Veneziano Amplitude for Any Intercept from Local Lagrangian.

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Some time ago, an operator formalism was devised which permits writing the Veneziano $(N+2)$-point function [1] in terms of vector creation and annihilation operators $a_\mu^{(n)\dagger}$, $a_\mu^{(n)}$ $(n=1, 2, 3, \ldots)$ in the form

$$V_{N+2} = \langle 0|V(p_1)\,D(\pi_1^2)\,V(p_2)\,D(\pi_2^2)\cdot\ldots\cdot V(p_N)|0\rangle \tag{1}$$

with vertex operators

$$V(p) = \exp\left[-pa^\dagger\right]\exp\left[pa\right], \qquad a_\mu^\dagger \equiv \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\,a_\mu^{(n)\dagger}, \tag{2}$$

and propagators

$$D(s) = \int_0^1 \mathrm{d}x\, x^{-\alpha(s)-\Sigma n a^\dagger a-1}(1-x)^{-c} \tag{3}$$

$(\pi_i = p_0 + \ldots + p_i,\ c = -\alpha_0 + 1,\ \alpha_0 = \text{intercept of the trajectory})$.

By expanding the factor $(1-x)^{-c}$, $D(s)$ can be integrated to give

$$D(s) = \sum_{l_0=0}^{\infty}\binom{l_0+c-1}{l_0}\frac{1}{-\alpha(s)-\sum\limits_{n=1}^{\infty} n a^{(n)\dagger}a^{(n)} + l_0}\,. \tag{4}$$

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(1) S. FUBINI, D. GORDON and G. VENEZIANO: *Phys. Lett.*, **29** B, 679 (1969); Y. NAMBU: talk presented at the *International Conference on Symmetries and Quark Models*, Wayne University, June 1969, Enrico Fermi Institute preprint EFI 69-64.

This representation suffers from several unaesthetical features:

1) The vertices are momentum dependent.

2) The propagator contains an infinite sum unless $c = 0$.

3) The $N+2$ external momenta $p_0, p_1, \ldots, p_{N+1}$ do not enter on equal footing (p_0 and p_{N+1} are missing).

It is the purpose of this letter to exhibit, for any intercept α_0, a local Lagrangian whose multiperipheral Feynman graphs coincide with the corresponding Veneziano functions and which has the properties that

1) all vertices become constants,

2) the propagator takes a conventional form.

We are not able, however, to remove the intrinsic asymmetry of particles in the multiperipheral configuration between the external legs and the particle propagating through the whole graph.

In addition, we briefly discuss the difficulties associated with using the local conserved current supplied by our Lagrangian to couple the electromagnetic field.

Let $b^\dagger, b$ create and destroy a scalar quantum. Then it is possible to build the representation $D^+_{c/2}$ of $O_{2,1}$ by means of the following generators:

$$S_3 = b^\dagger b + \frac{c}{2}, \qquad S^+ = b^\dagger \sqrt{b^\dagger b + c}\,, \qquad S^- = \sqrt{b^\dagger b + c}\; b\,, \tag{5}$$

satisfying the commutation rules

$$[S_3, S_\pm] = \pm S_\pm\,, \qquad [S_+ S_-] = -2S_3\,. \tag{6}$$

Consider the free Lagrangian

$$\mathscr{L}(x) = \psi^\dagger(x)\,\bar{L}(i\partial)\,\psi(x) \tag{7}$$

with (2)

$$\bar{L}(p) = -c_1(p^2 - 2S_3) - \sqrt{4c_1^2 - 1}\, S_1 - \frac{\alpha_0 + 1}{2} - \sum_{n=1} \sqrt{2n}\, \Gamma_\mu^{(n)} p^\mu - \sum_{n=1} n a^{(n)\dagger} a^{(n)}\,, \tag{8}$$

where

$$c_1 = \tfrac{1}{2} + \sum_{n=1} 1 \tag{8}$$

and $\Gamma_\mu^{(n)}$ is the vector operator

$$\Gamma_\mu^{(n)} = \frac{1}{\sqrt{2}}\,(a_\mu^{(n)\dagger} + a_\mu^{(n)})\,. \tag{9}$$

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(2) I we drop the zero-mode part and leave only one vector mode this Lagrangian is exactly the one discussed by H. LEUTWYLER: *Phys. Lett.*, **31** B, 214 (1970).

In order to keep track of divergences due to the number of vector quanta going to infinity, like that in c_1, let us truncate all $\sum\limits_{n=1}$ and use $\sum\limits_{n=1}^{\nu}$.

