1S_2+40S_2^2+16S_2\bar{L}_0+4\bar{L}_0^2$$
$$+(48S_1-8S_2-12+8\bar{L}_0)/\sqrt{\lambda_{0T}}+18/\lambda_{0T}-8/3\lambda_{0T}^{3/2}$$
$$+(d-2)^{-1}[16S_2^2+16S_2\bar{L}_0+4\bar{L}_0^2+8(\bar{L}_0+2S_2)/\sqrt{\lambda_{0T}}+4/\lambda_{0T}]\}\,. \quad (8)$$

Contact with our previous result [8] can be made as in the finite-R case: After a finite renormalization (see e.g., eq. (3.15) of ref. [3]) we get

$$\bar{\alpha}_0S_1(\lambda_{0T})=\lambda S_1(\Lambda^2)+\lambda^2S_2(\Lambda^2)+O(1/d,\lambda^3)\,, \quad (9a)$$
$$\bar{\alpha}_0\bar{L}_0=\lambda L+2\lambda^2L+O(1/d,\lambda^3)\,, \quad (9b)$$
$$\bar{\alpha}_0/\sqrt{\lambda_{0T}}=\lambda/\Lambda+\lambda^2/\Lambda+O(1/d,\lambda^3)\,, \quad (9c)$$

and the corresponding expression of ref. [8] is obtained, after taking the large-d limit.

The renormalized two-loop string tension and one-loop coupling constant are

$$M^2 = M_0^2[1+2\bar{\alpha}_0(1-\bar{l}_0)+4\bar{\alpha}_0^2\bar{l}_0^2+(d-2)^{-1}4\bar{\alpha}_0^2\bar{l}_0^2] , \qquad (10)$$

$$1/\alpha = (1/\alpha_0)(1-\tfrac{1}{2}d\alpha_0 L_0) , \qquad (11)$$

where, as before

$$\bar{\alpha}_0 \equiv (d-2)\alpha_0/16\pi , \quad \bar{l}_0 \equiv -4\pi L_0 . \qquad (12a,b)$$

Inverting eqs. (9) and (10) we get

$$M_0^2 = M^2[1-2\bar{\alpha}(1-\bar{l})-4\bar{\alpha}^2\bar{l}(1-\bar{l})+(d-2)^{-1}4\bar{\alpha}^2\bar{l}^2+\mathrm{O}(\bar{\alpha}^3)] , \qquad (13a)$$

$$\bar{\alpha}_0 = \bar{\alpha}[1+2\bar{\alpha}\bar{l}+(d-2)^{-1}4\bar{\alpha}\bar{l}+\mathrm{O}(\bar{\alpha}^2)] . \qquad (13b)$$

Thus the renormalized two-loop finite temperature string tension is

$$\begin{aligned} M^2(\beta) = {} & M^2+\bar{\alpha}M^2[8S_1+2(L_T-1)+4/\sqrt{\lambda_T}-4/3\lambda_T] \\ & + \bar{\alpha}^2M^2\{-\tfrac{16}{3}(1+2S_1-S_2)/\lambda_T+6-48S_1+16S_2+96S_1^2-96S_1S_2+40S_2^2+16S_2L_T-4L_T+4L_T^2 \\ & + (48S_1-8S_2-16+8L_T)/\sqrt{\lambda_T}+18/\lambda_T-8/3\lambda_T^{3/2} \\ & + (d-2)^{-1}[16S_2^2+16S_2L_T+4L_T^2+(8L_T+16S_2)/\sqrt{\lambda_T}+4/\lambda_T]\} . \end{aligned} \qquad (14)$$

Defining

$$\tilde{M}^2 = (d-2)^{-2}2M^2 , \quad \tilde{\alpha} = \tfrac{1}{2}(d-2)\alpha \qquad (15a,b)$$

we can construct a dimensionless function of $\tilde{M}\beta$ by introducing the following quantity:

$$\hat{M}^2(\beta) \equiv (d-2)^{-1}(2/\tilde{M}^2)M^2(\tilde{M},\beta) . \qquad (16)$$

After normalizing $\tilde{M}$ to unity we study numerically eq. (16) for the string tension. The function $\hat{M}^2(\beta)$ is displayed in fig. 1.

