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Abelian double-gauge-invariant continuous quantum field theory of electric charge confinement ☆

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We present a simple continuum field theory in the continuum which imitates the confinement mechanism of the standard U(1) lattice gauge model by forming strings between opposite electric charges in a condensate of Dirac monopoles. The monopoles are described by a gauge field $F^{\rm M}_{\mu\nu}$ which under string deformation changes locally by $\partial_\mu \Lambda^{\rm M}_\nu - \partial_\nu \Lambda^{\rm M}_\mu$ without changing physical observables. These gauge fields are transformed into a Higgs field which gives rise to flux tubes of electric field lines. The theory explains the phenomenon observed in many Monte Carlo simulations, also of nonabelian lattice gauge models, that the thermal deconfinement transition apparently restores at the same time also the spontaneously broken chiral symmetry of pion physics.

It has long been known that compact QED on a lattice shows quark confinement for a sufficiently strong charge e [1]. The system contains a grand-canonical ensemble of magnetic monopoles which condense at some critical value e_c. The condensate squeezes the electric field lines emerging from any charge into a thin tube giving rise to a confining potential [2]. It is possible to transform the partition function to the dual version of a standard Higgs model coupled minimally to the dual vector potential $\tilde{A}_\mu$ [3]. The Higgs field is the *disorder field* [4] of the magnetic monopoles, i.e., its Feynman graphs are the direct pictures of the monopole worldlines in the ensemble.

It is also well known that two electric charges in this model are connected by Abrikosov vortices producing the linearly rising potential between the charges and thus confinement. The system is a perfect dia-electric.

While there is no problem in taking the dual Higgs field description of quark confinement to the continuum limit [3], the same thing has apparently never been done in the original formulation in terms of the gauge field A_μ. The reason was the lack of an adequate continuum description of the integer-valued jumps in the electromagnetic gauge field A_μ across the worldsurfaces spanned by the worldlines of the magnetic monopoles which in the compact lattice formulation is straightforward. Such a continuum description has, however, recently been found. According to refs. [5,6] [#1], a fixed set of electric and magnetic charges is described by a euclidean action

$$\mathscr{A} = \frac{1}{16\pi} \int d^4x [F_{\mu\nu}(x) - F^{\rm M}_{\mu\nu}(x)]^2 + {\rm i} \int d^4x \, j_\mu(x) A_\mu(x) \,, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the usual field tensor,

$$j_\mu(x) \equiv e\delta_\mu(x; L) \tag{2}$$

is the charge distribution along closed worldlines L of the electric charges with $\delta_\mu(x; L)$ being δ-functions singular on the lines L

$$\delta_\mu(x; L) = \int d\tau \frac{d\bar{x}_\mu}{d\tau} \delta^{(4)}(x - \bar{x}(\tau)) \,, \tag{3}$$

while $F^{\rm M}_{\mu\nu}(x)$ is the *gauge field of monopoles*. It is defined as follows: Let $\tilde{L}$ be the worldline of a mono-

[#1] In refs. [5,6] I used the superscript P instead of M for the monopole gauge field $F^{\rm M}_{\mu\nu}$ since there I wanted to emphasize the analogy with the *plastic* gauge field used in the theory of plastic deformations (described in Vol. II of ref. [4]).

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pole and $\tilde{S}$ an arbitrary surface enclosed by $\tilde{L}$, then we take the δ-function on this surface,

$$\delta_{\mu\nu}(x;\tilde{S}) = \int d\sigma\, d\tau \left(\frac{\partial \bar{x}_\mu}{\partial \sigma} \frac{\partial \bar{x}_\nu}{\partial \tau} - (\mu \leftrightarrow \nu) \right) \times \delta^{(4)}(x - \bar{x}(\sigma, \tau)) , \tag{4}$$

and define $F^{\mathrm{M}}_{\mu\nu}$ in terms of the dual of this

$$F^{\mathrm{M}}_{\mu\nu}(x) \equiv 4\pi g \cdot \tfrac{1}{2} \epsilon_{\mu\nu\lambda\kappa} \delta_{\lambda\kappa}(x;\tilde{S}) . \tag{5}$$

