A Mixing Operator for the $SU_3 \times SU_3$ Chiral Algebra (*).

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Summary. — A saturation scheme within an infinite set of states is proposed for the chiral algebra. Mesons are classified in the ($\underline{35}$, any L) representations of $SU_6 \times O_3$, while the ($\underline{56}$, $L =$ even) and ($\underline{70}$, $L =$ odd) representations are chosen for baryons. A mixing operator $\exp[-i\theta Z]$ is proposed which transforms the axial charges of the SU_6 solution into the physical ones. The specific form we choose for Z gives rise to many predictions. All the axial couplings of the lower positive-parity meson states to π, ρ and ω are obtained in terms of only one parameter and the results are in fair agreement with experiment. In particular, the B-meson is predicted to decay transversely, while the g_1/g_0 ratio for the A_1 is found to be $-\frac{1}{2}$. For the $\frac{1}{2}^+$ baryon octet, we get $D/F = \frac{3}{2}$ and $G^* = \frac{4}{5} G_A$ (which implies $\Gamma_{\Delta \to \mathcal{N}\pi} = 125$ MeV) up to second order in the mixing angle θ. For each $SU_6 \times O_3$ multiplet all the decays into $\mathcal{N} + \pi$ are given in terms of only one parameter (at lowest possible order in the mixing angle) in general agreement with experiment. Finally, we are able to obtain strong restrictions on the chiral content of the $\frac{1}{2}^+$ octet, which are perfectly compatible with nature.

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1. – Introduction.

While duality people are dwelling in immense systems of particles (and ghosts), saturation attempts to the algebra of vector and axial-vector charges have, in the past, modestly limited themselves to considering only small sets of baryon and meson resonances ([1-3]). Following the original suggestion of GELL-MANN, inspired by the quark model, all saturation schemes are based on a finite number of representations of $SU_6 \times O_3$ (or $SU_4 \times O_3$). The most extensive discussion of baryons that can be found in the literature saturates with $SU_6 \times O_3$ multiplets $(\underline{56}, L = 0)$, $(\underline{70}, L = 1)$, $(\underline{56}, L = 2)$ ([2]), while meson considerations have worked their way up to $SU_4 \times O_3$ multiplets $(\underline{15}, L = 0)$ and $(\underline{15}, L = 1)$ ([3]). This limitation has a simple reason: particles have definite transformation properties only under the groups SU_2 (or SU_3), P and C, and therefore will, in general, consist of arbitrary mixtures of $SU_2 \times SU_2$ (or $SU_3 \times SU_3$) representations compatible with the selection rules of the axial charge. Thus baryons can mix arbitrarily, while mixed meson states possess the same G-parity and, at the helicity zero level, the same normality $N = P(-)^J$. In addition, elastic matrix elements have to be proportional to the helicity of the particles. These restrictions fortunately reduce the number of mixing angles in the case of the $(\underline{15}, L = 0)$ and $(\underline{15}, L = 1)$ meson scheme to two (π-A_1 at helicity 0 and ρ-B mixing at helicity ± 1) ([3]). In the baryon case, however, even with further restrictions, six angles are needed in order to describe the mixing between the $(\underline{56}, L = 0)$ and the $(\underline{70}, L = 1)$ ([2]).

It is well known that limited saturation schemes cannot account for many empirical facts and theoretically accepted ideas about particle properties. First, meson and baryon trajectories rise almost linearly up to high values of spin and possibly indefinitely. Second, Adler-Weisberger relations in which the highest members in a finite saturation scheme are taken as targets yield, in general, too large coupling constants, since neighbouring resonances of higher masses are missing in the intermediate states which would bring down the individual couplings. Finally, it is well known that local commutation rules of current densities require an infinite set of particle states if they are

([1]) R. DASHEN and M. GELL-MANN: *Coral Gables Conference on Symmetry Principles at High Energy*, Vol. **3** (1966); R. GATTO, L. MAIANI and G. PREPARATA: *Physics*, **3**, 1 (1967); *Phys. Rev. Lett.*, **16**, 377 (1966); *Nuovo Cimento*, **44** A, 1279 (1966); H. HARARI: *Phys. Rev. Lett.*, **16**, 964 (1966); N. CABIBBO and H. RUEGG: *Phys. Lett.*, **22**, 85 (1966).

([2]) F. BUCCELLA, M. DE MARIA and M. LUSIGNOLI: *Nucl. Phys.*, **6** B, 430 (1968); F. BUCCELLA, M. DE MARIA and B. TIROZZI: *Nucl. Phys.*, **8** B, 521 (1968); F. BUCCELLA, E. CELEGHINI and E. SORACE: *Lett. Nuovo Cimento*, **1**, 556 (1969); **2**, 571 (1969).

([3]) C. BOLDRIGHINI, F. BUCCELLA, E. CELEGHINI, E. SORACE and L. TRIOLO: Nota Interna No. 262, Istituto di Fisica dell'Università, Roma.

to yield realistic electromagnetic form factors of nucleons and their resonances (4).

For these reasons, any group classification scheme of the particle states will need, for the description of the internal excitations, at least a reducible unitary representation of $O_{3,1}$. A single irreducible representation corresponds to a quark model, in which an orbital angular momentum can be excited from some value L_0 to ∞. Such a spectrum, however, is too small to accomodate a possible occurrence of daughters of Regge trajectories. Therefore GELL-MANN and ZWEIG have proposed the group $O_{4,1}$ allowing for orbital excitations with H-atom degeneracy (5). At the present experimental situation, such a spectrum seems far too large to be taken seriously. A better candidate appears to us: the spectrum of the three-dimensional oscillator described by an irreducible representation of $U_{3,1}$. This spectrum can be labelled by a principal quantum number $n = 0, 1, 2, 3, \ldots$, for each of which L can take the values $L = 0, 2, \ldots, n$ for $n =$ even and $L = 1, 3, \ldots, n$ for $n =$ odd. Exciting a quark-antiquark system $\underline{15}$ with these orbits one finds in the π-π channel exactly the same multiplicity of particles as there are poles in the four-point Veneziano amplitude, except that every second scalar meson is missing. It is curious to note that the Veneziano amplitude for π-π does, in fact, not couple the second scalar meson and gives relatively small values for the fourth, sixth, etc., σ-meson (6). For the baryons, an $L = 0$ multiplet for $n = 2$ accounts comfortably for the Roper resonance with the quantum numbers of the nucleon.

Whatever the exact group of internal excitations of the quark model may be, the crucial problem is how to economize the mixing procedure without introducing infinitely many parameters. In this work, we propose to use a simple unitary «mixing» operator

$$T = \exp[-i\theta Z] \tag{1}$$

with the generator

$$Z = (\boldsymbol{W} \times \boldsymbol{M})_3 = i(W_+ M_- - W_- M_+) . \tag{2}$$

The operator $\boldsymbol{M}$ is the generator of the Lorentz group, while $W^\pm$ transform

(4) This is the content of the theorem by F. COESTER and G. ROEPSTORFF: *Phys. Rev.*, **155**, 1583 (1967). For a calculation of such form factors (as for example the dipole formula for the nucleons) see: H. KLEINERT: *Springer Tracts in Modern Physics*, Vol. **49** (1969).

