STIFFNESS DEPENDENCE OF DECONFINEMENT TRANSITION FOR STRINGS WITH EXTRINSIC CURVATURE TERM ☆

G. GERMÁN and H. KLEINERT
Institut für Theorie der Elementarteilchen, Freie Universität Berlin, Arnimallee 14, 1000 Berlin 33, Germany

Received 23 March 1989

We determine the stiffness dependence of the deconfinement temperature for a string with an extrinsic curvature term in the limit of infinite dimension d, where the system can be treated exactly. We also discuss the importance of the square-root singularity of the model in relation to previous perturbative calculations. This singularity invalidates perturbative results for the string tension at moderately high temperatures where the deconfinement takes place (and, by analogy, for the potential at short distances).

Recently there has been considerable interest in studying a model of strings with curvature stiffness proposed independently by Polyakov [1] and by one of the authors [2]. Such a string is apparently a good candidate for the string between quarks in QCD. To compare the two objects we must compare physically observable quantities which can be calculated in both cases. A good candidate is the temperature dependence of the string tension and, in particular, the thermal deconfinement temperature where the tension vanishes. The string with extrinsic curvature stiffness is characterized by two parameters, the total string tension M^2 and the stiffness $\tilde{\alpha}^{-1}$ which is a dimensionless number. For dimensional reasons the temperature behaviour of the tension $\hat{M}^2 \equiv M^2(T)/M^2(0)$ is a function of T/M and $\tilde{\alpha}$. The reduced deconfinement temperature $T^{\rm dec}/M$ depends only on $\tilde{\alpha}$ and the dimension in which the string moves. For a string in QCD, on the other hand, the dependence is on the symmetry group and the space dimension (see table 1). If there is to be any match between the two, the stiffness parameter $\tilde{\alpha}$ should contain the information on the symmetry group. This is why it is important to study the $\tilde{\alpha}$ dependence of the string in detail.

Actually, since the theory requires renormalization, one must specify the scale μ in which the renormalized $\tilde{\alpha}(\mu)$ is prescribed. This μ may be chosen to coincide with M. Another possibility to specify $\tilde{\alpha}$ indirectly is via the fraction of the total tension that is generated spontaneously from the curvature fluctuations. We shall use the parameter ν to denote the ratio $M^2_{\rm NG}/M^2_{\rm sp} \equiv$ Nambu–Goto tension/spontaneous tension. The total tension is then

$$M^2 = M^2_{\rm sp}(1+\nu). \tag{1}$$

As of now there exist several studies of the thermal behaviour of $M^2(T)$. First, there is a complete and exact solution for the case $d=\infty$ and $\nu=0$ (no Namby-Goto term, only spontaneous tension) [4]. Also for $\nu \neq 0$ the exact equations have been derived [5] but due to their complexity they were not evaluated explicitly. Further, there are two perturbative calculations in the limit of large stiffness, $\tilde{\alpha} \to 0$ for $d=\infty$ [6] and for any d [7]. Finally, there exists upper and lower bounds to the deconfinement temperature [8] calculated in the same small $\tilde{\alpha}$ limit

$$0.69 \leqslant T^{\rm dec}/M \leqslant 0.98, \tag{2}$$

which we do not agree with as will be explained later in this letter.

It is the purpose of the present letter to give a determination of $T^{\rm dec}$ for $d=\infty$ and all ν using the exact equations and to discuss and compare with previous results and with QCD data (see table 1). Particular attention is devoted to the relevance of the

☆ Work supported in part by Deutsche Forschungsgemeinschaft under grant no. Kl 256.

0370-2693/89/$ 03.50

Table 1
Deconfinement temperature over string tension $T^{\rm dec}/M$, for a stiff string model and comparison with QCD values obtained from Monte Carlo simulation (see e.g. ref. [3]). As of now, there is no direct comparison since the simulations include a grand canonical ensemble of closed strings in the vacuum. In contrast to the single string in our model.