The Lagrangian (7) yields a wave equation

$$\overline{L}(p)\,|\overline{p}\rangle = 0\,. \tag{10}$$

If we transform all states by a unitary transformation

$$|\overline{p}\rangle = T(p)\,|p\rangle'\,, \qquad T(p) = \exp\left[p_\mu(a^\dagger - a)^\mu\right], \tag{11}$$

we find for $|p\rangle'$

$$L'(p)\,|p\rangle' = 0 \tag{12}$$

with

$$L'(p) = -\frac{p^2}{2} + 2c_1 S_3 - \sqrt{4c_1^2 - 1}\, S_1 - \frac{\alpha_0 + 1}{2} - \sum n a^{(n)\dagger} a^{(n)}\,. \tag{13}$$

The zero-mode content in S_1 can be diagonalized by a second unitary transformation

$$|p\rangle' = t\,|p\rangle\,, \qquad t = \exp\left[i\zeta S_2\right], \qquad \cosh\zeta = 2c_1\,, \tag{14}$$

such that $|p\rangle$ solves a wave equation $L(p)\,|p\rangle = 0$ with

$$L(p) = -\frac{p^2}{2} - \alpha_0 - \sum n a^{(n)\dagger} a^{(n)} + b^\dagger b\,. \tag{15}$$

The states at rest are given by the basis states of the Hilbert space $|\alpha, l_0\rangle$:

$$|\boldsymbol{p} = 0, \alpha l_0\rangle = |\alpha, l_0\rangle \equiv a_{\mu_1}^{(n_1)\dagger} a_{\mu_2}^{(n_2)\dagger} \ldots a_{\mu_\alpha}^{(n_\alpha)\dagger} (b^\dagger)^{l_0}\,|0\rangle \tag{16}$$

with masses

$$\frac{m^2}{2} = -\alpha_0 + \sum n l_n + l_0\,, \tag{17}$$

where l_n are the number of quanta $a_\mu^{(n)\dagger}$ in the state $|\alpha, l_0\rangle$. The solution of eq. (10) is

$$|\overline{p\alpha, l_0}\rangle = T(p)\,t\,|p\alpha, l_0\rangle \equiv a^\dagger(p\alpha, l_0)\,|0\rangle\,. \tag{18}$$

We introduce spinors

$$u_{\alpha' l_0'}(p\alpha l_0) = \langle \alpha' l_0' | \overline{p\alpha l_0}\rangle \tag{19}$$

and can write down a local field

$$\psi_{\alpha' l_0'}(x) = \sum_{p,\alpha l_0} |Z_{\alpha l_0}^{\frac{1}{2}}|\, a(p\alpha l_0)\, u_{\alpha' l_0'}(p\alpha l_0) \exp\left[-ipx\right] + \text{neg. freq.} \tag{20}$$

satisfying the equal-time commutation rules

$$(21)\qquad \begin{cases} [\psi(x),\,\psi(y)]_{x_0=y_0}=0\,, & [\pi(x),\,\pi(y)]_{x_0=y_0}=0\,,\\ [\pi(x),\,\psi(y)]_{x_0=y_0}=-i\delta^3(\boldsymbol{x}-\boldsymbol{y})\,, & \pi(x)=\psi^\dagger\left(c_1\overleftarrow{\partial}_0+\dfrac{i}{2}\sum\sqrt{2n}\,\Gamma_0\right),\end{cases}$$

where $Z_{\alpha l_0}$ is a wave function renormalization constant determined by the condition

$$(22)\qquad u^\dagger(\boldsymbol{p}\alpha' l_0')(c_1 P_0+\sum\sqrt{2n}\,\Gamma_0)\,u(\boldsymbol{p}\alpha l_0)=Z^{-1}_{\alpha l_0}\,\delta_{\alpha' l_0',\alpha l_0}\,\frac{p_0}{m_{\alpha,l_0}}\,.$$

Clearly, the Wick contraction of the field $\psi(x)$ becomes

$$(23)\qquad \overline{\psi^\dagger(x)\,\psi}(y)=\frac{i}{(2\pi)^4}\int\mathrm{d}p\,\frac{1}{L(p)}\exp\left[-ip(x-y)\right].$$

Let us couple this infinite-component field to an external scalar field $\varphi(x)$ by a local interaction

$$(24)\qquad \mathscr{L}_{\text{int}}(x)=gc_1\sum_\alpha\psi^\dagger_{\alpha,0}(x)\,\psi_{\alpha,0}(x)\,\varphi(x)\,.$$

Then the N-th–order Feynman graph corresponding to the multiperipheral configuration shown in Fig. 1 is given by

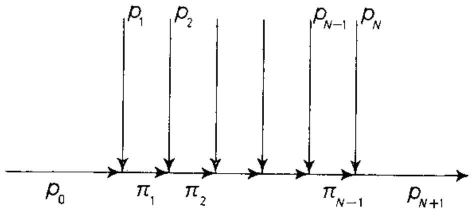

Fig. 1.

$$(25)\qquad T_{N+2}=(igc_1)^N\langle p_0\,00|\,\Omega\,\frac{1}{L(\pi_1)}\,\Omega\,\frac{1}{L(\pi_2)}\,\Omega\ldots\Omega\,\frac{1}{L(\pi_{N-1})}\,|p_{N+1}\,00\rangle\,,$$

where Ω is the operator projecting onto states with no scalar quantum ([3])

$$(26)\qquad \Omega=\sum_\alpha|\alpha l_0=0\rangle\langle\alpha l_0=0|\,.$$

Let us rewrite

$$(27)\qquad |p\alpha l_0\rangle=T(p)\,t\,|p\alpha l_0\rangle\,,$$

$$(28)\qquad \frac{1}{L(\pi)}=T(\pi)\,t\,\frac{1}{L(\pi)}\,tT^{-1}(\pi)\,,$$

and observe that

1) T commutes with Ω.

