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Temperature behavior and thermal deconfinement for the Nambu-Goto string occupied by Bose or Fermi fields

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We study the string tension at finite temperature for the Nambu-Goto string occupied by a set of massless or massive scalar or spin-$\frac{1}{2}$ Fermi fields. The temperature behavior of the tension is similar in shape to that of the pure Nambu-Goto string, with a square-root singularity indicating the thermal deconfinement transition. The presence of scalar or Fermi fields shifts the deconfinement temperature to lower or higher values, respectively. There is an upper bound on the number of fermions which the string can support, just as in quantum chromodynamics.

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I. INTRODUCTION

In the confined phase of quantum chromodynamics (QCD), quarks are held together by color-electric flux tubes forming hadrons. A phase transition to a deconfined quark-gluon plasma occurs when the temperature of the system is raised to a certain critical value $T^{\rm dec}$. This phenomenon has been studied analytically in a qualitative way [1] and quantitatively via numerous Monte Carlo simulations of lattice models [2] so that it is now reasonably well understood.

In the Nambu-Goto (NG) string model of hadrons [3], the color-electric flux tube is idealized to be an infinitely thin line whose dynamics is governed by the string tension. In this model our understanding of the deconfinement transition is far less satisfactory. One reason is the impossibility, at present, to take into account the many vacuum fluctuations of closed strings which in spacetime form a grand-canonical ensemble of world surfaces of arbitrary topology. Not even Monte Carlo simulations on finite lattices are presently able to do this. This is why, until now, the temperature behavior has been studied only for a single string and the same limitation applies to the present work. Fortunately, also the single string shows a deconfinement transition and it is hoped that this is not far from the transition in an ensemble. As the temperature is raised, a single string fluctuates with increasing amplitude and at the deconfinement temperature $T^{\rm dec}$ the fluctuations become catastrophic. The value of this temperature was calculated to be $T^{\rm dec}/M_{\rm NG}=0.69$ [4], where $M_{\rm NG}^2$ is the zero-temperature string tension. This number was obtained by evaluating exactly the finite-temperature string tension in the limit of large dimension d of spacetime. The deconfinement temperature is proportional to $1/\sqrt{(d-2)}$ [see Eq. (62)]. The large-d expansion had previously been proposed by Lüscher, Symanzik, and Weisz [5] as a calculational tool and used by Alvarez [6] to obtain the distance behavior of the quark potential in the NG model.

While the *ratio* between the deconfinement temperatures of the three- and four-dimensional Monte Carlo simulations is accounted for quite well by the Nambu-Goto string—the simulations show approximately the factor $\sqrt{2}$ in accordance with the $1/\sqrt{d-2}$ factor—the absolute size of the above value is too large in comparison with the Monte Carlo result for an SU(3) gauge theory [9] in four spacetime dimensions which is $T^{\rm dec}/M_{\rm NG}=0.48\pm0.05$ (see Table I). The size is in better agreement with an SU(2) gauge theory where $T^{\rm dec}/M_{\rm NG}=0.74\pm0.10$ (see Table I). Thus, if we assume the effect of the ensemble of closed strings to be small, then either the $1/d$ corrections [10] are large or the NG string is not a good candidate for the description of the behavior of SU(3) QCD close to the transition. Physically, the difference between the color-electric flux tubes formed by the different gauge fields lies obviously in the *internal structure* of the tube. In an SU(n) color gauge theory, the tube is filled with n^2-1 gluon excitations whose mass is zero at the center of the tube and increases to about 600 MeV at the walls [11]. In addition, there are vacuum fluctuations of a sea of colored and flavored quarks and antiquarks with various masses. In an attempt to understand the effect of such additional fluctuations within the string model of the color-electric flux tube we investigate the consequences of populating the NG string by massless or massive scalar or real spin-$\frac{1}{2}$ Fermi fields.

Our starting point is the functional integral over all Euclidean world surface configurations swept out by the string and over all fluctuations of the scalar or Fermi fields. Let $X^\mu(\xi^i)$ ($\mu=1,\ldots,d$; $i=1,2$) describe the points of the world surface in the d-dimensional spacetime parametrized by $\xi=(\xi^0,\xi^1)$. The scalar and Fermi fields living on the string depend only on ξ^i, i.e., $\phi^r=\phi^r(\xi^0,\xi^1)$. Then the string model to be studied is defined by the functional integral

$$Z=\int[\mathcal{D}\phi^r][\mathcal{D}X^\mu]e^{-A}\,, \qquad (1)$$

where A is the action which for N_s real scalar fields is given by

$$A=M_0^2\int d^2\xi\sqrt{g}+\int d^2\xi\sqrt{g}\left[\tfrac{1}{2}(D_i\phi^r)^2+\tfrac{1}{2}m^2(\phi^r)^2\right] \qquad (2)$$

TABLE I. Deconfinement temperature (normalized by the string tension) of the NG and the Polyakov-Kleinert model [7,8] in comparison with values obtained from Monte Carlo simulations of lattice gauge theories without quarks (see, e.g., Ref. [9]). The last column gives the SU(2) gauge theory in three dimensions, for comparison. Note that the ratio with respect to the four-dimensional result is explained by the $1/\sqrt{d-2}$ factor obtained in the NG string model.

Theory	SU(2)	SU(3)	SU(64)	$SU(2)_3$
Monte Carlo	0.74±0.10	0.48±0.05	0.49±0.04	0.94±0.03
Nambu-Goto		0.69		0.98
Stiff string		$0.67 \le T^{\rm dec}/M_{\rm NG} \le 0.69$		$0.95 \le T^{\rm dec}/M_{\rm NG} \le 0.98$

($r=1,\ldots,N_s$) and for spin-$\frac{1}{2}$ Fermi fields by

$$A_f=\int d^2\xi\sqrt{g}\,[M_0^2+\psi^{b\dagger}(\not{D}-im)\psi^b]\ . \tag{3}$$

Here D_i are the covariant derivatives on the world surface. We shall assume that there are N_F complex Fermi field components. Our final result will be able to account for both bosons and fermions and for their simultaneous presence. If they are degenerate in mass it will depend only on $N=N_S-N_F$, the difference between the number of scalar and Fermi field N_S-N_F.

Qualitatively, the results to be obtained in this paper are of quite a general character since they are merely based on counting fluctuation degrees of freedom. Quantitatively, however, we expect changes since the mode-mode interactions have been neglected. For the gluons this may not be too serious since most of the gluon-gluon interactions have been accounted for by the formation of the string which is assumed to exist *a priori* in the model and forms the background of the field fluctuations. For quark loops the mistake is more severe. A quark loop annihilates the string inside it since the quark terminates the color-electric flux. The quarks living on the string do not account for this. Indeed, the results obtained for the quark loops will pose a puzzle which will probably be resolved only after the annihilation effects have been included into the model.

The organization of the paper is as follows. In Sec. II we calculate the free energy of the string in the limit of large d and obtain an expression which still requires extremization with respect to a Lagrange multiplier. This presentation follows closely that given in Ref. [7]. In Secs. III to VII we perform the extremization exactly in certain approximations and various limiting situations. The general case is solved numerically. In Sec. VIII we give a brief discussion of the main aspects of our model and the conclusions. Some details of the calculations are relegated to the Appendixes.

II. FREE ENERGY AND GENERAL GAP EQUATIONS AT FINITE TEMPERATURE

The above actions describe a string evolving in a d-dimensional Euclidean space with N_S scalar fields or N_F Fermi fields which are functions of the two parameters ξ^i ($i=0,1$). We shall proceed first only for the case of N scalar fields. For fermions the result is obtained by simply replacing N by $-N$ in all formulas. In the case of equal masses, the combined system is described setting $N=N_S-N_F$.