This field has the property that its curl is singular on the boundary line $\tilde{L}$:

$$\tfrac{1}{2} \epsilon_{\mu\nu\lambda\kappa} \partial_\nu F^{\mathrm{M}}_{\lambda\kappa}(x) = 4\pi g \delta_\mu(x;\tilde{L}) = 4\pi \tilde{j}_\mu(x) , \tag{6}$$

this being a reformulation of Stokes' integral theorem in terms of distributions. The constant g is the magnetic charge of the monopoles which is assumed to satisfy *Dirac's charge quantization condition*

$$2eg = \text{integer} . \tag{7}$$

The euclidean quantum partition function of the system is found by summing, in a functional integral, the Boltzmann factor $\exp(-\mathscr{A})$ over all field configurations A_μ, all line configurations L in j^μ, and all surface configurations $\tilde{S}$ in $F^{\mathrm{M}}_{\mu\nu}$.

It was pointed out in ref. [5] that the action (1) is invariant under two types of gauge transformations, the ordinary *electromagnetic gauge transformations*

$$A_\mu \to A_\mu + \partial_\mu \Lambda , \tag{8}$$

and the completely independent *monopole gauge transformations*

$$A_\mu \to A_\mu + \Lambda^{\mathrm{M}}_\mu , \tag{9}$$

$$F^{\mathrm{M}}_{\mu\nu} \to F^{\mathrm{M}}_{\mu\nu} + \partial_\mu \Lambda^{\mathrm{M}}_\nu - \partial_\nu \Lambda^{\mathrm{M}}_\mu , \tag{10}$$

which involves an arbitrary superposition of δ-functions on three-volumes V,

$$\Lambda^{\mathrm{M}}_\mu(x) = 4\pi g \sum_{\mathrm{V}} \delta_\mu(x;\tilde{V}) , \tag{11}$$

with

$$\delta_\mu(x,\tilde{V}) \equiv \epsilon_{\mu\nu\kappa\delta} \times \int d\sigma\, d\tau\, d\lambda \frac{\partial \bar{x}_\nu}{\partial \sigma} \frac{\partial \bar{x}_\kappa}{\partial \tau} \frac{\partial \bar{x}_\delta}{\partial \lambda} \delta^{(4)}(x - \bar{x}(\sigma, \tau, \lambda)) . \tag{12}$$

The invariance of the gradient term in the action (1) is obvious. The current term, on the other hand, changes under the two gauge transformations by i $\times \int d^4x\, j_\mu \partial_\mu \Lambda$ and by i $\int d^4x\, j_\mu \Lambda^{\mathrm{M}}_\mu$, respectively. The first change vanishes after a partial integration for closed worldlines $\partial_\mu \delta_\mu(x;L) = 0$ ensuring the electric current conservation law $\partial_\mu j_\mu = 0$. The second change is irrelevant since $\int d^4x\, \delta_\mu(x;L) \delta_\mu(x;\tilde{V})$ is an integer, counting the number of times by which the line L pierces the volume $\tilde{V}$. The exponential $\exp(-\mathscr{A})$ governing the fluctuations in the functional integral changes by $\exp(-\mathrm{i} \cdot 4\pi egn)$ which is a trivial unit factor due to (7).

Certainly, the functional integrals over A_μ and the surfaces $\tilde{S}$ require gauge fixing to remove infinite degeneraties. The options for gauge fixing A_μ are well known; for $\tilde{S}$ one may fix the surface shapes in such a way that they are uniquely determined by their boundary lines $\tilde{L}$. This was the key for constructing a field theory of magnetic monopoles in ref. [6].