	Theory			
	$SU(2)_3$	$SU(2)$	$SU(3)$	$SU(64)$
Monte Carlo	0.94 ± 0.03	0.74 ± 0.10	0.48 ± 0.05	0.49 ± 0.04
Stiff string model		$0.670\leqslant T^{\rm dec}/M\leqslant 0.691$		

singularity of the model near the deconfinement transition which the perturbative approach misses. The reader interested in the details of the derivation of the $d=\infty$ equations may consult the elaborated presentation given in ref. [5]. All T's appearing in this letter correspond to "extrinsic temperatures", $T_{\rm ext}$, as introduced in ref. [5], defined by the temporal periodicity in the extrinsic space, $x^0\to x^0+\beta$.

The model of strings with extrinsic curvature stiffness has the following action [1,2]:

$$A=M_{\rm NG}^2\int d^2\xi\sqrt{g}+\frac{1}{2\alpha}\int d^2\xi\sqrt{g}\,(Dx^\mu)^2. \qquad (3)$$

As stated above, it will be studied in the large-d approximation. For this we introduce Lagrange multipliers λ_0, λ_1 in the usual way [5] so as to fix $g_{ij}=\rho_i\delta_{ij}$ to be the intrinsic metric. After integrating out the transverse fluctuations the action can be written as

$$A=\tfrac{1}{2}(d-2)R_{\rm ext}\beta_{\rm ext}\sqrt{\rho_0\rho_1}\,f_{\rm tot}^T, \qquad (4)$$

where the energy density $f_{\rm tot}^T$ is given by

$$f_{\rm tot}^T=\tilde{M}_{\rm NG}^2-\frac{\tilde\lambda}{4\pi}\ln\frac{\tilde\lambda}{\bar\lambda}+\Delta f^T(\tilde\lambda,T)+\Delta f^\delta(\tilde\lambda,T,\delta)$$
$$+\frac{1}{2\tilde\alpha}\left(\frac{\tilde\lambda_0}{\rho_0}+\frac{\tilde\lambda_1}{\rho_1}\right), \qquad (5)$$

and

$$\tilde M_{\rm NG}^2\equiv\tfrac{1}{2}(d-2)M_{\rm NG}^2,\quad \tilde\alpha=\tfrac{1}{2}(d-2)\alpha,$$

$$\tilde\lambda\equiv\tfrac{1}{2}(\tilde\lambda_0+\tilde\lambda_1),\quad \delta\equiv\frac{\lambda_1-\lambda_0}{\lambda_1+\lambda_0}.$$

The functions $\Delta f^T(\tilde\lambda,T)$ and $\Delta f^\delta(\tilde\lambda,T,\delta)$ contain the isotropic and anisotropic finite temperature corrections, respectively.

After introducing $\bar\rho=\rho_{0,1}$ $(T=0)$, $\hat\rho_0=\rho_{0,1}/\bar\rho$, the action becomes

$$A=R_{\rm ext}\beta_{\rm ext}M_{\rm tot}^2 \qquad (6)$$

where $M_{\rm tot}^2\equiv M_{\rm tot}^2(T=0)\cdot\hat M_{\rm tot}^2(T)$, with the total tension of the infinite size system

$$M_{\rm tot}^2(T=0)=\frac{d-2}{2}\frac{\bar\lambda_\nu(1+\nu)\bar\rho}{4\pi}, \qquad (6')$$

and the temperature dependence collected in the reduced tension

$$\hat M_{\rm tot}^2(T)=\sqrt{\hat\rho_0\hat\rho_1}\,\frac{4\pi}{\bar\lambda_\nu(1+\nu)}f_{\rm tot}^T \qquad (6'')$$

which is normalized so that $\hat M_{\rm tot}^2(T=0)=1$.