In the limit of large d, the functional integral can be done exactly for any N. A large value of N of the order of d is, however, necessary to see an effect of the additional scalar or Fermi particles in the limit. To do this limit, it is convenient to separate the extrinsic configurations of the string from the intrinsic ones by introducing the metric g_{ij} as an independent fluctuation field and using Lagrange multipliers to constrain it to the induced metric on the world sheet:

$$g_{ij}\equiv\partial_i X^\mu \partial_j X^\mu\ . \tag{4}$$

We choose the Gauss parametrization for the world surface in which only the transverse displacements of the string coordinates are dynamical field variables by writing

$$X^\mu(\xi^i)=(\xi^0,\xi^1,X^2,\ldots,X^{d-1})\equiv(r,t,U^a)\ , \tag{5}$$

with the $d-2$ vertical displacement fields $U^a=U^a(\xi^0,\xi^1)$ ($a=2,3,\ldots,d-1$). Then the metric is given by

$$g_{ij}=\delta_{ij}+\partial_i U^a \partial_j U^a\ , \tag{6}$$

and the action reads as

$$A=\int d^2\xi\sqrt{g}\left[M_0^2+\lambda^{ij}(\partial_i U^a\partial_j U^a+\delta_{ij}-g_{ij})+\tfrac{1}{2}(D_i\phi^r)^2+\tfrac{1}{2}m^2(\phi^r)^2\right]\ . \tag{7}$$

The partition function of the string becomes the functional integral

$$Z=\int[\mathcal{D}\phi^r][\mathcal{D}U^a][\mathcal{D}g_{ij}][\mathcal{D}\lambda^{ij}]e^{-A}\ . \tag{8}$$

A finite temperature of the system is introduced by imposing periodic boundary conditions for all fields in the "time" direction, e.g.,

$$U^a(r,t)=U^a(r,t+1/T)\ . \tag{9}$$

The advantage of the functional integral (8) is that the $d-2$ displacement fields and the extra fields ϕ^r occur quadratically in the exponent so that they can be integrated out. For the extra scalar (or Fermi) fields this can be done exactly and gives an additional effective action in the form of a trace log. The quadratic form of the displacement fields, however, is accompanied by ξ-dependent Lagrange multipliers $\lambda^{ij}(\xi)$ so that the resulting effective action in λ^{ij} is not known. Fortunately, since the trace log is accompanied by a factor $d-2$, the limit $d\rightarrow\infty$ can still be treated exactly. In this limit, the functional integral is dominated by the saddle point of the action and it can be shown [6] that this has a constant metric g_{ij} and a constant Lagrange multiplier λ^{ij}. For symmetry reasons these will have the form

$$g_{ij}=\begin{bmatrix}\rho_0 & 0\\ 0 & \rho_1\end{bmatrix}, \tag{10}$$

and

$$\lambda^{ij}=\begin{bmatrix}\lambda_0/\rho_0 & 0\\ 0 & \lambda_1/\rho_1\end{bmatrix}. \tag{11}$$

At such a constant saddle point, we can then integrate out the quadratic U^a and ϕ^r fields in (7) and obtain an effective action

$$A_{\text{eff}}=\int d^2\xi(\rho_0\rho_1)^{1/2}\left[M_0^2+\frac{d-2}{2}\text{Tr}\ln(-\lambda^{ij}\partial_i\partial_j) \right. \tag{12}$$

$$\left. +\frac{N_S}{2}\text{Tr}\ln\left[-\frac{1}{\rho_i}\partial_i^2+m^2\right] +\lambda^{ij}(\delta_{ij}-g_{ij})\right], \tag{13}$$

where ∂_i^2/ρ_i is the covariant Laplacian D^2 for the saddle-point metric (10):

$$D^2=\frac{1}{\sqrt{g}}\partial_i[\sqrt{g}g^{ij}\partial_j]=\left[\frac{1}{\rho_0}\partial_0^2+\frac{1}{\rho_1}\partial_i^2\right]\equiv\frac{1}{\rho_i}\partial_i^2 . \tag{14}$$

For fermions, the number N_S in Eq. (13) is simply replaced by $-N_F$.

The eigenvalues of the operator $-D^2$ in momentum space are $k_0^2/\rho_0+k_1^2/\rho_1$. Because of the boundary conditions, the time component of $\mathbf{k}$, the frequencies k_0, are discrete,

$$k_0=(k_0)_n=2\pi nT_{\text{ext}}=\frac{2\pi n}{\beta_{\text{ext}}}\quad(n=0,\pm1,\pm2,\cdots), \tag{15}$$

while k_1 is a continuous variable. We have attached to the physical temperature the subscript ext to emphasize that the inverse $\beta_{\text{ext}}=1/T_{\text{ext}}$ is the temporal extent in *extrinsic* space (in contradistinction to an auxiliary intrinsic temperature T to be introduced below for calculations). Thus the trace logs in the action Eq. (13) are sums and integrals over momenta k_0,k_1. We shall write the effective action as

$$A_{\text{eff}}=\frac{d-2}{2}R_{\text{ext}}\beta_{\text{ext}}(\rho_0\rho_1)^{1/2}f , \tag{16}$$

where f is a "free-energy density" which we separate into three terms: $f=f_0+f_1+f_2$. Setting the number of scalar fields N_S equal to $\nu(d-2)$, the f_i's are

$$f_0=\frac{1}{2\tilde{\alpha}}\left[-\lambda_0-\lambda_1+\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right], \tag{17}$$

$$f_1=\tilde{M}_0^2+\frac{1}{(\rho_0\rho_1)^{1/2}}T_{\text{ext}} \times\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dk_1}{2\pi}\ln\left[\frac{\lambda_0}{\rho_0}(k_0)_n^2+\frac{\lambda_1}{\rho_1}k_1^2\right], \tag{18}$$

$$f_2=\frac{\nu}{(\rho_0\rho_1)^{1/2}}T_{\text{ext}} \times\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dk_1}{2\pi}\ln\left[\frac{(k_0)_n^2}{\rho_0}+\frac{k_1^2}{\rho_1}+m^2\right]. \tag{19}$$

The parameters $\tilde{\alpha},\tilde{M}_0^2$, and $\tilde{m}^2$ are given by

$$\frac{1}{2\tilde{\alpha}}=\frac{2}{d-2},\quad \tilde{M}_0^2=\frac{2}{d-2}M_0^2,\quad \tilde{m}^2=\frac{N}{d-2}m^2=\nu m^2 . \tag{20}$$

To calculate the spectral sums in Eqs. (18) and (19) we introduce auxiliary intrinsic quantities (announced above). These are defined in terms of the extrinsic ones via metric factors as

$$T\equiv\frac{T_{\text{ext}}}{(\rho_0)^{1/2}}\ \text{or}\ \beta\equiv(\rho_0)^{1/2}\beta_{\text{ext}} , \tag{21}$$

$$R\equiv(\rho_1)^{1/2}R_{\text{ext}} , \tag{22}$$

$$\omega_n=\frac{(k_0)_n}{(\rho_0)^{1/2}}=\frac{2\pi n}{(\rho_0)^{1/2}}T_{\text{ext}}=2\pi nT , \tag{23}$$

$$q_1=k_1/(\rho_1)^{1/2} . \tag{24}$$

Then the f_i's become

$$f_0=-\frac{\lambda_0+\lambda_1}{2\tilde{\alpha}}+\frac{1}{2\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right], \tag{25}$$

$$f_1=\tilde{M}_0^2+T\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dq_1}{2\pi}\ln(\lambda_0\omega_n^2+\lambda_1q_1^2) , \tag{26}$$

$$f_2=\nu T\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dq_1}{2\pi}\ln(\omega_n^2+q_1^2+m^2) . \tag{27}$$