It was also shown in refs. [5,6] that a duality transformation brings $\mathscr{A}$ to the completely equivalent form

$$\tilde{\mathscr{A}} = \frac{1}{16\pi} \int d^4x [\tilde{F}_{\mu\nu}(x) - \tilde{F}^{\mathrm{E}}_{\mu\nu}(x)]^2 + \mathrm{i} \int d^4x\, \tilde{j}_\mu(x) \tilde{A}_\mu(x) , \tag{13}$$

where $\tilde{F}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$ is the dual field tensor $\tilde{F}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\lambda\kappa} F_{\mu\nu}$ and $\tilde{j}_\mu(x) = g\delta_\mu(x;\tilde{L})$ the dual current density singular on the magnetic monopole worldlines $\tilde{L}$. Now the electric charges are described by a *charge gauge field* $\tilde{F}^{\mathrm{E}}_{\mu\nu}$ which is singular on worldsurfaces S enclosed by the electric worldlines L:

$$\tilde{F}^{\mathrm{E}}_{\mu\nu}(x) = 4\pi e \cdot \tfrac{1}{2} \epsilon_{\mu\nu\lambda\kappa} \delta_{\lambda\kappa}(x;S) . \tag{14}$$

This action is, of course, invariant under the *magnetoelectric gauge transformations*

$$\tilde{A}_\mu \to \tilde{A}_\mu + \partial_\mu \tilde{\Lambda} \tag{15}$$

and under the discrete-valued *charge gauge transformations*

$$\tilde{A}_\mu \to \tilde{A}_\mu + \tilde{\Lambda}^{\mathrm{E}}_\mu ,$$

$$\tilde{F}^{\mathrm{E}}_{\mu\nu} \to \tilde{F}^{\mathrm{E}}_{\mu\nu} + \partial_\mu \tilde{\Lambda}^{\mathrm{E}}_\nu - \partial_\nu \tilde{\Lambda}^{\mathrm{E}}_\mu . \tag{16}$$

In my 1982 Erice lectures I have shown how to transform the lattice action of a compact U(1) lattice gauge model into a dual Higgs model which corresponds to a Ginzburg–Landau theory of a four-dimensional superconductor involving the dual vector potential $\tilde{A}_\mu$. The same method can now be applied to the present continuum formulation. The crucial observation is that sums over grand-canonical ensembles of fluctuating closed non-selfbacktracking worldlines of monopoles of any number and shape, to be denoted by

$$\sum_{\{\tilde{L}\}} \exp\left\{ i \int d^4x \tilde{j}(x) \tilde{A}_\mu(x) \right\}, \tag{17}$$

can be transformed into a *disorder field theory* [4] described by the functional integral

$$\int \mathscr{D}\psi^* \mathscr{D}\psi \times \exp\left\{ -\int d^4x (|\tilde{D}\psi|^2 + m^2|\psi|^2 + \lambda|\psi|^4) \right\}, \tag{18}$$

where $\tilde{D}_\mu \equiv \partial_\mu - g\tilde{A}_\mu$ is the covariant derivative involving the dual gauge field. When performing a perturbation expansion of this functional integral in powers of the coupling constant λ, the Feynman loop diagrams of the ψ field provide direct pictures for the different ways in which the closed monopole worldlines interact in the ensemble.

The mass parameter m^2 is proportional to $g/g_c - 1$ where g_c is some critical value of the magnetic charge [3]. For $g < g_c$, m^2 is negative and the disorder field ψ develops the nonzero expectation $|\psi|$ whose absolute value is equal to $\sqrt{|m^2|/2\lambda}$. From the derivative term $|\tilde{\mathscr{D}}\psi|^2$, the dual field $\tilde{A}_\mu$ receives a mass term $(\tilde{m}_A^2/8\pi)\tilde{A}_\mu^2$ with $\tilde{m}_A^2$ equal to $8\pi g^2|m^2|/2\lambda$. For small enough g the penetration depth $1/\tilde{m}_A$ of the vector potential is much larger than the coherence length $1/m$ of the disorder field and the system behaves like a dual superconductor of type II. Between charges of opposite sign, the electric field lines are squeezed into the four-dimensional analogs of the Abrikosov flux tubes. Within the present functional integral, the initially irrelevant surfaces S enclosed by the charge worldlines L acquire, via the phase transition, an energy proportional to their area which removes the charge gauge invariance of the action. They become physical fluctuating objects and generate the linearly rising static potential between the charges, thus causing charge confinement.