(5) For a discussion of this model, see: H. HARARI: talk in the *Proceedings of the XIV International Conference on High-Energy Physics* (Vienna, 1968).

(6) The Veneziano model for the π-π amplitude, with massless pions gives $R_2 = 0$, $R_3 = 0.14$, $R_4 = 0.013$, $R_5 = 0.07$, $R_6 = 0.013, \ldots$, where R_n is the ratio between the partial widths of $\sigma_n \to \pi\pi$ and $\sigma_1 \to \pi\pi$ and we see that the even σ-mesons are indeed coupled much less than the others.

like the $\sigma^{\pm}$ members of the $\underline{15}$ (or $\underline{35}$) for SU_4 (or SU_6), respectively. For the mesons, W is taken to connect only $\underline{15}$ and $\underline{15}$ (or $\underline{35}$ and $\underline{35}$) symmetrically [7], while for the baryons we assume it to contain only the transitions $\underline{56} \rightleftarrows \underline{70}$.

Our choice of Z makes the matrix elements of the axial charge [8] $X_{\beta\alpha}(h)$ (h = helicity) satisfy the physical constraints. The reason is, briefly, that Z is a positive G-parity, isosinglet, $J^P = 1^-$ operator. Therefore $X_{\beta\alpha}(h)$:

i) trivially will change the G-parity,

ii) fulfils the angular condition

$$X_{\beta\alpha}(h) = -\eta_\beta\eta_\alpha(-)^{J_\beta - J_\alpha} X_{\beta\alpha}(-h)\,, \tag{3}$$

which is a consequence of the $J^P=1^-$ character of the Lorentz generator in the multipole expansion of $X(h)$ [9].

In this paper we shall discuss the various general consequences of this prescription of mixing which are independent of the particular choice of the orbital group. We shall show how several mixing angles of earlier works [2,3] are predicted in excellent agreement with experiment. For example, the exper-

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[7] Here it coincides essentially with the W-spin of H. J. LIPKIN and S. MESHKOV: *Phys. Rev. Lett.*, **14**, 670 (1965).

[8] If particles α and β are moving collinearly in the z-direction with helicity h, $X(h)$ is defined as the invariant matrix element of the axial charge at $t=0$:

$$\langle\beta \boldsymbol{p}' h'|Q_5^a|\alpha \boldsymbol{p} h\rangle = 2(p_0 + |\boldsymbol{p}|)\,\delta_{hh'} X_{\beta\alpha}(h)\,.$$

In terms of $X(h)$ the decay of α into β and a massless pion π_a is given by

$$\Gamma_{\alpha\to\beta+\pi} = \lambda\,\frac{(m_\alpha^2 - m_\beta^2)^3}{m_\alpha^3}\,\frac{1}{2J_\alpha+1}\sum_h |X^a_{\beta\alpha}(h)|^2\,,$$

where $\lambda = (16\pi F_\pi^2)^{-1} = 2.2\ (\text{GeV})^{-2}$. The chiral algebra says that X_a is an SU_2 vector commuting according to $[X_a(h), X_b(h)] = i f_{abc} T_c$, where T is the isospin. For a detailed discussion of $X(h)$ see: S. WEINBERG: *Phys. Rev.*, **177**, 2604 (1968).

[9] The condition ii) follows by writing $X_{\beta\alpha}(h)$ for states at rest,

$$X_{\beta\alpha}(h) = \sqrt{\frac{m_\alpha}{m_\beta}}\,\langle\beta 0 h|Q_6^{0+3}\exp\left[iM_3\log\frac{m_\alpha}{m_\beta}\right]|\alpha 0 h\rangle$$

expanding the exponential and multipole analyzing the products $Q_5^{0+3}M_3^n$. In our case the mixing generator Z, which is the third component of a vector operator under $J = L + S$ obviously produces the same multipole properties as M_3.

In particular, the elastic matrix element $X_{\alpha\alpha}$ is a pure vector, which is clear from the equation above since $m_\alpha = m_\beta$, and $Q_5^{0+3} \to Q_5^0$, as a pseudoscalar, has no elastic transitions. That this property is fulfilled by our mixed states is not quite as trivial to see. Up to the lowest order we have verified this property.

imental polarization of the $A_1 \to \rho\pi$ decay ([10])

$$\left| \frac{X_{A_1\rho}(1)}{X_{A_1\rho}(0)} \right| = 0.48 \pm 0.13 \tag{4}$$

comes out to be $\frac{1}{2}$.

For the nucleons, the most important result is

$$G^* = \frac{4}{5} G_A \,, \qquad \frac{D}{F} = \frac{3}{2} \,, \tag{5}$$

which gives (with $G_A = \frac{5}{4}$) a width of

$$\Gamma_{\Delta \to \mathcal{N}\pi} = 125 \text{ MeV} \,,$$

if one calculates Γ by integrating over the Δ peak ([11]). For the higher meson and baryon resonance decays, see the text and Table II.

The specific infinite-component properties of our mixing operator will be discussed in a future paper.

2. – Meson system.

The quark-antiquark representation $\underline{15}$ of SU_4 consists of ω, ρ, π, the spin components of which decompose under the chiral $SU_2 \times SU_2$ subgroup

$$\boldsymbol{Q}^+ = \tfrac{1}{4} \boldsymbol{\tau}(1 + \sigma_3) \,, \qquad \boldsymbol{Q}^- = \tfrac{1}{4} \boldsymbol{\tau}(1 - \sigma_3) \,,$$

according to

$$\boldsymbol{\rho} = \begin{pmatrix} \boldsymbol{v}^1 \\ \boldsymbol{t} \\ \boldsymbol{v}^{-1} \end{pmatrix}, \qquad \omega = \begin{pmatrix} v_4^1 \\ s \\ -v_4^{-1} \end{pmatrix}, \qquad \pi_i = t_{4i} \,.$$

([10]) J. BALLAM *et al.*: SLAC publication 627, submitted to *Phys. Rev.*

([11]) The connection between G^* and the $\Delta\mathcal{N}\pi$ coupling constant defined by

$$\mathscr{L} = (g^*/\mu)\, \bar{\Delta}_\nu \mathcal{N} \partial_\nu \pi$$

is $G^{*2} = 4(g^{*2}/4\pi)$ where

$$\frac{g^{*2}}{4\pi} = \frac{3\mu^2}{\pi} \int\limits_{(M+\mu)^2}^{\infty} \frac{ds}{E + M} \, \frac{\operatorname{Im} f_{33}}{q^2} \,.$$

One finds $g^{*2}/4\pi \simeq 0.26$. See: J. ENGLES, G. HÖHLER and B. PETERSSON: Karlsruhe preprint, April (1968).