The divergencies contained in Eqs. (26) and (27) are controlled by a Euclidean cutoff $|\mathbf{k}|<\Lambda$ in momentum space (see also Appendix A). Introducing $\bar{\lambda}=(\lambda_0+\lambda_1)/2$ the resulting expressions are

$$f_0=-\frac{\bar{\lambda}}{2\tilde{\alpha}}+\frac{1}{2\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right], \tag{28}$$

$$f_1=\tilde{M}_0^2-\frac{\pi T^2}{3}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}, \tag{29}$$

$$f_2=\frac{\tilde{m}^2}{4\pi}\left[4\pi L_0+\bar{L}_T+4S_1+\frac{2}{(\lambda_T)^{1/2}}-\frac{1}{3\lambda_T}\right], \tag{30}$$

where L_0 is the divergent integral

$$L_0=\int \frac{d^2\mathbf{q}}{(2\pi)^2}\frac{1}{\mathbf{q}^2+m^2}=\frac{1}{4\pi}\ln\frac{\Lambda^2}{m^2}\,, \tag{31}$$

and Λ is an ultraviolet cutoff

$$\bar{L}_T=\ln\frac{\lambda_T}{4e^{-2\bar{\gamma}}},\quad \lambda_T\equiv\frac{m^2}{4\pi^2T^2} \tag{32}$$

with $\bar{\gamma}=0.577\,215\ldots$ being Euler's constant. The symbol S_1 denotes the convergent infinite sum

$$S_1=\frac{1}{\lambda_T}\sum_{n=1}^{\infty}[(n^2+\lambda_T)^{1/2}-n-\lambda_T/2n]\,. \tag{33}$$

Note that $f_0+f_1=f_{\rm NG}$ is the Nambu-Goto energy density. After absorbing the divergent integral

$$\int\frac{d^2q}{(2\pi)^2}\ln(\mathbf{q}^2+m^2)=\frac{m^2}{4\pi}(1+4\pi L_0) \tag{34}$$

into a renormalized string tension

$$\tilde{M}^2_{\rm NG}=\tilde{M}^2_0\left[1+\frac{\tilde{m}^2}{4\pi}(1+4\pi L_0)\right]\,, \tag{35}$$

the total-energy density becomes

$$f=\tilde{M}^2_{\rm NG}-\frac{\pi T^2}{3}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}-\frac{\tilde{\lambda}}{\tilde{\alpha}}+\frac{1}{2\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right]+B\,. \tag{36}$$

All the new contributions, as compared to the pure Nambu-Goto string, are contained in

$$B\equiv\frac{\tilde{m}^2}{4\pi}\left[\bar{L}_T-1+4S_1+\frac{2}{(\lambda_T)^{1/2}}-\frac{1}{3\lambda_T}\right]\,. \tag{37}$$

To obtain the extremality conditions, the "gap equations," for ρ_0,ρ_1,λ_0, and λ_1 we have to vary the action given by Eqs. (16) and (36) at fixed extrinsic temperatures $T_{\rm ext}=(\rho_0)^{1/2}T$. Introducing

$$\gamma\equiv\frac{\pi T^2\rho_0}{3}=\frac{\pi T^2_{\rm ext}}{3}=\frac{m^2}{4\pi}\frac{1}{3\lambda_T}\,, \tag{38}$$

we find the gap equations as follows:

$$\tilde{M}^2_{\rm NG}+\frac{\gamma}{\rho_0}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}-\frac{\tilde{\lambda}}{\tilde{\alpha}}+\frac{1}{2\tilde{\alpha}}\left[-\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right]=-A\,, \tag{39}$$

$$\tilde{M}^2_{\rm NG}-\frac{\gamma}{\rho_0}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}-\frac{\tilde{\lambda}}{\tilde{\alpha}}+\frac{1}{2\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0}-\frac{\lambda_1}{\rho_1}\right]=-B\,, \tag{40}$$

$$\frac{\lambda_0}{\tilde{\alpha}}-\frac{1}{\tilde{\alpha}}\frac{\lambda_0}{\rho_0}+\frac{\gamma}{\rho_0}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}=0\,, \tag{41}$$

$$\frac{\lambda_1}{\tilde{\alpha}}-\frac{1}{\tilde{\alpha}}\frac{\lambda_1}{\rho_1}-\frac{\gamma}{\rho_0}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}=0\,, \tag{42}$$

where

$$A\equiv\frac{\tilde{m}^2}{4\pi}\left[\bar{L}_T+1-4(S_1-S_2)+\frac{1}{3}\frac{1}{\lambda_T}\right] \tag{43}$$

with S_2 being another convergent sum

$$S_2=\sum_{n=1}^{\infty}\left[\frac{1}{(n^2+\lambda_T)^{1/2}}-\frac{1}{n}\right]\,. \tag{44}$$

The Nambu-Goto case is obtained for $A=B=0$. Equations (39)–(42) can be rewritten in a form which will be used later:

$$\tilde{M}^2_{\rm NG}-\tilde{\lambda}/\tilde{\alpha}=-\tfrac{1}{2}(A+B)\,, \tag{45}$$

$$\frac{\gamma}{\rho_0}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}+\frac{1}{2\tilde{\alpha}}\left[-\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right]=-\tfrac{1}{2}(A-B)\,, \tag{46}$$

$$\frac{\tilde{\lambda}}{\tilde{\alpha}}-\frac{1}{2\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right]=0\,, \tag{47}$$

$$(\lambda_0-\lambda_1)/2\tilde{\alpha}=\tfrac{1}{2}(A-B)\,. \tag{48}$$

We now write the general expression for the string tension at finite temperature. It is obtained after dropping the surface area factor $R_{\rm ext}\beta_{\rm ext}$ in Eq. (16). Let $\tau\equiv T_{\rm ext}/\tilde{M}_{\rm NG}$ be the reduced temperature, then the string tension is given by

$$M^2(\tau)=\tfrac{1}{2}(d-2)\tilde{M}^2(\tau)=\tfrac{1}{2}(d-2)(\rho_0\rho_1)^{1/2}f\,. \tag{49}$$

If we factorize out the temperature dependence,

$$\tilde{M}^2(\tau)=\tilde{M}^2(\tau=0)\hat{M}^2(\tau)\,, \tag{50}$$

we obtain what may be called the "normalized string tension" in the form

$$\hat{M}^2(\tau)=(\rho_0\rho_1)^{1/2}\hat{f}\,, \tag{51}$$

where $\hat{f}\equiv f/\tilde{M}^2(\tau=0)$.

Since the main ingredient of a hadronic string is gluons, we shall first study the above equations for massless scalar fields. Before doing so, however, it will be useful to repeat the calculation for the Nambu-Goto string tension since the massless result, in a certain useful approximation, will follow directly from this.