This mechanism is particularly simple to describe in the London limit. Then the size of the field ψ is frozen so it can be replaced by a constant $|\psi|$ multiplied by a spacetime-dependent phase factor $\exp[i\theta(x)]$ and the functional integral (18) reduces to

$$\int \mathscr{D}\theta \exp\left\{ -\frac{\tilde{m}_A^2}{8\pi g^2} \int d^4x (\partial_\mu\theta - g\tilde{A}_\mu)^2 \right\}, \tag{19}$$

Thus the action (13) reads, in the London limit,

$$\tilde{\mathscr{A}}_{LL} = \frac{1}{4\pi} \int d^4x \left(\tfrac{1}{4}(\tilde{F}_{\mu\nu} - \tilde{F}^E_{\mu\nu})^2 + \frac{\tilde{m}_A^2}{2g^2} (\partial_\mu\theta - g\tilde{A}_\mu)^2 \right). \tag{20}$$

Integrating out the θ fluctuations in the functional integral generates a transverse mass term

$$\frac{\tilde{m}_A^2}{8\pi} \tilde{A}_\mu^{T2}, \tag{21}$$

where $\tilde{A}_\mu^T \equiv (g_{\mu\nu} - \partial_\mu\partial_\nu/\partial^2)\tilde{A}_\nu$. This causes the celebrated Meissner effect, now appearing in the dual superconductor. In the London limit the action becomes, therefore,

$$\tilde{A}_{LL} = \frac{1}{4\pi} \int d^4x [\tfrac{1}{4}(\tilde{F}_{\mu\nu} - \tilde{F}^E_{\mu\nu})^2 + \tfrac{1}{2}\tilde{m}_A^2 \tilde{A}_\mu^{T2}]. \tag{22}$$

Integrating out the $\tilde{A}_\mu$ fields gives the interaction between the worldlines of electric charges L and the surfaces S enclosed by them

$$\mathscr{A}_{LL,int} = \int d^4x \left(\frac{1}{16\pi} \times [(\tilde{F}^E_{\mu\nu})^2 - 2\,\partial_\mu \tilde{F}^E_{\mu\nu}(-\partial^2 + \tilde{m}_A^2)^{-1} \partial_\lambda \tilde{F}^E_{\lambda\nu}] + \tfrac{1}{2}\,\partial_\mu \tilde{F}^E_{\mu\nu}(-\partial^2 + \tilde{m}_A^2)^{-1} j_\nu + \frac{4\pi}{2} j_\mu(-\partial^2 + \tilde{m}_A^2)^{-1} j_\mu \right). \tag{23}$$

The functional integral is now to be taken over all L and S fluctuations with no more gauge fixing required. Using $\epsilon_{\mu\nu\lambda\kappa}\,\partial_\nu \tilde{F}^E_{\lambda\kappa} = 2(\partial_\nu \tilde{F}^E_{\lambda\kappa})^2 - 4(\partial_\mu \tilde{F}^E_{\mu\nu})^2$ this becomes

$$\tilde{A}_{\rm LL,int} = \int d^4x \int d^4x' \times\left(\frac{1}{16\pi}\tilde{m}_A^2 \tilde{F}^{\rm E}_{\mu\nu}(x) G_{\tilde{m}_A}(x-x') \tilde{F}^{\rm E}_{\mu\nu}(x') + \tfrac{1}{2}\partial_\mu \tilde{F}^{\rm E}_{\mu\nu} G_{\tilde{m}_A}(x-x') j_\mu(x-x') + \frac{4\pi}{2} j_\mu(x) G_{\tilde{m}_A}(x-x') j_\mu(x')\right), \quad (24)$$

with the massive correlation function

$$G_{\tilde{m}_A}(x) = \int \frac{d^4k}{(2\pi)^4} \exp(-ikx) \frac{1}{k^2+\tilde{m}_A^2} . \quad (25)$$

The last term is a short-range interaction between the electric charges. The first term gives the desired energy to the previously irrelevant surfaces S enclosed by the L and thus the confining part of the potential. The second term is a short-range interaction between the surfaces and the boundary lines.