Here v_μ, $t_{\mu\nu}$, and s denote the representations $(\frac{1}{2},\frac{1}{2})$, $(1,0)\mp(0,1)$ and $(0,0)$ respectively, and $\bar{t}$ is the dual tensor $\bar{t}_i=\frac{1}{2}\varepsilon_{ijk}t_{jk}$.

The matrix elements of the axial charge

$$\boldsymbol{X}=\boldsymbol{Q}^+-\boldsymbol{Q}^-$$

are given by

$$(6)\qquad \begin{cases} \langle v_4|X_i|v_j\rangle=\langle v_j|X_i|v_4\rangle=\delta_{ij}\,, \\ \langle \bar{t}_i|X_j|t_{4k}\rangle=-\langle t_{4k}|X_j|\bar{t}_i\rangle=i\varepsilon_{ijk}\,, \end{cases}$$

all other transitions being zero.

Suppose that the orbital excitations of the quark-antiquark system can be classified by some reducible representation of $O_{3,1}$, labelled by $|nLL_3\rangle$ (where the principal quantum number n allows the same L to occur more than once). Then the eigenstates of total angular momentum are given by

$$(7)\qquad \begin{cases} \left|nL\begin{pmatrix}\rho\\ \omega\end{pmatrix}Jh\right\rangle=\langle 1S_3LL_3|Jh\rangle\begin{pmatrix}\rho\\ \omega\end{pmatrix}^{S_3}|nLL_3\rangle\,, \\ |nL\pi Jh\rangle=\pi|nLL_3\rangle\,\delta_{JL}\,\delta_{hL_3}\,. \end{cases}$$

Clearly the orbital wave functions $|nLL_3\rangle$ have to be eigenstates of P and C. Since the generator $\boldsymbol{M}$ is odd under both transformations, while $\boldsymbol{L}$ is even, the scalar product $\boldsymbol{L}\cdot\boldsymbol{M}$ vanishes. Hence only triangular representations with lowest spin zero can be mixed in $|nLL_3\rangle$. In this case $\boldsymbol{M}$ has no matrix elements between states of equal L.

Explicitly, one finds at the $L=1$ level the following particles (h=helicity).

$h=2$	$h=1$	$h=0$
$A_2=\boldsymbol{v}^1$	$A_2=(1/\sqrt{2})(\boldsymbol{v}^1+\bar{\boldsymbol{t}})$	$A_2=(1/\sqrt{3})[(1/\sqrt{2})(\boldsymbol{v}^1+\boldsymbol{v}^{-1})+\sqrt{2}\,\bar{\boldsymbol{t}}]$
$f\;=v_4^1$	$f\;=(1/\sqrt{2})(v_4^1+s)$	$f\;=(1/\sqrt{3})[(1/\sqrt{2})(v_4^1-v_4^{-1})+\sqrt{2}\,s]$
	$A_1=(1/\sqrt{2})(\boldsymbol{v}^1-\bar{\boldsymbol{t}})$	$A_1=(1/\sqrt{2})(\boldsymbol{v}^1-\boldsymbol{v}^{-1})$
	$D\;=(1/\sqrt{2})(v_4^1-s)$	$D\;=(1/\sqrt{2})(v_4^1+v_4^{-1})$
	$\boldsymbol{B}\;=\boldsymbol{t}$	$\boldsymbol{B}\;=\boldsymbol{t}$
		$A_0=(1/\sqrt{3})[(\boldsymbol{v}^1+\boldsymbol{v}^{-1})-\bar{\boldsymbol{t}}]$
		$\sigma\;=(1/\sqrt{3})[(v_4^1-v_4^{-1})-s]$

In order to allow any of these states to decay into the ground states $(+\pi)$, mixing has to be introduced. Neglecting the contribution of higher states, the most one can mix is, at the $h=1$ level,

$$(8) \qquad \rho' = \sqrt{1-\beta^2-\dots}\,\rho + \beta B\,, \qquad B' = -\beta\rho + \sqrt{1-\beta^2-\dots}\,B\,,$$

and at the $h=0$ level

$$(9) \qquad \pi' = \sqrt{1-\alpha^2-\dots}\,\pi + \alpha A_1\,, \qquad A_1' = -\alpha\pi + \sqrt{1-\alpha^2-\dots}\,A_1\,.$$

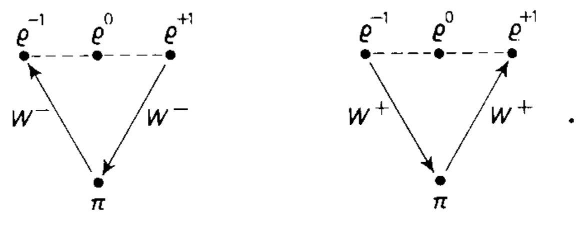
This gives for the reduced matrix elements of the axial charge at first order in the mixing angles [12]

$$(10) \qquad \left\{\begin{array}{ll} G_{t\pi}(0) = \dfrac{\alpha}{\sqrt{3}}\,, & G_{B\omega}(0) = 0\,, \\[2ex] G_{\sigma\pi}(0) = \sqrt{\dfrac{2}{3}}\,\alpha\,, & G_{B\omega}(1) = -\beta\,, \\[2ex] G_{A_1\rho}(0) = -\alpha\,, & G_{A_2\rho}(1) = \dfrac{\beta}{\sqrt{2}}\,. \\[2ex] G_{A_1\rho}(1) = \dfrac{-\beta}{\sqrt{2}}\,, & \end{array}\right.$$

In order to connect the parameters α and β consider the mixing prescribed by the operator T:

$$T = \exp\left[-i\theta Z\right]\,, \qquad Z = i(W_+M_- - W_-M_+)\,.$$

The matrix elements of $W^\pm$ can be given most easily in terms of the transition diagrams



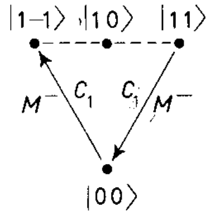
([12]) They are defined by

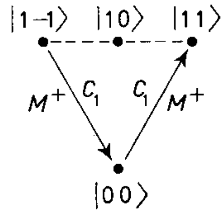
$$\langle\beta|X_a(h)|\alpha\rangle = \delta_{t_\beta,a}G_{\beta\alpha}(h)\,, \qquad \text{for } I_\alpha = 0,\ I_\beta = 1$$

$$\langle\beta|X_a(h)|\alpha\rangle = i\varepsilon_{t_\beta,at_\alpha}G_{\beta\alpha}(h)\,, \qquad \text{for } I_\alpha = I_\beta = 1\,,$$

with $G_{\beta\alpha}$ Hermitian.