III. THE NAMBU-GOTO CASE

The Nambu-Goto case is obtained by setting $A=B=0$. Thus f, as given by Eq. (36), becomes

$$f^{\rm NG}=\tilde{M}^2_{\rm NG}-\frac{\gamma}{\rho_0}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}-\frac{\tilde{\lambda}}{\tilde{\alpha}}+\frac{1}{2\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right] \tag{52}$$

and the gap equations are

$$\tilde{M}^2_{\rm NG}-\frac{\tilde{\lambda}}{\tilde{\alpha}}=0\,, \tag{53}$$

$$\frac{\gamma}{\rho_0}\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}+\frac{1}{2\tilde{\alpha}}\left[-\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right]=0\,, \tag{54}$$

$$\frac{\tilde{\lambda}}{\tilde{\alpha}}-\frac{1}{2\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right]=0\,, \tag{55}$$

$$\frac{\lambda_0-\lambda_1}{2\tilde{\alpha}}=0 . \qquad (56)$$

One learns from Eq. (56) that $\lambda_0=\lambda_1\equiv\tilde{\lambda}$. Inserting Eq. (55) into Eq. (52) we obtain, for the energy density,

$$\hat{f}^{\rm NG}=1-\frac{\gamma}{\rho_0}\frac{1}{\tilde{M}_{\rm NG}^2} , \qquad (57)$$

and the solutions of the gap equations are

$$\tilde{\lambda}=\tilde{\alpha}\tilde{M}_{\rm NG}^2 , \qquad (58)$$

$$\rho_0=1-\frac{\gamma}{\tilde{M}_{\rm NG}^2} , \qquad (59)$$

$$\rho_1=\frac{\rho_0}{2\rho_0-1} . \qquad (60)$$

The total string tension Eq. (51) is therefore given in this case by

$$\hat{M}_{\rm NG}^2(\tau)=\left[1-\frac{2\gamma}{\tilde{M}_{\rm NG}^2}\right]^{1/2}=\left[1-\left[\frac{T_{\rm ext}}{T_{\rm NG}^{\rm dec}}\right]^2\right]^{1/2} , \qquad (61)$$

where we have introduced the deconfinement temperature $T_{\rm NG}^{\rm dec}$ as the value of the temperature $T_{\rm ext}$ at which the total string tension vanishes. In the Nambu-Goto case it is given by

$$\frac{T_{\rm NG}^{\rm dec}}{M_{\rm NG}}=\left[\frac{3}{(d-2)\pi}\right]^{1/2} . \qquad (62)$$

This follows from Eq. (38). For $d=4$ this gives the deconfinement temperature

$$\left.\frac{T_{\rm NG}^{\rm dec}}{M_{\rm NG}}\right|_{d=4}\simeq 0.69 , \qquad (63)$$

which is larger than the SU(3) value

$$\frac{T_{\rm MC}^{\rm dec}}{M_{\rm NG}}=0.48\pm0.05 , \qquad (64)$$

and close to the SU(2) value

$$\frac{T_{\rm MC}^{\rm dec}}{M_{\rm NG}}=0.74\pm0.10 \qquad (65)$$

(see Table I). A plot of the temperature dependence of the string tension Eq. (61) is given by the solid line in Fig. 2.

We are now ready to consider the massless case $m=0$.

IV. THE MASSLESS CASE IN THE ISOTROPIC APPROXIMATION

Since the zero-temperature string is isotropic and the deconfinement happens at a rather low temperature, the confined system does not become very anisotropic and it is useful to study the equations in the approximation of a completely isotropic gap with $\lambda_0=\lambda_1\equiv\tilde{\lambda}$, where they allow for a simple analytic solution. For $m\to 0$ one has

$$A=-B=\frac{\tilde{m}^2}{4\pi}\left[\frac{1}{3\lambda_T}\right]\to\nu\frac{\gamma}{\rho_0} . \qquad (66)$$

The energy density obtained from the general expression Eq. (51) is now

$$f^{\rm iso}=\tilde{M}_{\rm NG}^2-\frac{\gamma}{\rho_0}-\frac{\tilde{\lambda}}{\tilde{\alpha}}+\frac{\tilde{\lambda}}{2\tilde{\alpha}}\left[\frac{1}{\rho_0}+\frac{1}{\rho_1}\right]-\nu\frac{\gamma}{\rho_0} . \qquad (67)$$

We observe that this formula can be obtained from the NG expression Eq. (52) (with $\lambda_0=\lambda_1\equiv\tilde{\lambda}$) by replacing $\gamma\to\gamma(1+\nu)$. This is also true for the gap equations and the final result can be obtained from Eq. (61) after replacing γ by $\gamma(1+\nu)$. Thus we find

$$\hat{M}_{\rm iso}^2(\tau)=\left[1-\frac{2\gamma(1+\nu)}{\tilde{M}_{\rm NG}^2}\right]^{1/2}=\left[1-\left[\frac{T_{\rm ext}}{T_{\rm iso}^{\rm dec}}\right]^2\right]^{1/2} , \qquad (68)$$

where $T_{\rm iso}^{\rm dec}$ denotes the deconfinement temperature in this isotropic approximation. It has the value

$$\frac{T_{\rm iso}^{\rm dec}}{M_{\rm NG}}=\left[\frac{1}{1+\nu}\right]^{1/2}\left[\frac{3}{(d-2)\pi}\right]^{1/2}=\left[\frac{1}{1+\nu}\right]^{1/2}\frac{T_{\rm NG}^{\rm dec}}{M_{\rm NG}} . \qquad (69)$$

For $\nu=1$, i.e., for $N=d-2=2$ massless scalar fields in four dimensions, the deconfinement temperature is given by

$$T_{\rm iso}^{\rm dec}/M_{\rm NG}\simeq 0.49 \qquad (70)$$

and is in agreement with the Monte Carlo number of four-dimensional SU(3) gauge theory in Eq. (64).

The color-electric flux tube contains eight gluons. In four dimensions each has two polarization degrees of freedom. This would imply $N_S=16$. For this value we find the much too small deconfinement temperature

$$T_{\rm iso}^{\rm dec}/M_{\rm NG}\simeq 0.23 . \qquad (71)$$

One may try and correct this value by assuming that the gluons are truely massless only at the core of the string while their mass increases towards the walls where it is of the order of the 600 MeV [11]. This is about twice as large as the dimensionally transmuted coupling constant

$$\Lambda_{\rm QCD}\approx 300 \text{ MeV} . \qquad (72)$$

It might be that the full four-dimensional number of polarization degrees of freedom cannot fluctuate freely inside the flux tube. This is not astonishing. One should expect the boundary conditions to strongly restrict the polarizations of the fields. This is known from work on the MIT bag model [12]. Inside an infinitely thin string the entire field lives on the boundary. The most suggestive idealization is that the gluon field inside the flux tube is almost a true two-dimensional gauge field. Then no dynamical freedom remains and the Nambu-Goto value of $T^{\rm dec}$ prevails. Because of the strong dependence of $T^{\rm dec}$ on N_S, the closeness of the NG value of $T^{\rm dec}$ to the computer value, it appears that the latter scenario is close to reality. The SU(N) dependence of the deconfinement temperature must therefore be a subtle finite tube thickness effect of the gluon fluctuations.