([3]) That such a projection operator is needed to include the sum $\sum_{l_0}$ in (4) via a scalar mode was first noticed by D. Amati, C. Bouchiat and J. L. Gervais: *Lett. Nuovo Cimento*, **2**, 399 (1969).

2)

$$T^{-1}(\pi_i)\,T(\pi_{i+1}) = V(\pi_i - \pi_{i+1})\exp\left[\frac{\mu^2}{2}\sum\frac{1}{n}\right] = V(p_{i+1})\exp\left[\frac{\mu^2}{2}\sum\frac{1}{n}\right]. \tag{29}$$

3) The matrix elements of the $O_{2,1}$ rotation $t=\exp[i\zeta S_2]$ are (4)

$$\langle\alpha' l_0'|t|\alpha l_0\rangle = \delta_{\alpha'\alpha}\, v^{c/2}_{l_0' l_0}(\zeta)\,, \tag{30}$$

where for $l_0' \geqslant l_0$

$$v^{c/2}_{l_0' l_0}(\zeta) = \frac{1}{(l_0'-l_0)!}\sqrt{\frac{l_0'!\,(l_0'+c-1)!}{l_0!\,(l_0+c-1)!}}\cosh\frac{\zeta^{-(c+l_0+l_0')}}{2}\sinh\frac{\zeta^{l_0'-l_0}}{2}\cdot \tag{31}$$

$$\cdot F\left(-l_0,\, 1-c-l_0,\, 1+l_0'-l_0,\, -\sinh^2\frac{\zeta}{2}\right).$$

In particular,

$$\langle\alpha' l_0|t|\alpha\, 0\rangle = \delta_{\alpha'\alpha}\sqrt{\binom{l_0+c-1}{l_0}}\left[\cosh\frac{\zeta}{2}\right]^{-c-l_0}\left[\sinh\frac{\zeta}{2}\right]^{l_0}. \tag{32}$$

This matrix element behaves asymptotically, for large $\zeta \approx \log 4c_1$, like

$$\delta_{\alpha'\alpha}\sqrt{\binom{l_0+c-1}{l_0}}\, c_1^{-c}\,. \tag{33}$$

Then the Feynman amplitude (25) becomes

$$T_{N+2} = (igc_1)^N\,(v^{c/2}_{00})^2 \sum_{l_0^{(1)},l_0^{(2)},\dots,l_0^{(N-1)}} (v^{c/2}_{l_0^{(1)}0})^2\cdot(v^{c/2}_{l_0^{(2)}0})^2\cdot\ldots\cdot(v^{c/2}_{l_0^{(N-1)},0})^2\cdot \tag{34}$$

$$\cdot\exp\left[\frac{N\mu^2}{2}\sum\frac{1}{n}\right]\langle p_0\,00|\,V(p_1)\,\frac{1}{-p^2/2-\alpha_0-\sum n a^{(n)\dagger}a + l_0}\,V(p_2)\cdot\ldots\cdot V(p_{N-1})\,|p_N\,00\rangle\,.$$

Let us now go to the limit of infinitely many vector modes, $\nu\to\infty$. In this limit $\sum 1/n \approx \log\nu$ and since $\mu^2/2 = c-1$ we see that all infinities cancel and T_{N+2} reduces to the Veneziano function V_{N+2}.

Our local Lagrangian suggests the introduction of an electromagnetic coupling via the local conserved current

$$j_\mu(x) = \psi^\dagger(x)\big(c_1 i\overleftrightarrow{\partial}_\mu + \sum\sqrt{2n}\,\Gamma^{(n)}_\mu\big)\,\psi(x)\,. \tag{35}$$

(4) U. BARGMANN: *Ann. Math.*, 48, 568 (1947).

Unfortunately this current has two bad properties:

1) It is only defined as long as the number of vector modes ν is finite (5). The reason is that every form factor appears as a finite function in t multiplied by $\exp\left[t\sum 1/n\right]$.

2) The form factors contain no poles in t at the position of the vector particles of the theory as one would expect from a dual current.

Both properties can only be remedied by multiplying the matrix elements of the current by a common form factor $G(t)\exp\left[-t\sum 1/n\right]$, where $G(t)$ has the structure of the standard Veneziano-type form factors (6). If one does this one looses, however, almost all predictive power and our current $j^\mu(x)$ can merely serve to determine the ratios of the form factors for different external states.

In conclusion we see that we do not escape the introduction of two different types of particles. As already discussed by NAMBU (1), this may be due to an incompatibility of Feynman-graph techniques with duality. For the construction of a dual current the minimal local current is no possible candidate. A form factor is needed ruining locality.

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(5) For finite ν it shares all nice properties of canonical currents. In particular, if one introduces SU_3, it satisfies the algebra of charge densities and Compton amplitudes pick up fixed poles at $J = 1$.

(6) R. JENGO and E. REMIDDI: CERN preprint; P. DI VECCHIA and F. DRAGO: Frascati preprint; P. H. FRAMPTON: EFI preprint.