Similar diagrams hold for $M_{\pm}$ between $L=0,1$ states, where C_1 is an arbitrary reduced matrix element [13]





Obviously Z annihilates ρ^0, while for $\rho^{\pm 1}$ and π one finds

$$(11) \qquad -iZ\pi = (W_+ M_- - W_- M_+)\, \pi|00\rangle = C_1(\rho^1|1-1\rangle - \rho^{-1}|11\rangle) = \sqrt{2}\, C_1 A_1^0 ,$$

while

$$(12) \qquad -iZ\rho^1 = (W_+ M_- - W_- M_+)\, \rho^1|00\rangle = -C_1\pi|11\rangle = -C_1 \boldsymbol{B}^1 .$$

Hence to lowest order in the mixing angle θ we find

$$(13) \qquad \frac{\beta}{\alpha} = -\frac{1}{\sqrt{2}} .$$

From the experimental value 0.11 ($\simeq 1/9$) for $G^2_{f\pi}$, α is determined to be $1/\sqrt{3}$. As a consequence, one predicts

$$(14) \qquad \begin{cases} \sum_h G^2_{A_1\rho}(h) = \frac{1}{2} & (\text{exp} \leqslant 0.48) , \\ G^2_{\sigma\pi} = \frac{2}{9} & (\text{exp.} \gg 0.1) , \\ \sum_h G^2_{B\omega}(h) = \frac{1}{3} & (\text{exp. } 0.33) , \\ \sum_h G^2_{A_2\rho}(h) = \frac{1}{6} & (\text{exp. } 0.13) , \end{cases}$$

and the polarization ratio of the A_1 decay

$$(15) \qquad \left| \frac{G_{A_1\rho}(1)}{G_{A_1\rho}(0)} \right| = \frac{1}{2} \qquad (\text{exp. } 0.48 \pm 0.13)\ ({}^{10}) .$$

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(13) The coefficients C are defined by

$$\langle L+1, L_3 \pm 1 | M^{\pm} | L L_3 \rangle = \frac{1}{\sqrt{2}} \sqrt{(L \pm L_3 + 1)(L \pm L_3 + 2)}\, C_{L+1} .$$

To lowest order in Z it is also easy to derive directly a set of branching ratios among odd-L isospin-zero particles f, σ and their recurrences. Since an odd power of Z applied to π always yields a state of the form $(\boldsymbol{v}^1 - \boldsymbol{v}^{-1})$, and f_J, σ_J ($f_2 \equiv \mathrm{f}$, $\sigma_0 = \sigma$) have the form

$$\text{(16)} \qquad \begin{cases} f_{L+1} = \dfrac{1}{\sqrt{2L+1}} \left[\sqrt{\dfrac{L}{2}} (v_4^1 - v_4^{-1}) + \sqrt{L+1}\, S \right], \\[2ex] \sigma_{L-1} = \dfrac{1}{\sqrt{2L+1}} \left[\sqrt{\dfrac{L+1}{2}} (v_4^1 - v_4^{-1}) - \sqrt{L}\, S \right], \end{cases}$$

we find

$$\text{(17)} \qquad \frac{\langle f_{L+1} | X | \pi \rangle}{\langle \sigma_{L-1} | X | \pi \rangle} = \sqrt{\frac{L}{L+1}}\,.$$

Similarly, for the odd parity, odd $J \geqslant 3$ mesons $\rho_{J=L+1}$ (recurrences of ρ) and $\rho'_{J=L-1}$ (ρ' and its recurrences), we find

$$\text{(18)} \qquad \frac{\langle \varrho_{L+1} | X | \pi \rangle}{\langle \varrho'_{L-1} | X | \pi \rangle} = -\sqrt{\frac{L+1}{L}}\,.$$

If we compare this with the coupling given by the Veneziano model for π-π scattering

$$\text{(19)} \qquad \begin{cases} G^2_{\pi\sigma_{L-1}} \overset{\text{odd } L}{=} \dfrac{L(2L-1)^{L-2}}{4^{L-1}(2L-1)!!} \dfrac{1}{\pi} \overset{\text{even } L}{=} G^2_{\pi\rho'_{L-1}}, \\[2ex] G^2_{\pi f_{L+1}} \overset{\text{odd } L}{=} \dfrac{(L+1)(2L+1)^{L-1}}{4^{L}(2L+1)!!} \dfrac{1}{\pi} \overset{\text{even } L}{=} G^2_{\pi\rho_{L+1}}\,. \end{cases}$$

We find for the ratios $G^2_{\pi f_{L+1}}/G^2_{\pi\sigma_{L-1}}$ and $G^2_{\pi\rho_{L+1}}/G^2_{\pi\rho'_{L-1}}$:

	$L=1$	$L=2$	$L=3$	$L=4$	$L=\infty$
Veneziano	1/6	3/8	7/15	405/784	$e/4$
Lowest order in Z	1/2	3/2	3/4	5/4	1

3. – Baryons.