In the fermionic case, Eq. (69) becomes

$$\frac{T^{\rm ded}_{{\rm iso},f}}{M_{\rm NG}}=\left[\frac{1}{1-\nu}\right]^{1/2}\left[\frac{3}{(d-2)\pi}\right]^{1/2} =\left[\frac{1}{1-\nu}\right]^{1/2}\frac{T^{\rm dec}_{\rm NG}}{M_{\rm NG}} . \qquad (73)$$

Thus we see that while the deconfinement temperature decreases with ν in the bosonic case, it increases for fermions. For $\nu=-1$, i.e., for $N=-(d-2)$, the deconfinement transition disappears completely. The case $N>-(d-2)$ is not permissible. This means that for a certain number of scalar fields, the number of Fermi fields is bounded from above. Note that asymptotic freedom of an SU(n) gauge theory at zero temperature gives an upper bound for the number of quarks,

$$N_F<11n^2/2 ,$$

determined by the positive sign of the first coefficient of the β function [13]:

$$\beta(g)=-g^3(11n/3-2N_Fn/3)/16\pi^2 .$$

In our case, if all fields are massless, the number of fermions in four dimensions is bounded by

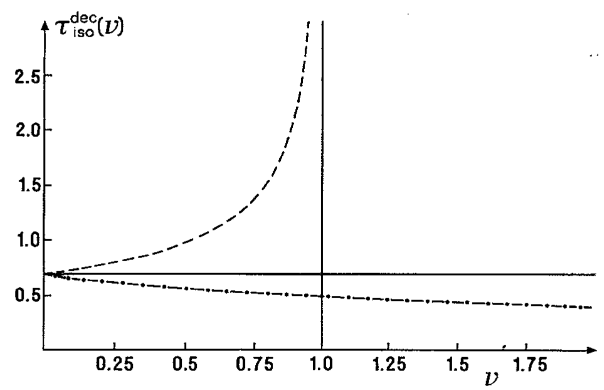

FIG. 1. The deconfinement temperature as a function of ν for the massless case in the isotropic approximation. The dashed curve represents the Nambu-Goto string occupied by massless spin-$\frac{1}{2}$ fermions while the dash-dotted curve shows the case of scalar fields. Scalar fields lower and fermions increase the deconfinement temperature. There is an upper bound for the number of fermions, $N_F<N_S+2$.

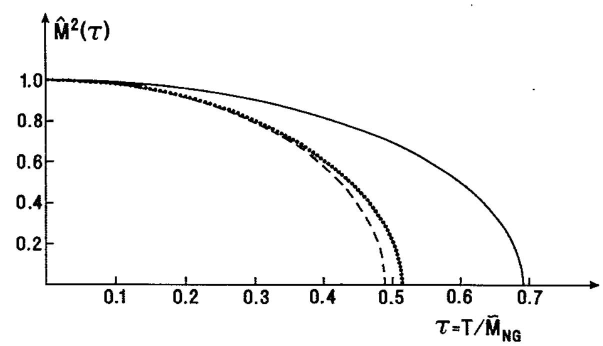

FIG. 2. The string tension $\hat{M}^2(\tau)$ as a function of the reduced temperature $\tau\equiv T_{\rm ext}/\tilde{N}_{\rm NG}$, once for the Nambu-Goto model (solid line) and once in the presence of $N=d-2$ massless scalar fields. In the isotropic approximation (dashed line) the deconfinement temperature is $T^{\rm dec}_{\rm iso}/M_{\rm NG}=0.49$, much smaller than the Nambu-Goto value 0.69, and agrees with the Monte Carlo number for an SU(3) lattice gauge theory. The dotted line shows the full anisotropic case where $T^{\rm dec}_{\rm aniso}/M_{\rm NG}=0.51$.

$$N_F<N_S+2 . \qquad (74)$$

Although the asymptotic freedom at zero temperature is closely related to the infrared confinement, the deconfinement transition at a finite temperature is a fundamentally different phenomenon. While asymptotic freedom is certainly independent of the quark masses, the deconfinement temperature is expected to depend crucially on them [1,2]. This is borne out by our model. For massive quarks living on the string, the above bound is a complicated function of the field numbers and the masses and cannot be written in closed form (see Secs. VI and VII).

We plot Eqs. (69) and (73) as a function of ν in Fig. 1 and temperature behavior of the string tension, Eq. (68) for $\nu=1$ in Fig. 2 (dashed line), where it is compared with the NG curve (solid line).

V. THE MASSLESS CASE WITH FULL ANISOTROPY

Now f is given by

$$f^{\rm aniso}=\tilde{M}^2_{\rm NG}-\frac{\gamma}{\rho_0}\left[\nu+\left[\frac{\lambda_0}{\lambda_1}\right]^{1/2}\right]-\frac{\tilde{\lambda}}{\tilde{\alpha}} +\frac{1}{\tilde{\alpha}}\left[\frac{\lambda_0}{\rho_0}+\frac{\lambda_1}{\rho_1}\right] . \qquad (75)$$

The gap equations are identical to Eqs. (53)–(56) with the exception of Eq. (54) which has $-\nu\gamma/\rho_0$ on the right-hand side. The solutions are

$$\lambda_0=\tilde{\alpha}\tilde{M}^2_{\rm NG}\left[1+\nu\hat{\gamma}/\rho_0\right] , \qquad (76)$$

$$\lambda_1=\tilde{\alpha}\tilde{M}^2_{\rm NG}\left[1-\nu\hat{\gamma}/\rho_0\right] , \qquad (77)$$

$$\rho_1=\frac{\rho_0(1-\nu\hat{\gamma}/\rho_0)}{2\rho_0-1-\nu\hat{\gamma}/\rho_0}\ , \qquad (78)$$

where the symbols with a caret denote the corresponding quantities normalized by a factor $1/\tilde{M}_{\rm NG}^2$, i.e., $\hat{O}\equiv O/\tilde{M}_{\rm NG}^2$. The energy density is

$$\hat{f}^{\rm aniso}=1-\frac{\hat{\gamma}}{\rho_0}\left[\nu+\left[\frac{1+\nu\hat{\gamma}/\rho_0}{1-\nu\hat{\gamma}/\rho_0}\right]^{1/2}\right]\ , \qquad (79)$$

and ρ_0 is the solution of the quartic equation

$$\rho_0^4-2\rho_0^3+(1-\hat{\gamma}^2-\nu^2\hat{\gamma}^2)\rho_0^2+2\nu^2\hat{\gamma}^2\rho_0-\nu^2\hat{\gamma}^2=0\ . \qquad (80)$$

Although we have worked with the analytic solutions of Eq. (80), these are not very instructive and it is not useful to write them here. From the four solutions of Eq. (80) we choose the one which extremizes $\hat{f}^{\rm aniso}$. The resulting expression for the string tension is shown for the case $\nu=1$ as a dotted curve in Fig. 2. It does not differ much from the isotropic approximation thus justifying its study. The deconfinement temperature in four dimensions with $\nu=1$ is found to be

$$T_{\rm aniso}^{\rm dec}/M_{\rm NG}\simeq 0.51\ , \qquad (81)$$

close to the isotropic number [and thus to the Monte Carlo result Eq. (64)].

It is interesting to write an analytic result for the string tension in the neighborhood of the NG string, i.e., for small ν, where it reads as

$$\hat{M}_{\rm aniso}^2(\tau)\simeq\sqrt{1-2\hat{\gamma}}-\nu\frac{\hat{\gamma}}{\sqrt{1-2\hat{\gamma}}}-\frac{1}{2}\nu^2\frac{\hat{\gamma}^2}{(\sqrt{1-2\hat{\gamma}})^3}\left[\frac{\hat{\gamma}}{1-\hat{\gamma}}\right] \qquad (82)$$

$$=\hat{M}_{\rm NG}^2-\nu\frac{\hat{\gamma}}{\hat{M}_{\rm NG}^2}-\frac{1}{2}\nu^2\frac{\hat{\gamma}^2}{\hat{M}_{\rm NG}^6}\left[\frac{\hat{\gamma}}{1-\hat{\gamma}}\right]\ . \qquad (83)$$

This can be compared with the corresponding expansion of the isotropic result Eq. (68):

$$\hat{M}_{\rm iso}^2(\tau)\simeq\sqrt{1-2\hat{\gamma}}-\nu\frac{\hat{\gamma}}{\sqrt{1-2\hat{\gamma}}}-\frac{1}{2}\nu^2\frac{\hat{\gamma}^2}{(\sqrt{1-2\hat{\gamma}})^3} \qquad (84)$$

$$=\hat{M}_{\rm NG}^2-\nu\frac{\hat{\gamma}}{\hat{M}_{\rm NG}^2}-\frac{1}{2}\nu^2\frac{\hat{\gamma}^2}{\hat{M}_{\rm NG}^6}\ . \qquad (85)$$

We see that the effect of the anisotropy begins showing up in the ν^2 term.