Thus $M_{\rm tot}^2(T=0)$ is the sum of the Nambu–Goto tension and the spontaneously generated string tension [9]. Varying the action with respect to $\hat\rho_0$, $\hat\rho_1$, $\tilde\lambda$, and δ we find the two equations for $\hat\rho_0$, $\hat\rho_1$

$$\hat\rho_0=(1+\nu)(1-\delta)\left(2-\frac{4\pi}{\tilde\lambda}f_{\rm tot}^T+2\frac{\bar\lambda_\nu}{\tilde\lambda}\nu\right)^{-1}, \qquad (7a)$$

$$\hat\rho_1=(1+\nu)(1+\delta)\left(\frac{4\pi}{\tilde\lambda}f_{\rm tot}^T\right)^{-1}. \qquad (7b)$$

Substituting this into $f_{\rm tot}^T$ we obtain $(\lambda_T=\tilde\lambda\rho_0/4\pi T^2)$

$$f_{\rm tot}^T=\frac{\tilde\lambda}{4\pi}\left(2\frac{\bar\lambda_\nu}{\tilde\lambda}\nu-\ln\frac{T^2}{\bar T^2}-\frac{2}{3\lambda_T}+4S_1\right)$$
$$+\frac{\tilde\lambda}{4\pi}\left(\frac{2\sqrt{1+\delta}}{\sqrt{\lambda_T}}\right.$$
$$\left.+\frac{4}{\lambda_T}\sum_{m=1}^{\infty}(\hat A^++\hat A_m^--\sqrt{m^2+\lambda_T}-m)-\delta\right), \qquad (8)$$

where

$$\bar T=\sqrt{\bar\rho\bar\lambda_\nu(1+\nu)/4\pi}\,e^{-\gamma}$$
$$=\sqrt{1/2\pi(d-2)}\,e^\gamma M_{\rm tot}(0),$$

and

$$S_1 = \frac{1}{\lambda_T} \sum_{m=1}^{\infty} (\sqrt{m^2+\lambda_T} - m - \lambda_T/2m), \quad (8')$$

$$\hat{A}_m^{\pm} = \sqrt{m^2+\lambda_T(1+\delta)(1 \pm B_m)/2}, \quad (8'')$$

$$B_m = \sqrt{1+8\delta m^2/[\lambda_T(1+\delta)]^2}. \quad (8''')$$

All quantities of interest are now given in terms of $\tilde{\lambda}$, δ and the parameter ν. The stationary point of the action is given by the two gap equations for λ and δ: The first being

$$\frac{\tilde{\lambda}_\nu}{\tilde{\lambda}} \nu - \ln\frac{T^2}{\tilde{T}^2} + 2S_2 + \frac{\sqrt{1+\delta}}{\sqrt{\lambda_T}} + 2\sum_{m=1}^{\infty} (2\hat{a}_m - 1/\sqrt{m^2+\lambda_T}) - \delta = 0, \quad (9)$$

with

$$\hat{a}_m = \frac{1+\delta}{4}\left[\frac{1}{\hat{A}_m^+} + \frac{1}{\hat{A}_m^-} + \frac{1}{2}\left(B_m + \frac{1}{B_m}\right)\left(\frac{1}{\hat{A}_m^+} - \frac{1}{\hat{A}_m^-}\right)\right]. \quad (9')$$

The second being

$$\frac{1}{(\sqrt{1+\delta}\sqrt{\lambda_T})} + 8\sum_{m=1}^{\infty} \hat{b}_m - 1 = \frac{1}{1-\delta^2}\left[\delta\left(1+\frac{\tilde{\lambda}_\nu}{\tilde{\lambda}}\nu\right) - \frac{4\pi}{\tilde{\lambda}} f_{\rm tot}^T + \frac{\tilde{\lambda}_\nu}{\tilde{\lambda}}\nu + 1\right], \quad (10)$$

with

$$\hat{b}_m = \frac{1}{4\lambda_T(1+\delta)B_m} \times \left[\hat{A}_m^+ - \hat{A}_m^- + m^2\left(\frac{1}{\hat{A}_m^+} - \frac{1}{\hat{A}_m^-}\right)\right]. \quad (10')$$