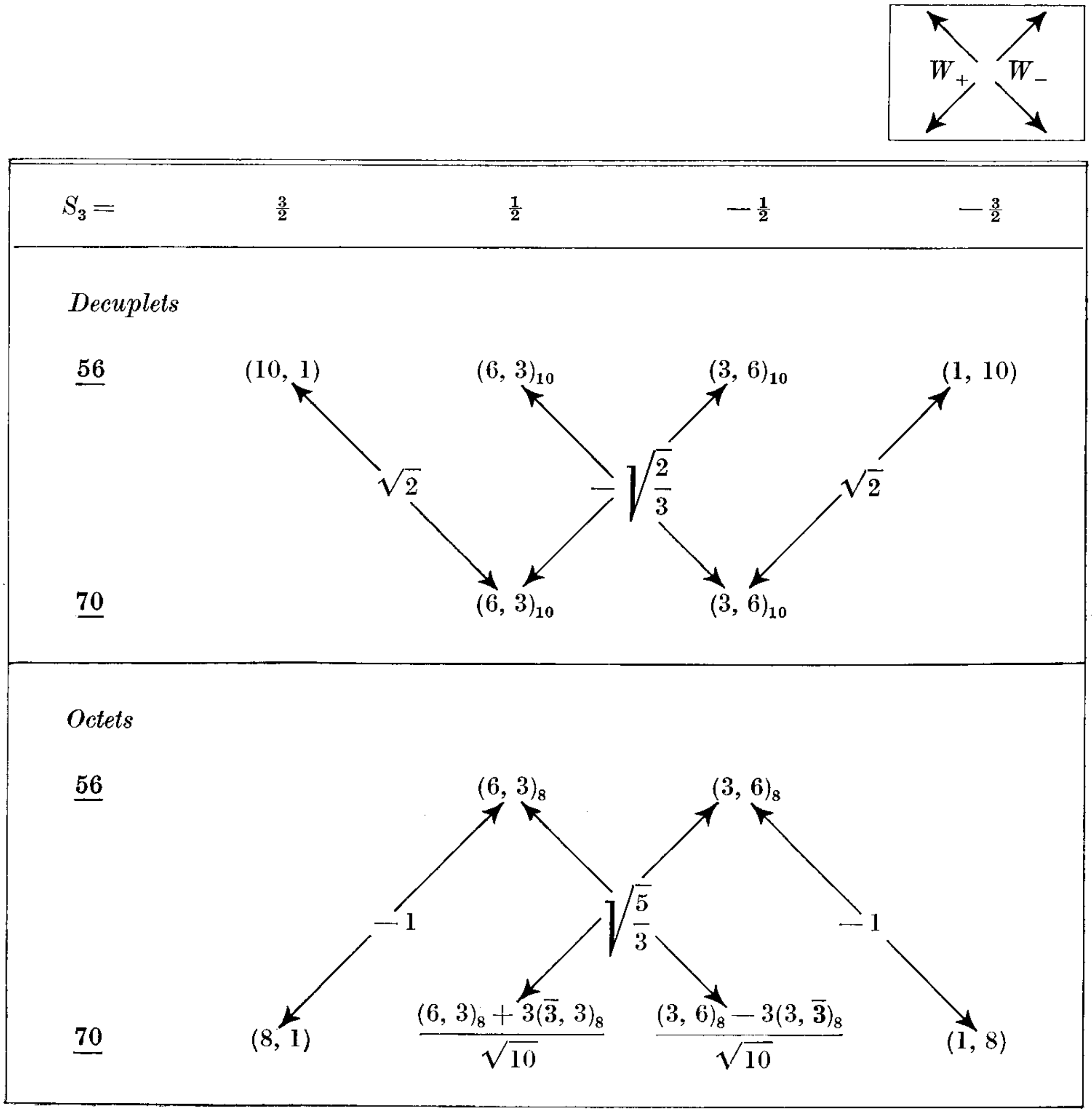
The experimental quantum numbers of the baryons can be classified according to the $SU_6 \times O_3$ representations ($\underline{56}$, $L=$ even) for positive parity and according to ($\underline{70}$, $L=$ odd) for negative parity ([14]).

([14]) R. H. Dalitz: *Proceedings of the Oxford Conference on Elementary Particles* (1965).

The decomposition of $\underline{56}$ and $\underline{70}$ into their chiral contents is given in Table I. States of total angular momentum J are constructed analogously to the meson case. For example, the $(\underline{70}, L=1)$ states of total J, $J_3(\equiv h) = \frac{1}{2}$,

$$
(20) \qquad \begin{cases}
|\underline{70}\,1\rangle_{L=1} = |\underline{70}\,[8, \tfrac{3}{2}]\,\tfrac{5}{2}\rangle_1 & (i.e.,\ S=\tfrac{3}{2}, J=\tfrac{5}{2})\,, \\
|\underline{70}\,2\rangle_{L=1} = |\underline{70}\,[8, \tfrac{3}{2}]\,\tfrac{3}{2}\rangle_1\,, \\
|\underline{70}\,3\rangle_{L=1} = |\underline{70}\,[8, \tfrac{3}{2}]\,\tfrac{1}{2}\rangle_1\,, \\
|\underline{70}\,4\rangle_{L=1} = |\underline{70}\,[8, \tfrac{1}{2}]\,\tfrac{3}{2}\rangle_1\,, \\
|\underline{70}\,5\rangle_{L=1} = |\underline{70}\,[8, \tfrac{1}{2}]\,\tfrac{1}{2}\rangle_1\,,
\end{cases}
$$

TABLE I. – *Matrix elements of* $W_{\pm}$.



are given in terms of the $SU_3 \times SU_3$ states

$$
(21) \qquad \begin{cases} |\underline{70}\, 1\} = (8,1)\,, \\ |\underline{70}\, 2\} = (6,3)\,, \\ |\underline{70}\, 3\} = (\bar{3},3)\,, \\ |\underline{70}\, 4\} = (3,6)\,, \\ |70\, 5\} = (3,\bar{3})\,, \end{cases}
$$

by

$$
(22) \qquad |\underline{70}\, a\rangle_{L=1} = A_{ab} |\underline{70}\, b\}\,,
$$

$$
A = (A^T)^{-1} = \begin{pmatrix} \frac{1}{\sqrt{10}} & \sqrt{\frac{3}{10}} & -\sqrt{\frac{3}{10}} & \sqrt{\frac{3}{20}} & \sqrt{\frac{3}{20}} \\ -\sqrt{\frac{2}{5}} & -\frac{1}{\sqrt{30}} & \frac{1}{\sqrt{30}} & \frac{2}{\sqrt{15}} & \frac{2}{\sqrt{15}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}.
$$

The W operator is a little more complicated in this case. Its transitions have been recorded in Table I.

As far as orbital excitations are concerned, we shall limit ourselves here to states of ($\underline{56}$, $L=0$), ($\underline{70}$, $L=1$) and ($\underline{56}$, $L=0,2$). The relevant transitions of the operator $M_\pm$ are pictured in Fig. 1 [13].

With these diagrams it is easy to compute the mixed chiral states contained in ($\underline{56}$, $L=0$), ($\underline{70}$, $L=1$), ($\underline{56}$, $L=0,2$). By means of the transformation from physical states of total J to their chiral contents we can reduce all physical matrix elements to those of

$$
(23) \qquad \begin{cases} \langle (8,1)_8 | X_a | (8,1)_8 \rangle = F_a\,, \\ \langle (6,3)_8 | X_a | (6,3)_8 \rangle = D_a + \frac{2}{3} F_a\,, \\ \langle (\bar{3},3)_8 | X_a | (\bar{3},3)_8 \rangle = -D_a\,, \end{cases}
$$

$$(23)\quad\begin{cases}\langle(\,\bar{3},3)_8|X_a|(\,\bar{3},3)_1\rangle &= -\dfrac{\sqrt{2}}{\sqrt{3}}\,,\\ \langle(10,1)\,|X_a|(10,1)\,\rangle &= E_a\,,\\ \langle(\,6,3)_{10}|X_a|(\,6,3)_{10}\rangle &= \dfrac{1}{3}\,E_a\,,\\ \langle(\,6,3)_{10}|X_a|(\,6,3)_8\rangle &= \dfrac{4}{3}\,C_a\,,\end{cases}$$