We now study the asymptotic expansions of eq. (16). For the large-β limit we use a convenient representation of the sums $S_{1,2}$ in terms of modified Bessel functions [3]

$$S_1 = \tfrac{1}{4}-\tfrac{1}{4}L_T-\frac{1}{2}\frac{1}{\sqrt{\lambda_T}}+\frac{1}{12}\frac{1}{\lambda_T}-\frac{1}{\pi}\frac{1}{\sqrt{\lambda_T}}\sum_{\tilde{m}=1}^{\infty}\frac{K_1(2\pi\tilde{m}\sqrt{\lambda_T})}{\tilde{m}} , \qquad (17a)$$

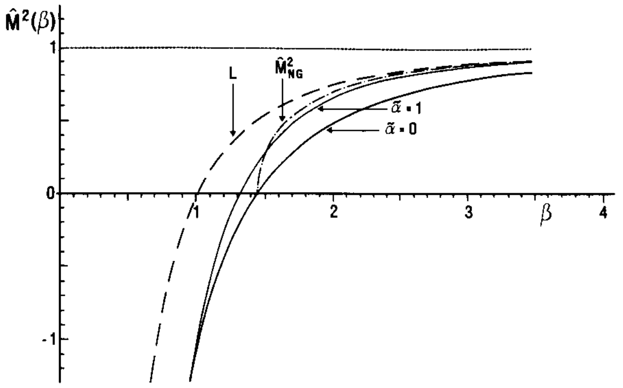

Fig. 1. The string tension $\hat{M}^2(\beta)$ as defined by eq. (16) is shown as a function of the inverse temperature β. The solid lines correspond to our tension eq. (16) for two values of the coupling constant. We also show the Nambu–Goto solution (dash-dotted line) given by eq. (19) as well as the first two terms of eq. (18) (dashed line).

$$S_2 = -\tfrac{1}{2}L_T - \frac{1}{2}\frac{1}{\sqrt{\lambda_T}} + 2\sum_{\tilde{m}=1}^{\infty} K_0(2\pi\tilde{m}\sqrt{\lambda_T}) \,. \tag{17b}$$

Thus the large-β limit of $\hat{M}^2(\beta)$ is given by

$$\hat{M}^2 = 1 - \tfrac{1}{3}\pi/\beta^2 - \tfrac{1}{2}(\tfrac{1}{3}\pi)^2/\beta^4 + O(\exp(-\beta)) \,, \tag{18}$$

where the last term accounts for the exponentially small corrections coming from the large-β expansion of the Bessel functions introduced by eqs. (17). These powers agree with two-loop terms in the power series of the exactly known Nambu–Goto result

$$\hat{M}^2(\beta) = \sqrt{1 - (\beta_d/\beta)^2} \,. \tag{19}$$

It was pointed out in the exact solution [6] for $M^2(T)$ of spontaneous strings at large d, that also there the power-series expansion agrees with that of the Nambu–Goto tension. Thus we see that, at the two-loop level, finite-d corrections have no extra effects to the infinite-d results. They only influence the renormalization of the zero-temperature string tension and drop out after going to the renormalized expression. In particular, eq. (18) shows that the $1/\beta^2$-term, which corresponds to Lüscher's universal $1/R$-term in the quark potential, receives no corrections at two loops. For small β we use the formulae [3]

$$S_1 = -\tfrac{1}{8}\zeta(3)\lambda_T + \tfrac{1}{16}\zeta(5)\lambda_T^2 - \tfrac{5}{128}\zeta(7)\lambda_T^3 + \dots \,, \tag{20a}$$

$$S_2 = -\tfrac{1}{2}\zeta(3)\lambda_T + \tfrac{3}{8}\zeta(5)\lambda_T^2 - \tfrac{5}{16}\zeta(7)\lambda_T^3 + \dots \,. \tag{20b}$$

Thus, the normalized string tension at high temperature is given by

$$\hat{M}^2(\beta) = -\sqrt{\tilde{\alpha}}\left(\frac{\pi}{3}\right)\frac{1}{\beta^3} - \frac{2\pi}{3}\left(1 - \frac{\tilde{\alpha}}{2\pi}\frac{19d-32}{8d-16}\right)\frac{1}{\beta^2} + \sqrt{\tilde{\alpha}}\left[1 + \frac{\tilde{\alpha}}{4\pi}\left(-2 + \frac{d-1}{d-2}L_T\right)\right]\frac{1}{\beta}$$
$$+ \left\{1 - \frac{\tilde{\alpha}}{4\pi}\left[1 - L_T - \frac{\tilde{\alpha}}{4\pi}\left(\frac{3}{2} - \frac{\zeta(3)}{2} - L_T + \frac{d-1}{d-2}L_T^2\right)\right]\right\} - \frac{\tilde{\alpha}^2\zeta(3)}{32\pi^3}\beta^2 \,. \tag{21}$$

Finally, we want to discuss the important issue of the singularity of the string tension and its consequences on the validity of any asymptotic approximation to eq. (16). In the pure Nambu–Goto case the string tension has a square root singularity (see eq. (19) and fig. 1), the solution becoming imaginary for values of T bigger than a certain value T_d. In the purely spontaneous string [6] there is an analytic approximation to the exact result which in fact reduces to the Nambu–Goto solution [9]. In the limit of low temperature. The exact numerical result shows the very same pattern and there is a singularity at the transition point [6]. Note that a loop expansion like the one presented here is unable to locate the singularity. The existence of such a singularity in the exact solution would strongly suggest a similar singularity in the not exactly soluble problem of the static quark–antiquark potential.

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