We solve eqs. (9), (10) numerically. Starting with eq. (9)at $\delta=0$ and $\tilde{\lambda}=\tilde{\lambda}_\nu$ for some fixed ν, the value of $\tilde{\lambda}$ is then fed into eq. (10) to obtain a better approximation for δ. After a few iterations we see that the procedure converges. Then we evaluate the normalized string tension eq. (6″) as a function of T (in units of $M_{\rm tot}^2(0)$) for a range of values of the parameter ν (solid lines in fig. 1). Defining the deconfinement temperature $T^{\rm dec}$ as the value of T for which $\hat{M}^2(T)=0$, we are then able to extract and plot all these values as a function of ν (fig. 2). The parameter ν is related to the stiffness or inverse coupling as follows:

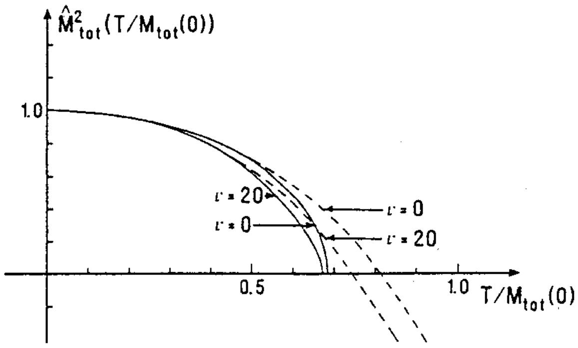

Fig. 1. The normalized string tension as a function of T (in units of $M_{\rm tot}^2(0)$). Solid lines represent plots of the exact solution eq. (6″) described in the text. Dashed lines come from a two-loop calculation of the string tension done by the authors [7]. Note how both approaches differ for T close to $T^{\rm dec}$. In particular, perturbative results miss out on the square-root singularity of $\hat{M}_{\rm tot}^2$ at $T=T^{\rm dec}$ and crosses over the $T^{\rm dec}$ axis. Thus, a determination of $T^{\rm dec}$ from a perturbative calculation is only reliable in the extreme $\nu\to\infty$ limit.

$$\tilde{\alpha} = \frac{4\pi}{1+\nu}. \quad (11)$$

This can be found by studying the infinite size system gap equations [9].

Thus, for very weak and very strong couplings ($\nu\to\infty$, $\nu\to-1$ respectively) we find that the deconfinement temperature has in either case the Nambu–Goto value of $T^{\rm dec}=0.691$. Between the two extremes the curve is lower than that limit (see fig. 2). There is a minimum which constitutes a lower bound, $T^{\rm dec}\geqslant 0.670$. This occurs for $\nu\approx 5$. Thus we set the following limits on $T^{\rm dec}$:

$$0.670 \leqslant T^{\rm dec}/M_{\rm tot}(0) \leqslant 0.691 = T_{\rm NG}^{\rm dec}/M_{\rm tot}(0). \quad (12)$$

Note that these limits for $T^{\rm dec}$ are in disagreement with the earlier ones of eq. (2) given by Xiaoan and Viswanathan [8]. For $\tilde{\alpha}=0$ we find the Nambu-Goto deconfinement $T^{\rm dec}=0.691$ although the string has quite different physical properties, being infinitely stiff. This somewhat surprising feature was observed earlier in a calculation of the string tension [6,7]. For $\tilde{\alpha}=\infty$ which is the point $a=\infty$ of ref. [8] we are back