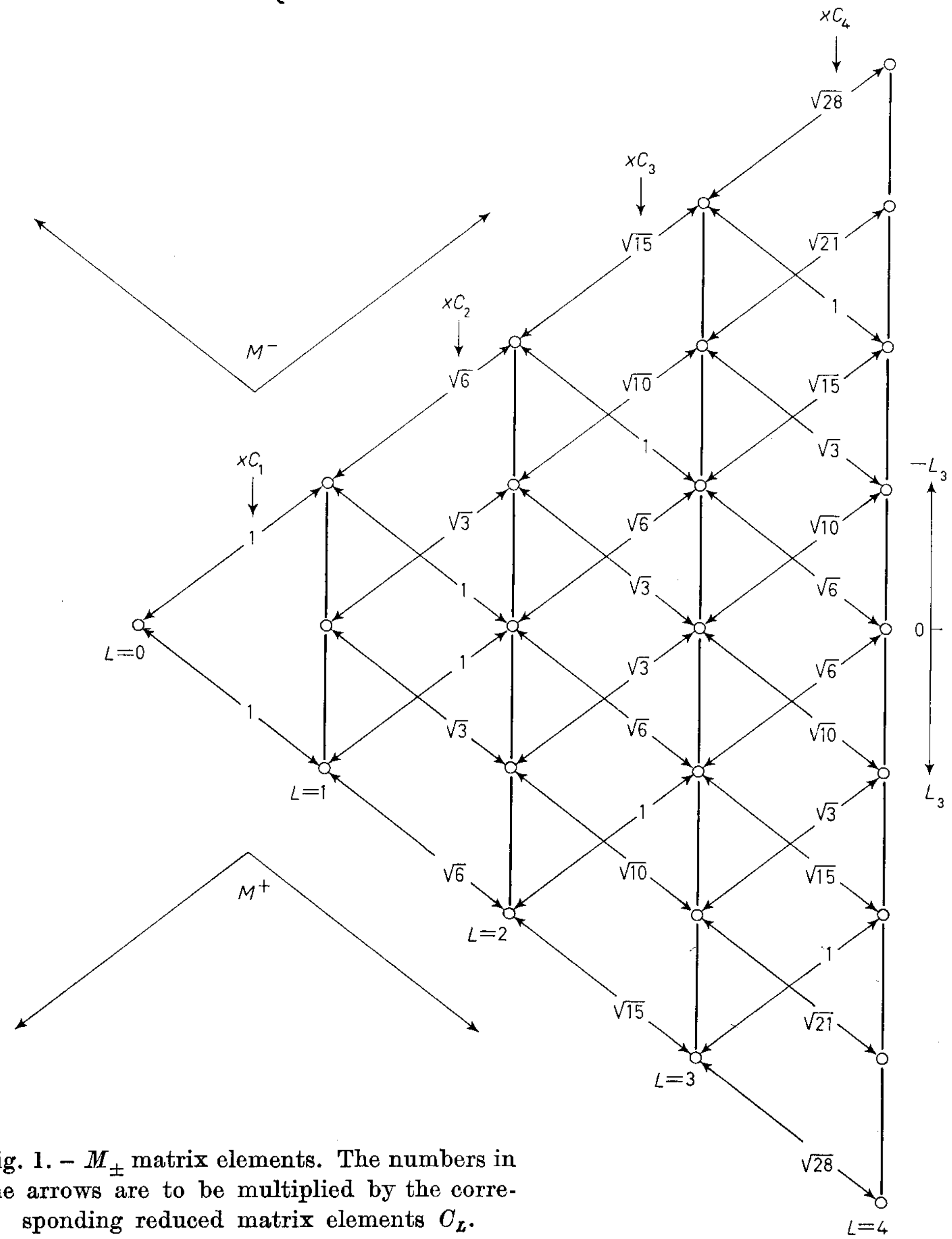


Fig. 1. – $M_\pm$ matrix elements. The numbers in the arrows are to be multiplied by the corresponding reduced matrix elements C_L.

where F, D, E, C are the usual SU_3 Clebsch-Gordan coefficients $\sqrt{3}(8\,8\,8_a)$, $\sqrt{\frac{5}{3}}(8\,8\,8_s)$, $\sqrt{6}(10\,8\,10)$ and $\sqrt{\frac{3}{2}}(8\,8\,10)$, respectively.

Let us normalize the axial coupling by

$$\langle\beta T_\beta=\tfrac{1}{2}|X_a|\alpha T_\alpha=\tfrac{1}{2}\rangle=\chi^\dagger\frac{\tau_a}{2}\chi G_{\beta\alpha}\,,$$

$$\langle\beta T_\beta=\tfrac{3}{2}|X_a|\alpha T_\alpha=\tfrac{1}{2}\rangle=\sqrt{\tfrac{3}{2}}\,\psi_a^\dagger\chi G_{\beta\alpha}\,,$$

such that the Adler-Weisberger relation becomes

$$\sum_{T_\gamma=\frac{1}{2}}G_{\beta\gamma}(h)\,G_{\gamma\alpha}(h)-\sum_{T_\gamma=\frac{3}{2}}G_{\beta\gamma}(h)\,G_{\gamma\alpha}(h)=\delta_{\beta\alpha}\,. \tag{24}$$

We then find for the couplings at the $h=\frac{1}{2}$ level, up to second order in the mixing angle,

$$G^*=\tfrac{4}{5}G_A$$

(where $G^*\equiv G_{\Delta\mathcal{N}}$, $G_A\equiv G_{\mathcal{N}\mathcal{N}}$) and

$$D/F=\tfrac{3}{2}\,.$$

Inserting the experimental value $G_A=1.25$ one obtains $\Gamma_{\Delta\to\mathcal{N}\pi}=125$ MeV ([11]), in excellent agreement with experiment.

The results for the ($\underline{70}$, $L=1$) resonances can best be discussed by comparing the mixing angles predicted by our mixing operator with those obtained in ref. ([2]). If one writes, analogously to the meson case, the mixed nucleon state of $h=\frac{1}{2}$ in the form

$$|\underline{56}[8,\tfrac{1}{2}]\rangle_{L=0}=\sqrt{1-\alpha_1'^2-\beta_1^2-\beta_1'^2-\gamma_1^2}\,|(6,3)_8\rangle_{L=0}+ \\ +\gamma_1|(8,1)\rangle_{L=1}+\beta_1'|(\bar{3},3)_8\rangle_{L=1}+\beta_1|(3,\bar{3})_8\rangle_{L=1}+\alpha_1'|(3,6)_8\rangle_{L=1}\,, \tag{25}$$

the various mixing angles are given in terms of only one parameter, and one gets

$$\beta_1'=0\,,\qquad \beta_1=-3\alpha_1'=-\sqrt{\tfrac{3}{2}}\gamma_1\,, \tag{26}$$

which compare rather well with the numbers found in ref. ([2])

$$\begin{cases}\alpha_1'=0.116\,, & \beta_1=-0.285\,,\\ \beta_1'=-0.060\,, & \gamma_1=0.402\,.\end{cases} \tag{27}$$