We are now going to derive a completely equivalent formulation within the original A_μ field theory (1). The crucial observation is that in the London limit the monopoles are so prolific that the discrete-valued monopole gauge field $F^{\rm M}_{\mu\nu}(x)$ can be replaced by an ordinary continuum field $f_{\mu\nu}(x)$ [4]. The fluctuations are kept finite by the *core energy* of the monopole worldlines arising from the short-range part of the magnetoelectric interactions between the monopoles (which is *not* screened as the long-range parts are, see the lattice derivation in my Erice lectures [4]). This core energy is proportional to their length. It can therefore be written in the form

$$\frac{1}{8\pi\tilde{m}_A^2}\int d^4x (\tfrac{1}{2}\epsilon_{\mu\nu\lambda\kappa}\,\partial_\nu f_{\lambda\kappa})^2 . \quad (26)$$

With the continuum limit of the monopole gauge field the action for the London limit of the dual theory becomes then

$$\mathscr{A} = \frac{1}{4\pi}\int d^4x\left(\tfrac{1}{4}(F_{\mu\nu}-f_{\mu\nu})^2 + \frac{1}{2\tilde{m}_A^2}(\tfrac{1}{2}\epsilon_{\mu\nu\lambda\kappa}\,\partial_\nu f_{\lambda\kappa})^2\right) + i\int d^4x\, j_\mu(x) A_\mu(x) . \quad (27)$$

Note that the monopole gauge transformations are now given by

$$A_\mu \to A_\mu + \Lambda_\mu , \quad f_{\mu\nu} \to f_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu , \quad (28)$$

with an *arbitrary vector function* $\Lambda_\mu(x)$ [not restricted to the superpositions (11)].

The equivalence with (22) is established by introducing an auxiliary field $\tilde{f}_{\mu\nu}$ and replacing the first term in the brackets by

$$\tfrac{1}{4}\tilde{f}^2_{\mu\nu} + \tfrac{1}{2}i\tilde{f}_{\mu\nu}(F_{\mu\nu}-f_{\mu\nu}) ,$$

integrating out the A_μ field in the functional integral to obtain the condition $\partial_\mu \tilde{f}_{\mu\nu} = 0$, solving it by the dual gauge field ansatz

$$\tilde{f}_{\mu\nu} = \epsilon_{\mu\nu\lambda\kappa}\,\partial_\lambda \tilde{A}_\kappa , \quad (29)$$

and integrating out the $f_{\mu\nu}$ gauge field in the resulting action

$$\frac{1}{4\pi}\left(i\tilde{A}_\mu(\tfrac{1}{2}\epsilon_{\mu\nu\lambda\kappa}\,\partial_\nu f_{\lambda\kappa}) + \frac{1}{2\tilde{m}_A^2}(\tfrac{1}{2}\epsilon_{\mu\nu\lambda\kappa}\,\partial_\nu f_{\lambda\kappa})^2\right) \quad (24)$$

(after fixing the monopole gauge, of course). This renders precisely the transverse mass term $\tilde{A}^{\rm T2}_\mu$ in (22) (dual Meissner effect) which, in turn, gives rise to charge confinement.

Note that while the new type of gauge invariance is still present for the monopoles, the irrelevance of the surfaces S enclosed by the charge worldlines L is destroyed in the confined phase.

A continuum model can also be written down without going to the London limit. For this we take the action (1) and liberate the field $F^{\rm M}_{\mu\nu}$ from its discreteness properties with the help of an arbitrary fluctuating auxiliary field $\tilde{A}_\mu$ by writing $\mathscr{A}$ as

$$\mathscr{A} = \frac{1}{16\pi}\int d^4x (F_{\mu\nu}-f_{\mu\nu})^2 + i\int d^4x\, j_\mu A_\mu - i\int d^4x\, \tilde{A}_\mu\left(\frac{1}{8\pi}\epsilon_{\mu\nu\lambda\kappa}\,\partial_\nu f_{\lambda\kappa} - \tilde{j}_\mu\right) . \quad (31)$$