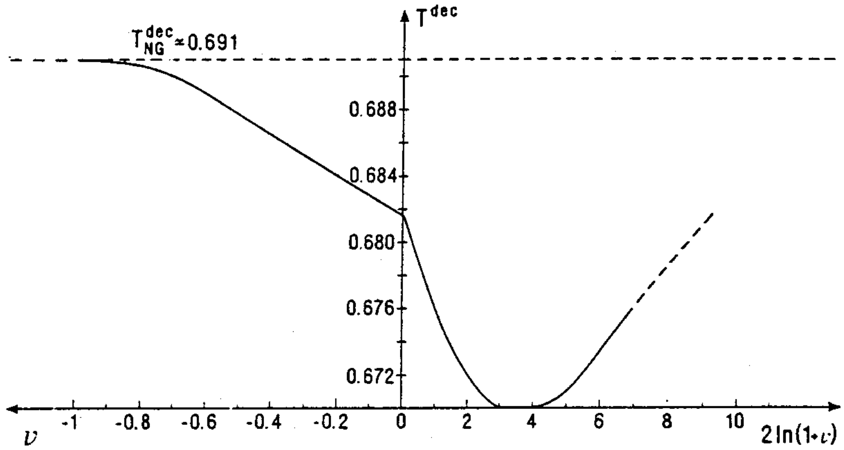

Fig. 2. The deconfinement temperature as a function of ν. Note how the negative and positive section of the "x" axis have different scales. For $\nu \to -1$ and $\nu \to \infty$ the curve approaches the Nambu–Goto value. The large-ν limit was obtained analytically from the equation $\hat{M}^2_{tot} = 0$.

at the Nambu–Goto value while they [8] find a higher deconfinement temperature by a factor of $\sqrt{2}$. The result of ref. [8] must be wrong since, for $\tilde{\alpha} \to \infty$, the effects of the extrinsic curvature term must disappear (see eq. (3)), being left with the old Nambu–Goto model. Moreover, apart from the value of T^{dec} at $\tilde{\alpha} = 0$ a plot of $T^{dec} = 1/\beta_c$ of their eq. (28) would disagree completely with our fig. 2. The reason for this is that their result is obtained from a one-loop approximation. To prove that this is indeed the case we use our previous work on the two-loop string tension (eq. (14) of ref. [7]). We discard the $\tilde{\alpha}^2$ piece and plot the remaining tree + one-loop contribution as a function of T (fig. 3). The plot reproduces their limits [8] (see eq. (2) above). Thus, we observe that a perturbative calculation fails to determine the deconfinement temperature reliably. The reason is that the deconfinement occurs at the singularity, which the perturbative calculation cannot determine. As in other models with a critical point the region beyond the transition require a different approach. In particular the deconfined state will certainly be filled with a grand canonical ensemble of closed strings. In refs. [7,10] we have performed perturbative two-loop calculations of the finite-temperature string tension and the static quark–antiquark potential respectively. Naturally one may wonder about the range of validity of these results. In the case of the finite-temperature string tension we have now a good understanding of the behaviour of the exact solution. In particular we know that this solution has a square-

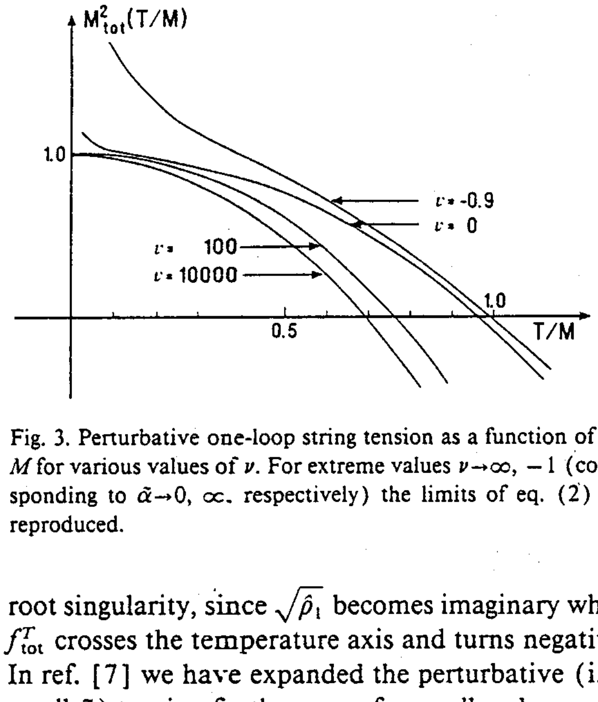

Fig. 3. Perturbative one-loop string tension as a function of T/M for various values of ν. For extreme values $\nu \to \infty$, -1 (corresponding to $\tilde{\alpha} \to 0$, ∞, respectively) the limits of eq. (2) are reproduced.