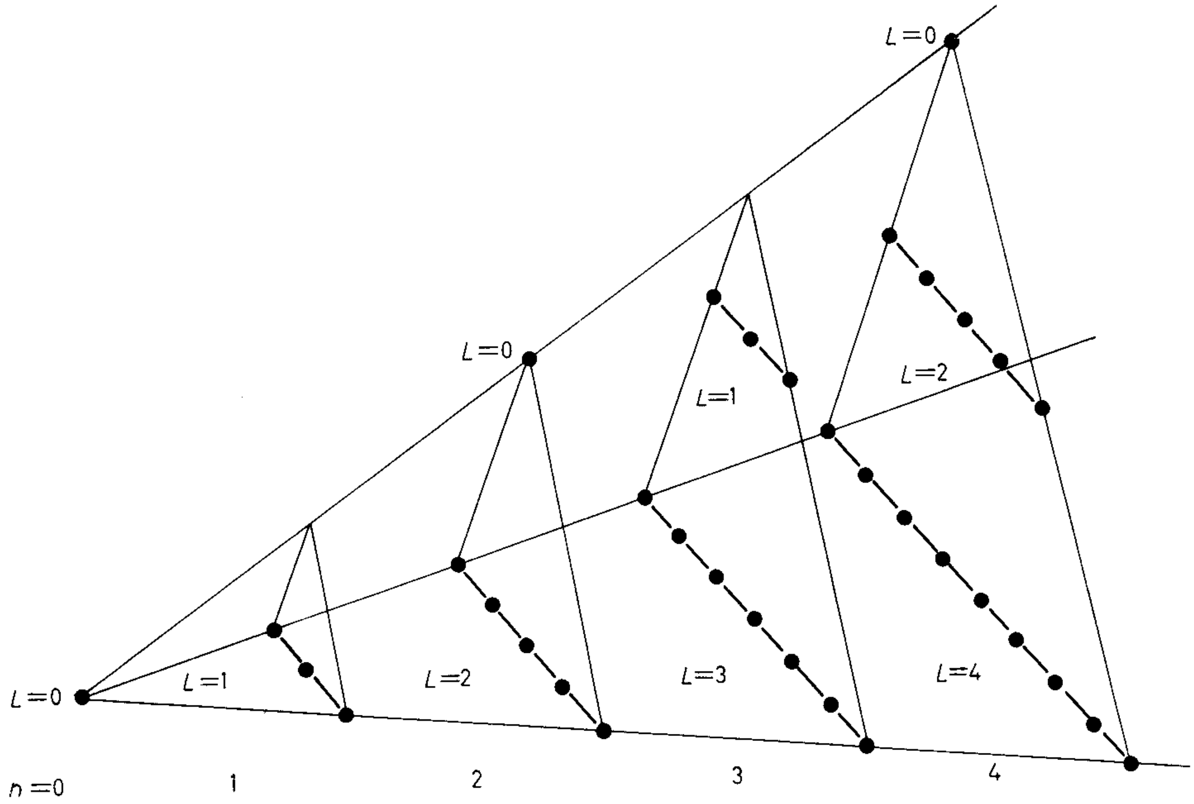


Fig. 2. – (n, L, L_3) states in $U_{3,1}$.

A detailed numerical comparison of these results and those for the higher states ($\underline{56}$, $n=2$; $L=0,2$) with experiment is given in Table II.

For the mixing of the decuplet, we obtain by mixing at $h=\frac{3}{2}$

$$|\underline{56}[10, \tfrac{3}{2}]\rangle_{L=0} = \sqrt{1-12\alpha_1'^2}\,|(10, 1)\rangle_{L=0} + 2\sqrt{3}\,\alpha_1'|(6, 3)_{10}\rangle_{L=1}\,, \tag{28}$$

and at $h=\frac{1}{2}$

$$|\underline{56}[10, \tfrac{3}{2}]\rangle_{L=0} = \sqrt{1-4\alpha_1'^2}\,|(6, 3)_{10}\rangle_{L=0} - 2\alpha_1'|(3, 6)_{10}\rangle_{L=1}\,. \tag{29}$$

This has two important consequences:

1) The opposite signs of the octet and decuplet mixing angles with the $|(3,6)\rangle_{L=1}$ representation imply small transition rates for the negative-parity decuplet decays into nucleon and pion, as observed experimentally.

2) The ratio of the mixing angles at the helicity $h=\frac{3}{2}$ and $h=\frac{1}{2}$ is such that the diagonal matrix elements of the decuplet are proportional to the helicity up to the second order in the mixing angle, as is required from the angular conditions (9).

As in the meson case, we get some relations between the axial couplings of the baryon octet to the higher states with $s=\frac{1}{2}$ and equal L, again at

TABLE II.

$\mathcal{N}_i$	J^P	Γ_{tot} (MeV)	$\eta_{\pi\mathcal{N}}$	$G^2_{\mathcal{N}\mathcal{N}_i}\|_{exp}$	$G^2_{\mathcal{N}\mathcal{N}_i}\|_{th}$	$F/D\|_{exp}$ (15)	$F/D\|_{th}$
		1) $\underline{56}$, $n=0$, $L=0$ (ground state)					
$\mathcal{N}(939)$	$\frac{1}{2}^+$			1.562	1.562	0.67	$\frac{2}{3}$
Δ (1236)	$\frac{3}{2}^+$	120		1.04	1.0		
		2) $\underline{56}$, $n=2$, $L=0$					
$\mathcal{N}(1470)$	$\frac{1}{2}^+$	300	0.6	0.162	0.162		$\frac{2}{3}$
Δ (?)	$\frac{3}{2}^+$				0.104		
		3) $\underline{56}$, $n=2$, $L=2$					
$\mathcal{N}(1688)$	$\frac{5}{2}^+$	140	0.6	0.097	0.097	1.07	$\frac{2}{3}$
$\mathcal{N}(1860)$	$\frac{3}{2}^+$	380	0.27	0.047	0.064		$\frac{2}{3}$
Δ (1890)	$\frac{5}{2}^+$	255	0.17	0.026	0.01		
Δ (1910)	$\frac{1}{2}^+$	325	0.25	0.016	0.020		
Δ (1950)	$\frac{7}{2}^+$	180	0.45	0.058	0.053		
Δ (?)	$\frac{3}{2}^+$				0.020		
		4) $\underline{70}$, $n=1$, $L=1$					
$\mathcal{N}(1520)$	$\frac{3}{2}^-$	125	0.5	0.091	0.091	1.2	$\frac{5}{3}$
$\mathcal{N}(1535)$	$\frac{1}{2}^-$	105	0.34	0.024	0.045	-0.34	$-\frac{1}{3}$
$\mathcal{N}(1670)$	$\frac{5}{2}^-$	140	0.42	0.070	0.021	-0.13	$-\frac{1}{3}$
$\mathcal{N}(1700)$	$\frac{1}{2}^-$	250	0.70	0.063	0.182		$\frac{5}{3}$
$\mathcal{N}(?)$	$\frac{3}{2}^-$				0.002		$-\frac{1}{3}$
Δ (1650)	$\frac{1}{2}^-$	190	0.27	0.021	0.023		
Δ (1670)	$\frac{3}{2}^-$	235	0.13	0.024	0.012		
Λ (1405)	$\frac{1}{2}^-$	40	1	0.200	0.154		
Λ (1520)	$\frac{3}{2}^-$	16	0.41	0.02	0.07		

lowest orders in the mixing angle; in fact, it is easy to see that for both decuplets and singlets of negative parity one finds the ratio

$$\frac{G^2_{L+\frac{1}{2}}}{G^2_{L-\frac{1}{2}}} = \frac{L}{L+1},$$

while for positive-parity octets

$$\frac{G^2_{L+\frac{1}{2}}}{G^2_{L-\frac{1}{2}}} = \frac{L+1}{L}$$

and $D/F = \frac{3}{2}$.