VI. THE MASSIVE CASE IN THE ISOTROPIC APPROXIMATION

We start by writing the energy density which will be distinguished from the massless case by a subscript μ which is related to the scalar field mass by $\mu=m/\tilde{M}_{\rm NG}$. From Eq. (36),

$$f_\mu^{\rm iso}=\tilde{M}_{\rm NG}^2-\frac{\gamma}{\rho_0}-\frac{\tilde{\lambda}}{\tilde{\alpha}}+\frac{\tilde{\lambda}}{2\tilde{\alpha}}\left[\frac{1}{\rho_0}+\frac{1}{\rho_1}\right]+B\ , \qquad (86)$$

where B has already been defined in Eq. (37). The gap equations are

$$\tilde{M}_{\rm NG}^2+\frac{\gamma}{\rho_0}-\frac{\tilde{\lambda}}{\tilde{\alpha}}+\frac{\tilde{\lambda}}{2\tilde{\alpha}}\left[-\frac{1}{\rho_0}+\frac{1}{\rho_1}\right]=-A\ , \qquad (87)$$

$$\tilde{M}_{\rm NG}^2-\frac{\gamma}{\rho_0}-\frac{\tilde{\lambda}}{\tilde{\alpha}}+\frac{\tilde{\lambda}}{2\tilde{\alpha}}\left[\frac{1}{\rho_0}-\frac{1}{\rho_1}\right]=-B\ , \qquad (88)$$

$$1-\frac{1}{2}\left[\frac{1}{\rho_0}+\frac{1}{\rho_1}\right]=0\ , \qquad (89)$$

with A defined by Eq. (43). We can manipulate these equations into a more convenient form,

$$\tilde{M}_{\rm NG}^2-\tilde{\lambda}/\tilde{\alpha}=-\tfrac{1}{2}(A+B)\ , \qquad (90)$$

$$\frac{\gamma}{\rho_0}+\frac{\tilde{\lambda}}{2\tilde{\alpha}}\left[-\frac{1}{\rho_0}+\frac{1}{\rho_1}\right]=-\tfrac{1}{2}(A+B)\ , \qquad (91)$$

$$1-\frac{1}{2}\left[\frac{1}{\rho_0}+\frac{1}{\rho_1}\right]=0\ . \qquad (92)$$

From Eq. (92) it follows that

$$\hat{f}_\mu^{\rm iso}=1+\hat{B}-\hat{D}\ , \qquad (93)$$

where

$$\hat{D}=\frac{\mu^2}{12\pi}\frac{1}{\lambda_T}\quad\text{and}\quad \mu^2\equiv\frac{m^2}{\tilde{M}_{\rm NG}^2}\ . \qquad (94)$$

μ is a dimensionless parameter which scales with d like $\sqrt{d-2}$. Both A and B involve the infinite sums S_1 and S_2 given by Eqs. (33) and (44), respectively. For this reason we cannot write an equation for ρ_0 as in the massless case ($\lambda_T=\rho_0\lambda_{Text}$). Instead, we write the quantities of interest as functions of λ_T and μ. This (λ_T,μ) pair then becomes an input for the numerical treatment of the problem. From Eqs. (90)–(92) it follows that

$$\rho_0=\frac{\hat{A}+\hat{B}+2}{2(\hat{A}+\hat{B}+2-\hat{f})}\ , \qquad (95)$$

$$\rho_1=\frac{\hat{A}+\hat{B}+2}{2\hat{f}}\ . \qquad (96)$$

Together with Eq. (93) the normalized string tension becomes

$$\hat{M}_{{\rm iso},\mu}^2(\tau)=\frac{1}{2}\left[\frac{(\hat{A}+\hat{B}+2)^2}{(\hat{A}+\hat{B}+2-\hat{f})\hat{f}}\right]^{1/2}\hat{f}=\frac{1}{2}\left[\frac{(\hat{A}+\hat{B}+2)^2\hat{f}}{(\hat{A}+\hat{B}+2-\hat{f})}\right]^{1/2}\ . \qquad (97)$$

Note that (i) $\hat{A}+\hat{B}$ is always bigger or equal to zero (see Appendix B), (ii) $\hat{f}$ is always less or equal to one, and (iii) $\hat{f}$ already appears as an overall factor under the square root through the term $(\rho_0\rho_1)^{1/2}$, in particular, through

$(\rho_1)^{1/2}$. Together this makes $\hat{M}^2$ zero for vanishing $\hat{f}$ and imaginary only when $\hat{f}$ becomes negative. Although we cannot solve the problem analytically we see right away that $\hat{M}^2_{\rm iso,\mu}$ has a square-root singularity as in the Nambu-Goto model. The string tension (97) is plotted in Fig. 3 for various values of the parameter m.

For a large mass m or low temperatures, $\hat{A}$ and $\hat{B}$ contribute very little and can be approximated by their leading asymptotic behavior for large arguments [see the Appendix, Eqs. (A13) and (A14)]. The string tension takes the simple form

$$\tilde{M}^2_{\rm iso,\tau\simeq 0} \simeq \tilde{M}^2_{\rm NG} + \frac{1}{2}\nu \frac{\mu^2}{\tilde{M}^2_{\rm NG}} \frac{\exp[-2\pi(\lambda_T)^{1/2}]}{\pi(\lambda_T)^{1/4}} , \tag{98}$$

where

$$\lambda_T \simeq \frac{\mu^2}{4\pi^2}\frac{1-\hat{\gamma}}{\tau^2} \tag{99}$$

and

$$\hat{\gamma} = \pi\tau^2/3 . \tag{100}$$

For small μ we are close to the massless case and we can study this limit analytically. As we can see from Eqs. (93)–(97) all relevant μ-dependent quantities are contained in the terms denoted $\hat{A}$ and $\hat{B}$. In the limit of vanishing μ, $\hat{A}$ and $\hat{B}$ are given by

$$\hat{A} = -\hat{B} = \nu\hat{D} \tag{101}$$

with $\hat{D}$ given by Eq. (94). If we recall that $\lambda_T = \mu^2\rho_0/4\pi^2\tau^2$ we see that $\hat{D}$ is independent of μ, i.e.,

$$\hat{D} = (\pi/3\rho_0)\tau^2 = \hat{\gamma}/\rho_0 . \tag{102}$$

This will become useful shortly.

Thus using Eq. (101) with the value of $\hat{D}$ given above we get

$$\rho_0 = \frac{1}{2-\hat{f}^{\rm iso}_{\mu=0}} = 1-\hat{\gamma}(1+\nu) , \tag{103}$$

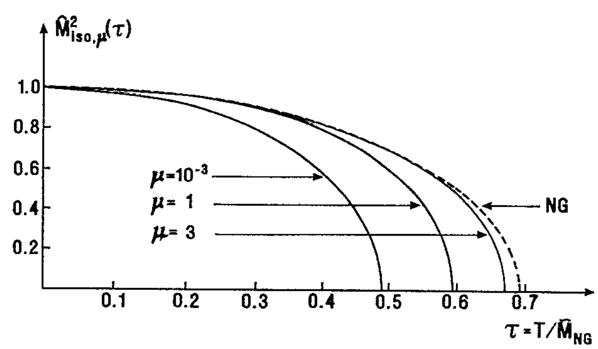

FIG. 3. The string tension for the massive case in the isotropic approximation for different values of $\mu = m/\tilde{M}_{\rm NG}$ in comparison with the Nambu-Goto case. For small μ, $T^{\rm dec}_{\rm iso,\mu}$ is very close to the massless ($\mu=0$) case Eq. (69). This plot was done for $N=d-2$ scalar fields.

$$\rho_1 = \frac{1}{\hat{f}^{\rm iso}_{\mu=0}} = \frac{1-\hat{\gamma}(1+\nu)}{1-2\hat{\gamma}(1+\nu)} , \tag{104}$$

so that the string tension becomes

$$\tilde{M}^2_{\rm iso,\mu=0}(\tau) = \left[\frac{\hat{f}}{2-\hat{f}}\right]^{1/2} = \sqrt{1-2\hat{\gamma}(1+\nu)} . \tag{105}$$

This is precisely the string tension Eq. (68) for the massless isotropic case.