In this expression, the $f_{\mu\nu}$ fields are arbitrary fields but are forced by the $\tilde{A}_\mu$ fluctuations to become proper monopole gauge fields. The sum over the grand-canonical monopole worldlines turns the last term in this action into a Higgs model as in (18), (17) so that the general charge confining theory has the action

$$\mathscr{A}=\frac{1}{16\pi}\int d^4x(F_{\mu\nu}-f_{\mu\nu})^2+i\int d^4x\, j_\mu A_\mu$$

$$+\int d^4x\left(-i\tilde{A}_\mu\frac{1}{8\pi}\tfrac{1}{2}\epsilon_{\mu\nu\lambda\kappa}\,\partial_\nu f_{\lambda\kappa}\right.$$

$$\left.+|\tilde{D}\psi|^2+m^2|\psi|^2+\lambda|\psi|^4\right). \tag{32}$$

This model is triple-gauge-invariant, under the two gauge transformations (8), (28) and under the dual gauge transformation

$$\tilde{A}_\mu\to\tilde{A}_\mu+\partial_\mu\tilde{A}\,,$$
$$\psi\to\exp(ig\tilde{A})\psi\,. \tag{33}$$

In the London limit, the Higgs partition function reduces again to (19) and produces a transverse mass term (21) for $\tilde{A}_\mu$ which, together with the other $\tilde{A}_\mu$ term in (32), generates the gradient term of $f_{\mu\nu}$ in the action (27).

It should be noted that before going to the London limit, the functional integral over the Higgs part of the partition function associated with (32) can also be replaced by a form more closely related to (19):

$$\int\mathscr{D}\theta\sum_{\{\tilde{V}\}}\exp\left(-\frac{\tilde{m}_A^2}{8\pi g^2}\int d^4x[\partial_\mu\theta(x)\right.$$

$$\left.-g\tilde{A}_\mu(x)-2\pi\delta_\mu(x;\tilde{V})]^2\right). \tag{34}$$

The softening of the size fluctuations of the Higgs field ψ allows for jumps of the phase fluctuations by 2π. In (34) these are accounted for by the δ_μ-functions on all possible volume elements $\tilde{V}$. A quadratic completion brings (34) to

$$\int\mathscr{D}l_\mu\int\mathscr{D}\theta\exp\left[-\int d^4x\left(\frac{4\pi g^2}{2\tilde{m}_A^2}l_\mu^2(x)\right.\right.$$

$$\left.\left.-il_\mu(x)[\partial_\mu\theta-g\tilde{A}_\mu-2\pi\delta_\mu(x;\tilde{V})]\right)\right]. \tag{35}$$

Integrating out the $\theta(x)$ field makes $l_\mu(x)$ satisfy $\partial_\mu l_\mu(x)\equiv 0$. The sum over all $\tilde{V}$ enforces for $l_\mu(x)$ the form $l_\mu=\delta_\mu(x;\tilde{L})$ (continuous version of Poisson's summation formula). The singular lines $\tilde{L}$ are the monopole worldlines described before by the Higgs field ψ. It is convenient to write $l_\mu(x)=(1/g)\tilde{j}_\mu(x)$ as in (6) so that (35) becomes

$$\sum_{\{\tilde{L}\}}\exp\left[-\int d^4x\left(\frac{4\pi}{2\tilde{m}_A^2}\tilde{j}_\mu^2(x)+i\tilde{j}_\mu(x)\tilde{A}_\mu\right)\right]. \tag{36}$$

Setting $j_\mu(x)=(1/4\pi)\epsilon_{\mu\nu\lambda\kappa}\partial_\nu F^{M}_{\lambda\kappa}(x)$ and integrating out the $\tilde{A}_\mu$ field we find the initial action (1), but with the additional core energy for the monopoles, the same that was introduced when deriving the Higgs action from the sum over all line configurations $\tilde{L}$.

In the London limit, the monopole lines $\tilde{L}$ are prolific so that the restriction of $j_\mu(x)$ to the form $g\delta_\mu(x;\tilde{L})$ becomes irrelevant and $F^{M}_{\mu\nu}(x)$ can be replaced by $f_{\mu\nu}(x)$ thus establising contact with (2).