---

(15) See, R. LEVI-SETTI: talk in *Proceedings of the Lund International Conference on Elementary Particles* (1969).

There is another property of the mixing operator $\exp[-i\theta Z]$, which we would like to emphasize. First we write the most general expression for the chiral content of the octet $\frac{1}{2}^+$ in the absence of exotic states, at $h=\frac{1}{2}$ ([2]),

$$
\begin{aligned}
(30)\qquad |\underline{56}\,[8, \tfrac{1}{2}]\rangle_{L=0} = \alpha|(6, 3)_8\rangle_{\text{all } L} + \alpha'|(3, 6)_8\rangle_{\text{all } L} + \\
+ \beta|(3, \bar{3})_8\rangle_{\text{all } L} + \beta'|(\bar{3}, 3)_8\rangle_{\text{all } L} + \gamma|(8, 1)\rangle_{\text{all } L} + \gamma'|(1, 8)\rangle_{\text{all } L}\,, \\
\alpha^2 + \alpha'^2 + \beta^2 + \beta'^2 + \gamma^2 + \gamma'^2 = 1\,.
\end{aligned}
$$

The following sum rules can be derived for the mixing angles:

$$
(31)\qquad \begin{cases}
\alpha^2 + \alpha'^2 = \dfrac{9}{16}\sum_r G^2_{\mathcal{N}\Delta_r} = \dfrac{9}{16}\left(\sum_r G^2_{\mathcal{N}\mathcal{N}_r} - 1\right), \\
\beta^2 + \beta'^2 = \dfrac{3}{4}\sum_r G^2_{\Sigma Y_{0r}}\,, \\
\alpha^2 + \beta^2 - \alpha'^2 - \beta'^2 = D\,, \\
\dfrac{2}{3}\alpha^2 + \gamma^2 - \dfrac{2}{3}\alpha'^2 - \gamma'^2 = F\,.
\end{cases}
$$

Using the experimental widths and the diagonal matrix elements of the axial charges, the right-hand sides are determined to be

$$
(32)\qquad \begin{cases}
\alpha^2 + \alpha'^2 = 0.67\,, & \alpha^2 + \beta^2 - \alpha'^2 - \beta'^2 = 0.75\,, \\
\beta^2 + \beta'^2 = 0.18\,, & \frac{2}{3}\alpha^2 + \gamma^2 - \frac{2}{3}\alpha'^2 - \gamma'^2 = 0.50\,.
\end{cases}
$$

This implies

$$
(33)\qquad \begin{cases}
\alpha'^2\beta'^2 = 0.05\,, \\
\frac{2}{3}\alpha'^2 + \gamma'^2 = 0.05\,.
\end{cases}
$$

The exclusion of the $(1, 8)$ and $(\bar{3}, 3)_8$ $SU_3\times SU_3$ representations in the baryon octet wave function at $h=\frac{1}{2}$ is a specific property of our mixing operator $\exp[-i\theta Z]$. This happens since the two quoted representations appear only in the $(\underline{70}, L=\text{odd})$ and will never be reached by repeated application of the Z-operator due to its property $\Delta S_3 = \pm 1$. Moreover the amount of $(3, 6)_8$ is expected to be small (see Table I). Both these predictions are in agreement with eq. (33).

Finally, we want to stress that α^2 (which gives the amount of the mixing of the pion to the higher states) and $\alpha_1'^2 + \beta_1^2 + \gamma_1^2$ (which plays the same role for the baryon octet) are almost equal; in fact, we obtain

$$\alpha^2 = \tfrac{1}{3}\,, \qquad \alpha_1'^2 + \beta_1^2 + \gamma_1^2 = 0.31\,,$$

from the experimental widths of the f and the $\mathcal{N}(1520)$, respectively. This universal character makes our mixing operator still more appealing.

4. – Concluding remarks.

In this work we have been concerned only with the axial coupling constants of the particles. We have completely ignored the restrictions imposed upon the mass spectrum by means of the superconvergence of the $I_t = 2$ $\pi\alpha \to \pi\beta$ scattering amplitude, which gives ([8])

$$[X_a, [X_b, m^2]] = \tfrac{1}{3}\delta_{ab}[X_c, [X_c, m^2]]\,. \tag{34}$$

The fact that Z transforms like a $(\frac{1}{2}, \frac{1}{2})$ (or $(3, \overline{3}) \pm (\overline{3}, 3)$) representation of $SU_2 \times SU_2$ (or $SU_3 \times SU_3$) makes us expect also the masses to behave reasonably well. This problem will be studied in detail elsewhere.

RIASSUNTO

Si propone uno schema di saturazione dell'algebra chirale all'interno di un insieme infinito di stati. I mesoni sono classificati nelle rappresentazioni ($\underline{35}$, L qualsiasi) di $SU_6 \times O_3$, mentre per i barioni si scelgono le ($\underline{56}$, L pari) e ($\underline{70}$, L dispari). Un operatore di mistura $\exp[-i\theta Z]$ viene proposto per trasformare le cariche assiali di SU_6 in quelle fisiche. La forma specifica scelta per Z dà luogo a molte predizioni. Gli accoppiamenti dei mesoni più bassi di parità positiva a π, ρ ed ω si esprimono in termini di un solo parametro in buon accordo con i valori sperimentali. In particolare si prevede il decadimento traversale del mesone B, mentre par l'A_1 il rapporto g_1/g_0 risulta $-\frac{1}{2}$. Per quanto concerne l'ottetto dei barioni $\frac{1}{2}^+$, otteniamo $\frac{3}{2}$ per D/F e $\frac{4}{5}$ per G^*/G_A (che corrisponde a 125 MeV per $\Gamma_{\Delta\to\mathcal{N}\pi}$) sino al secondo ordine nel parametro θ. Per ciascun multipletto di $SU_6 \times O_3$ tutti i decadimenti in $\mathcal{N}+\pi$ si ottengono in termini di un solo parametro (all'ordine più basso possibile nell'angolo di mistura) in accordo generale con i dati sperimentali. Infine siamo in grado di ottenere forti restrizioni sul contenuto chirale dell'ottetto $\frac{1}{2}^+$, del tutto compatibile con la realtà fisica.

Резюме не получено.