VII. THE MASSIVE CASE WITH FULL ANISOTROPY

The energy density and gap equations for the massive case with no approximation are given by Eq. (36) and Eqs. (45)–(48), respectively. After inserting Eq. (47) into f this becomes

$$\hat{f}^{\rm aniso}_{\mu} = 1 + \hat{B} - \hat{D}(\lambda_0/\lambda_1)^{1/2} , \tag{106}$$

and the gap equations are solved by

$$\lambda_0 = \alpha\tilde{M}^2_{\rm NG}(1+\hat{A}) , \tag{107}$$

$$\lambda_1 = \alpha\tilde{M}^2_{\rm NG}(1+\hat{B}) , \tag{108}$$

together with the metric factors

$$\rho_0^{-1} = 1 + \frac{\hat{D}}{\sqrt{(1+\hat{A})(1+\hat{B})}} , \tag{109}$$

$$\rho_1 = \frac{\rho_0(1+\hat{B})}{\rho_0(\hat{A}+\hat{B}+2)-(1+\hat{A})} . \tag{110}$$

In analogy with Eq. (96) these can also be written in the form

$$\rho_0 = \frac{1+\hat{A}}{\hat{A}+\hat{B}+2-\hat{f}} , \tag{111}$$

$$\rho_1 = (1+\hat{B})/\hat{f} , \tag{112}$$

and the resulting string tension is

$$\tilde{M}^2_{\rm aniso,\mu}(\tau) = \left[\frac{(1+\hat{A})(1+\hat{B})}{(\hat{A}+\hat{B}+2-\hat{f})\hat{f}}\right]^{1/2}\hat{f} = \left[\frac{(1+\hat{A})(1+\hat{B})\hat{f}}{(\hat{A}+\hat{B}+2-\hat{f})}\right]^{1/2} . \tag{113}$$

The second equality in Eq. (113) is explained by considerations similar to those given in the paragraph following Eq. (97). Thus as in the previous cases, the string tension has a square-root singularity when $\hat{f}$ becomes negative. For large mass or low temperatures this expression is well approximated by

$$\hat{M}^2_{\rm aniso,\tau\simeq 0} \simeq \hat{M}^2_{\rm NG} - \frac{1}{2}\nu\frac{\mu^2}{\hat{M}^2_{\rm NG}}\frac{\exp[-2\pi(\lambda_T)^{1/2}]}{\pi^2(\lambda_T)^{3/4}} . \tag{114}$$

A plot of $\hat{M}^2_{\rm aniso,\mu}(\tau)$ for various values of μ is shown in Fig. 4. Again, in the limit of small μ we approach the result of the massless case. As before we can study this lim-

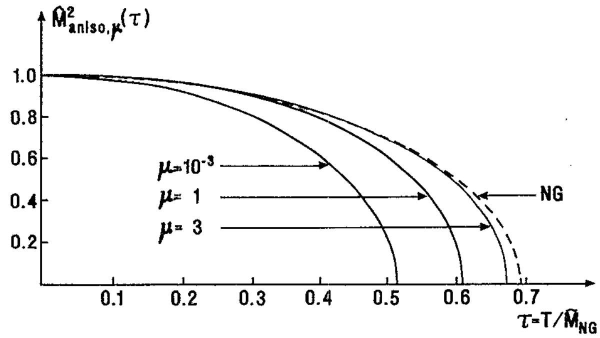

FIG. 4. The string tension for the massive anisotropic case for different values of $\mu = m/\hat{M}_{NG}$ and $N = d-2$ scalar fields in four dimensions. Again, the Nambu-Goto curve is plotted for comparison. In the regime of small μ, the deconfinement temperature is very close to the massless result $T^{\text{dec}}_{\text{aniso}}/M_{NG} = 0.51$.

it analytically. Using Eqs. (101) and (102), we get

$$\hat{f}^{\text{aniso}}_{\mu=0} = 1 - \hat{D}\left[\nu + \left[\frac{1+\nu\hat{D}}{1-\nu\hat{D}}\right]^{1/2}\right], \tag{115}$$

$$\rho_0 = \frac{1+\nu\hat{D}}{2-\hat{f}}, \tag{116}$$

$$\rho_1 = \frac{1-\nu\hat{D}}{\hat{f}}. \tag{117}$$

The string tension becomes

$$\hat{M}^2_{\text{aniso},\mu=0}(\tau) = \left[\frac{(1-\nu^2\hat{D}^2)\hat{f}}{(2-\hat{f})}\right]^{1/2}. \tag{118}$$

To go further we have to solve Eq. (116) for ρ_0 as we did in the isotropic case. Thus we insert $\hat{D}$ as given by Eq. (102) into Eq. (116) and obtain the following equation for ρ_0:

$$\rho_0^4 - 2\rho_0^3 + (1 - \hat{\gamma}^2 - \nu^2\hat{\gamma}^2)\rho_0^2 + 2\nu^2\hat{\gamma}^2\rho_0 - \nu^2\hat{\gamma}^2 = 0. \tag{119}$$

This is precisely Eq. (80) which ρ_0 obeys in the massless case.

VIII. DISCUSSION AND CONCLUSION

We have studied the effects of massive scalar or Fermi fields living on a Nambu-Goto string where "living on" means that the fields depend only on the (ξ^0, ξ^1) parameters and not on the coordinates of the Euclidean embedding space. As a function of temperature, the behavior of the string tension is quite similar to the NG case. There is always a square-root singularity at a certain temperature which signals the onset of a deconfined phase. Our main result are equations which describe how much the deconfinement temperature decreases with the number of scalar particles and increases for fermions. There is a simple approximate analytic formula for N_S massless scalar and N_F Fermi field components in which the deconfinement temperature is proportional to $1/\sqrt{1+\nu}$ where $\nu = (N_S - N_F)/(d-2)$. Thus the number of fermions which allows for a confining string is bounded, just as in QCD.

Of course, there is no problem in finding a value of the parameters m, ν at which the model's deconfinement temperature coincides with the values obtained by Monte Carlo simulations of an SU(3) pure gauge theory (for instance, $m=0$, $\nu=1$). As argued before, these values show that the boundary conditions on the gluon field make it essentially two dimensional and that the SU(N) dependence is mainly due to finite tube thickness effects. In the limit of two-dimensional gluons where the number of dynamical gluons is zero so that $N_S = 0$, the deconfinement transition would be moved upwards and disappear completely for only $N_F \approx 0.4$ massless Fermi field components. This is much too small to be physically correct. Since colored up and down quarks in two dimensions have already $3\times2\times2$ components it is very hard to understand, within this simple model, how nature manages to still have a deconfinement transition in spite of the existence of so many sea-quark–antiquark pairs inside hadrons. Even if the quark fields are allowed to have a constituent mass of ≈ 300 MeV their component number is still bounded by $N_F \approx 0.5$. The explanation for this contradiction must lie in the basic defect of the model to ignore the annihilation of the string inside the quark loop. To incorporate this into the theoretical description should greatly improve the model.