It goes without saying that in order to apply the model to quarks, the electric current term in (27) has to be replaced by a Dirac action with the usual gauge-invariant coupling

$$\mathscr{A}_D=\int d^4x\,\bar{\psi}(\not{D}-\mathscr{M})\psi\,, \tag{37}$$

where $\mathscr{M}$ is some small current mass matrix in flavor space:

$$\mathscr{M}=\begin{pmatrix}\mathscr{M}_u & & & \\ & \mathscr{M}_d & & \\ & & \mathscr{M}_s & \\ & & & \ddots\end{pmatrix}. \tag{38}$$

The action (24) contains then a four-Fermi interaction of precisely the kind that has been transformed a long time ago [7] into an action of pseudoscalar, scalar, vector, and axial-vector mesons via a functional integral technique which we have called *hadronization*. It reproduces successfully many of the low-energy properties of these hadrons, in particular their chiral symmetry, its spontaneous breakdown, and the relation between current and constituent masses of the quark fields. This technique has since then been the subject of numerous investigations and generalizations (in particular by the color degree of freedom). It has recently been used to describe the low-lying baryons and the restoration of chiral symmetry by thermal effects [8]. The inclusion of the surface energy will be an important task for the future.

An interesting aspect of (24) is that the local part of the four-Fermi interaction, which is proportional to $1/\tilde{m}_A^2$, arises by the same mechanism as the confining potential, whose tension is proportional to $\tilde{m}_A^2 \log(\Lambda^2/\tilde{m}_A^2)$, with Λ being some ultraviolet cutoff parameter. One would therefore predict that at an increased temperature of the order of $\tilde{m}_A$ the spontaneous symmetry breakdown, which is caused by the four-Fermi interaction, takes place at the same temperature at which the potential loses its deconfinement properties. This initially surprising coincidence has long been observed in Monte Carlo simulations of lattice gauge theories.

In a forthcoming publication [9] we shall investigate the static and fluctuation properties of strings implied by this term and the resulting potential between quarks. It is immediately obvious that for short distances, the $1/r$ term in the Yukawa interaction between the quarks due to the second part in eq. (24) will be dominant. As discussed in ref. [7], this explains the success of the pure four-Fermi theory as far as the low-lying mesons are concerned since they are bound mainly by the short-distance part of the potential. At larger distances, the linearly rising potential due to the first term will become most important. There exists, as yet, no efficient scheme for including it into the hadronized action.

One fluctuation property of the surface S is obvious from earlier investigations of composite strings: The extrinsic curvature stiffness which was introduced some time ago by Polyakov [10] and the author [11] to smoothen string fluctuations near a critical point has, surprisingly, the opposite sign [12] from what was expected from the analogy with membranes in physical chemistry [11].

It is an important open problem to generalize the above London limit to the case of colored gluons. In particular, the existence of three- and four-string vertices must be accounted for in a simple way. A promising possibility may open up via the recently much discussed 't Hooft [13] hypothesis of dominance of abelian monopoles [14]. Alternatively, a clever assignment of color indices to the gluons and surfaces which in the London limit avoids introducing the complexity of all gluonic self-interactions may eventually be found [15] [#2].

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[#2] Attempts in this direction have been made by Hoček [15]. This approach has, however, stability problems and does not possess the double-gauge invariance which is essential for the irrelevance of the surfaces $\tilde{S}$ enclosed by the monopole worldlines $\tilde{L}$.

References

[1] K.G. Wilson, Phys. Rev. D 10 (1974) 2445;
T. Banks, R. Myerson and J. Kogut, Nucl. Phys. B 129 (1977) 493;
M. Peskin, Ann. Phys. 113 (1978) 122;
J.L. Cardy, Nucl. Phys. B 170 (1980) 369;
T.A. De Grand and D. Toussaint, Phys. Rev. D 22 (1980) 2478;
P. Cea and L. Cosmai, Phys. Lett. B 249 (1990) 114;
J.D. Stack and R.J. Wensley, preprint ILL-(TH)-91-11.

[2] S. Mandelstam, Phys. Rev. D 19 (1979) 2391;
G. 't Hooft, Nucl. Phys. B 153 (1979) 141;
M. Baker, J.S. Ball and F. Zachariasen, Phys. Rep. 209 (1991) 73.