root singularity, since $\sqrt{\rho_1}$ becomes imaginary when f^T_{tot} crosses the temperature axis and turns negative. In ref. [7] we have expanded the perturbative (i.e., small $\tilde{\alpha}$) tension further, once for small and once for large λ_T ($\lambda_T \equiv \tilde{\lambda}\rho_0/4\pi^2T^2 \approx \tilde{\alpha}\tilde{M}^2/4\pi^2T^2 + ...$). From the present finding we see that of these two the large-λ_T expansion is reliable (see fig. 1) since for $T \to 0$ we are far away from the singularity. The small-λ_T expansion, however, cannot simply be understood as the limit $T \to \infty$ because the perturbative curves miss out on the singularity. For small enough $\tilde{\alpha}$, however, we may trust the small $\tilde{\lambda}_T$ expansion only for a certain intermediate range of T which lies sufficiently below the singularity. We have studied this competition of limits numerically and in fig. 1 we compare the string tension $\hat{M}^2_{tot}(T/M^2_{tot}(0))$ for two values of ν. Solid lines are curves obtained by plotting $\hat{M}^2_{tot}$ as given by eq. (6″). Dashed lines give the string tension from the perturbative two-loop calculation reported by the authors [7].

In analogy with this findings we can expect similar problems to exist for the quark potential of the model. In particular, small distance results are not to be trusted without further studies. Unfortunately, the problem is in that case far more difficult to analyze, even with the help of a computer, since the gaps (Lagrange multipliers) become r-dependent [11]. However, an analogous situation would indicate that perturbative small $\lambda_R(\equiv\tilde{\alpha}M^2R^2/\pi^2+...)$ results (i.e. eq. (5.8) of ref. [10]) are reliable at best at intermediate distances, breaking down for small R. Unfortunately, this casts doubts on one of the more attractive predictions of the model: The $1/R$ dependence of the static quark potential at small R being $-\pi/6R$ [11], which agreed so nicely with the analysis of Eichten et al. [12] ($-0.52/R$).

In conclusion we have determined the deconfinement temperature for stiff strings as a function of the stiffness parameter for $d=\infty$. This has been achieved by a numerical evaluation of the exact solution of the model. Our main results are given by eq. (12) and fig. 2.

References

[1] A.M. Polyakov, Nucl. Phys. B 268 (1986) 406.
[2] H. Kleinert, Phys. Lett. B 174 (1986) 335.
[3] M. Flensburg and C. Petersen, Nucl. Phys. B 283 (1987) 141.
[4] H. Kleinert, Phys. Lett. B 189 (1987) 187.
[5] H. Kleinert, Phys. Rev. D, to appear.
[6] G. Germán and H. Kleinert, Comment on "Effective string tension in the finite-temperature smooth-string model", Berlin preprint (1988).
[7] G. Germán and H. Kleinert, Phys. Lett. B 220 (1989) 133.
[8] Z. Xiaoan and K.S. Viswanathan, Mod. Phys. Lett. A 4 (1989) 99.
[9] H. Kleinert, Phys. Rev. Lett. 58 (1987) 1915.
[10] G. Germán and H. Kleinert, Phys. Rev. D, to appear.
[11] E. Braaten and S.M. Tse, Phys. Rev. D 36 (1987) 3102.
[12] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane and T.M. Yan, Phys. Rev. D 21 (1980)203.