For all parameters m, ν, the low-temperature behavior of the string tension approaches the NG case since the quantities $\hat{A}$ and $\hat{B}$ tend to zero for $T/m \to 0$. The same thing happens, of course, for all temperatures in the limit of large mass m.

On the basis of the present model it is hard to understand how the string can accommodate as many quarks on a string as mentioned above. In conclusion, it appears that a full incorporation of the internal gluon and quark fluctuations into the NG string may lead to the correct model for the color-electric flux tube of QCD which is much better than the NG string model alone.

APPENDIX A: EVALUATION OF FINITE-TEMPERATURE CONTRIBUTIONS TO FREE ENERGY

In this appendix we illustrate how to calculate f_1 and f_2 as given by Eqs. (26) and (27) by working out f_2 in detail. For the finite system under consideration we write f_2 as

$$f_2^F = \nu T \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dq_1}{2\pi} \ln(q_1^2 + \omega_n^2 + m^2). \tag{A1}$$

The integral has already been calculated in [14]. The result is

$$f_2^F=\nu T\sum_{n=-\infty}^{\infty}(\omega_n^2+m^2)^{1/2}=\frac{\bar{m}^2}{2\pi}\frac{1}{\lambda_T}\sum_{n=-\infty}^{\infty}(n^2+\lambda_T)^{1/2}. \tag{A2}$$

We regularize f_2^F by adding and subtracting the infinite system result, denoted by f_2^I,

$$f_2^I=\nu\int\frac{d^2q}{(2\pi)^2}\ln(q^2+m^2)=\frac{\bar{m}^2}{4\pi}(1+4\pi L_0) \tag{A3}$$

with L_0 given by Eq. (31). Thus

$$f^F=f^I+(f^F-f^I)=\frac{\bar{m}^2}{4\pi}(1+4\pi L_0)+\frac{\bar{m}^2}{2\pi\lambda_T}\left[\sum_{n=-\infty}^{\infty}-\int_{-\infty}^{\infty}dn\right](n^2+\lambda_T)^{1/2}$$

$$=\frac{\bar{m}^2}{4\pi}\left[1+4\pi L_0+\frac{2}{(\lambda_T)^{1/2}}+\frac{4}{\lambda_T}\left[\sum_{n=1}^{\infty}-\int_0^{\infty}dn\right](n^2+\lambda_T)^{1/2}\right]. \tag{A4}$$

We evaluate this by carrying the integral up to some large but finite value $n=N$ and then taking the limit for $N\to\infty$. In the process terms of order $O(1/N)$ and higher are eliminated. The result is [14]

$$\left[\sum_{n=1}^{\infty}-\int_0^{\infty}dn\right](n^2+\lambda_T)^{1/2}=\frac{1}{12}+\frac{\lambda_T}{4}(\bar{L}_T-1)+\lambda_T S_1\ . \tag{A5}$$

Collecting terms in Eq. (A5) we arrive at Eq. (30).

APPENDIX B: ASYMPTOTIC EXPANSIONS FOR $\hat{A}$ AND $\hat{B}$

We give here the asymptotic expressions for the quantities

$$\hat{A}=\frac{\bar{\mu}^2}{4\pi}\left[\bar{L}_T+1-4(S_1-S_2)+\frac{1}{3\lambda_T}\right], \tag{A6}$$

$$\hat{B}=\frac{\bar{\mu}^2}{4\pi}\left[\bar{L}_T-1+4S_1+\frac{2}{(\lambda_T)^{1/2}}-\frac{1}{3\lambda_T}\right], \tag{A7}$$

where $\bar{\mu}=\bar{m}/\bar{M}_{\mathrm{NG}}$. For large λ_T there is a particularly useful representation of S_i in terms of modified Bessel functions [14]:

$$S_1=\frac{1}{4}-\frac{1}{4}\bar{L}_T-\frac{1}{2(\lambda_T)^{1/2}}+\frac{1}{12\lambda_T}-\frac{1}{\pi(\lambda_T)^{1/2}}\sum_{\bar{n}=1}^{\infty}\frac{K_1(2\pi\bar{n}(\lambda_T)^{1/2})}{\bar{n}}, \tag{A8}$$

$$S_2=-\frac{1}{2}\bar{L}_T-\frac{1}{2(\lambda_T)^{1/2}}+2\sum_{\bar{n}=1}^{\infty}K_0(2\pi\bar{n}(\lambda_T)^{1/2})\ . \tag{A9}$$

Thus $\hat{A}$ and $\hat{B}$ become simply

$$\hat{A}=\frac{\bar{\mu}^2}{4\pi}\left[\frac{4}{\pi(\lambda_T)^{1/2}}\sum_{\bar{n}=1}^{\infty}\frac{K_1(2\pi\bar{n}(\lambda_T)^{1/2})}{\bar{n}}+8\sum_{\bar{n}=1}^{\infty}K_0(2\pi\bar{n}(\lambda_T)^{1/2})\right], \tag{A10}$$

$$\hat{B}=\frac{\bar{\mu}^2}{4\pi}\left[-\frac{4}{\pi(\lambda_T)^{1/2}}\sum_{\bar{n}=1}^{\infty}\frac{K_1(2\pi\bar{n}(\lambda_T)^{1/2})}{\bar{n}}\right]. \tag{A11}$$

Thus we observe that, while $\hat{A}$ is always bigger or equal to zero, $\hat{B}$ is always less or equal to zero. This, of course, follows from the positivity of the modified Bessel functions K_0 and K_1. Also, because both K_0 and K_1 decay exponentially fast for large arguments we see that

$$\lim_{\lambda_T\to\infty}\hat{A}=-\lim_{\lambda_T\to\infty}\hat{B}=0\ . \tag{A12}$$

From Eqs. (A10) and (A11) it follows that $\hat{A}+\hat{B}\geq 0$.

For large masses or low temperatures, the modified Bessel functions can be approximated by their leading exponential behavior and we have

$$\hat{A}\approx\frac{\bar{m}^2}{2\pi}\left[\frac{1}{\pi(\lambda_T)^{3/4}}+\frac{2}{(\lambda_T)^{1/4}}\right]\exp[-2\pi(\lambda_T)^{1/2}]\ , \tag{A13}$$

$$\hat{B}\approx-\frac{\bar{m}^2}{2\pi}\frac{1}{\pi(\lambda_T)^{3/4}}\exp[-2\pi(\lambda_T)^{1/2}]\ . \tag{A14}$$

For small λ_T, S_1 and S_2 are approximated by

$$S_1=-\tfrac{1}{8}\zeta(3)\lambda_T+\tfrac{1}{16}\zeta(5)\lambda_T^2-\tfrac{5}{128}\zeta(7)\lambda_T^3+\cdots\ , \tag{A15}$$

$$S_2=-\tfrac{1}{2}\zeta(3)\lambda_T+\tfrac{3}{8}\zeta(5)\lambda_T^2-\tfrac{5}{16}\zeta(7)\lambda_T^3+\cdots\ . \tag{A16}$$

Thus, the leading term in $\hat{A}$, $\hat{B}$ for small λ_T is

$$\hat{A}\sim-\hat{B}\sim\frac{\bar{\mu}^2}{4\pi}\left[\frac{1}{3\lambda_T}\right]\equiv\nu\hat{D}\ . \tag{A17}$$

Because of $\lambda_T=\mu^2\rho_0/4\pi^2\tau^2$ the behavior of λ_T can be understood as a competition of limits between μ and τ with ρ_0 being determined. This is a familiar situation from an earlier investigation of possible effects of an extrinsic curvature stiffness term [15,16].

ACKNOWLEDGMENTS

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