[3] H. Kleinert, Higgs particles from pure gauge fields, Lecture presented at the Erice Summer School (1982), in: Gauge interactions, ed. A. Zichichi (Plenum, New York, 1984).

[4] H. Kleinert, Gauge fields in condensed matter, Vols. I, II (World Scientific, Singapore, 1989).

[5] H. Kleinert, J. Mod. Phys. A 7 (1992) 1693.

[6] H. Kleinert, Phys. Lett. B 246 (1990) 127.

[7] H. Kleinert, On the hadronization of quark theories, Lectures presented at the Erice Summer Institute (1976), Understanding the fundamental constituents of matter, ed. A. Zichichi (Plenum, New York, 1978) pp. 289–390.

[8] R. Cahill, C.D. Roberts and J. Praschifka, Aust. J. Phys. 42 (1989) 129;
R. Cahill, J. Praschifka and C.J. Burden, Aust. J. Phys. 42 (1989) 161;
R. Cahill, Aust. J. Phys. 42 (1989) 171; Nucl. Phys. A 543 (1992) 63c;
H. Reinhardt, Phys. Lett. B 244 (1990) 316;
V. Christov, E. Ruiz Arriola and K. Goeke, Phys. Lett. B 243 (1990) 191;
D. Ebert and L. Kaschluhn, Phys. Lett. B, to be published.

[9] M. Kiometzis and H. Kleinert, to be published.

[10] A.M. Polyakov, Nucl. Phys. B 268 (1986) 406.

[11] H. Kleinert, Phys. Lett. B 174 (1986) 335.

[12] N. Turok and K. Maeda, Phys. Lett. B 202 (1988) 376;
S.M. Barr and D. Hochberg, Phys. Rev. D 39 (1989) 2308.

[13] G. 't Hooft, Nucl. Phys. B 190 (1981) 455.

[14] A.S. Kronfeld, G. Schierholz and U.-J. Wiese, Nucl. Phys. B 293 (1987) 461;
A.S. Kronfeld, M.L. Laursen, G. Schierholz and U.-J. Wiese, Phys. Lett. B 198 (1987) 516;
F. Brandstaeter, G. Schierholz and U.-J. Wiese, DESY preprint 91-040 (1991);
T. Suzuki and I. Yotsuyanagi, Phys. Rev. D 42 (1991) 4257;

S. Hioki, S. Kitahara, S. Kiura, Y. Matsubara, O. Miyamura, S. Ohuo and T. Suzuki, preprint DPKU-9106 (1991);
J. Smit and A. Van der Sijs, Nucl. Phys. B 355 (1991) 603;
V.G. Bornyakov, E.M. Ilgenfritz, M.L. Laursen, V.K. Mitrijushkin, M. Müller-Preußker, A.J. Van der Sijs and A.M. Zadorozhny, Phys. Lett. B 261 (1991) 116;
J. Greensite and J. Winchester, Phys. Rev. D 40 (1989) 4167;
J. Greensite and J. Iwasaki, Phys. Lett. B 255 (1991) 415;
A. Di Giacomo, M. Maggiore and Š. Olejník, Nucl. Phys. B 347 (1990) 441;
M. Campostrini, A. Di Giacomo, H. Panagopoulos and E. Vicari, Nucl. Phys. B 329 (1990) 683;
M. Campostrini, A. Di Giacomo, M. Maggiore, Š. Olejník, H. Panagopoulos and E. Vicari, Nucl. Phys. B (Proc. Suppl.) 17 (1990) 563;
L. Del Debbio, A. Di Giacomo, M. Maggiore and Š. Olejník, Phys. Lett. B 267 (1991) 254;
L. Polley and U.-J. Wiese, Nucl. Phys. B 356 (1991) 629;
H.G. Evertz, K. Jansen, J. Jersák, C.B. Lang and T. Neuhaus, Nucl. Phys. B 285 (1987) 590;
T.L. Ivanenko, A.V. Pochinsky and M.I. Polikarpov, preprint LNF-91/086;
P. Cea and L. Cosmai, Bari preprint TH 89/91.
[15] J. Hoček, Czech. J. Phys. B 41 (1991) 1037; Prague preprints REZ-TH-91/5, 92/1.