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Vortex line nucleation of first-order phase transitions in early universe

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Abstract

With nonabelian Gauge Theories apparently possessing an abelian projection, the first-order phase transitions in the early universe should proceed similarly to those in $U(1)$ -symmetric field systems, where they are known to be nucleated by vortex lines, not bubbles, as often assumed. © 1999 Published by Elsevier Science B.V. All rights reserved.

1. Since Langer's historic paper on bubble nucleation [1,2] of first-order transition in a real scalar field system, field theorists have assumed this mechanism to cause transitions in a large variety of physical systems. This belief was enhanced a rigorous proof of Coleman at al., that the dominant classical solutions of rotationally-invariant field equations in instanton calculations are bubble-like [3]. Most importantly for our very existence, the first-order transitions in the early universe is commonly supposed to be nucleated by bubbles [4–6].

To an unbiased observer, this assumption comes as a surprise, since the evolution of the early universe is described by a nonabelian generalization of the Ginzburg-Landau theory of superconductivity which, moreover, contains an ordinary Ginzburg-Landau substructure of fields.

In addition, work on strong-interaction physics suggests, that also the nonabelian color gauge theory of the strong forces between quarks can be approximated, via a technique called abelian projection [7,8], by a simple modification of a Ginzburg-Landau theory which is similar to that of an ordinary supercon-

ductor, except for the exchange of electric and magnetic fields.

For a superconductor, however, bubbles play no role in the phase transition. This holds in both the type-II and the type-I regime which are distinguished by the ratio of the two length scales κ = magnetic penetration depth/coherence length. In the second-order regime, where κ is large and the transition is of second order, the transition can be understood completely as a proliferation of magnetic vortex lines. This can be shown convincingly in a lattice field theory of the system [12]. There exists a dual description of the theory which is a simple XY-model. The high-temperature expansion of the partition function of this model can be rewritten as a sum of closed lines which are direct pictures of the magnetic vortex lines in the superconductor at low temperatures. In this grand-canonical line ensemble one can easily calculate the temperature of proliferation [12].

In the continuous limit, this XY-model can be transformed via functional techniques into a $|\psi|^4$ -field theory with a complex *disorder field* [12]. In this formulation, the Feynman diagrams in the per-

turbation expansion of the vacuum energy are the direct pictures of the magnetic vortex lines, which proliferation as the mass term becomes negative.

When lowering the parameter κ into the regime of weak first-order transitions, there still exists a generalization of the XY-model describing this system, which has the same type of high-temperature expansion in terms of closed loops, thus showing again that only vortex lines can be relevant for understanding the transition [9]. Thus we conclude that in a superconductor and related field theories which possess vortex lines as topological excitation, these drive both second and first-order phase transitions.

It the sequel we give some simple arguments for the superior efficiency of vortex line over bubble nucleation, thus casting doubts on all studies of first-order phase transition in the early universe based on bubble nucleation.

2. The generalized XY-model which provides us with a disorder description of a superconductor on a lattice has the partition function

$$Z = \int \frac{\mathcal{D}\theta}{2\pi} e^{\beta \sum_{x,i} [\cos \nabla_i \theta + \delta \cos 2\nabla_i \theta]}, \quad (1)$$

where ∇_i are lattice gradients, and β, δ model parameters. The phase structure of this model has been studied in detail in the literature [10–12]. For $\delta = 0$ the model is known to describe the critical behavior of superfluid helium near the λ -transition. The same thing is true for a small interval around zero $\delta \in (0.1, 0.2)$. In addition, there exists a regime of δ where the transition is of first order. In the disordered phase, the partition function (1) can be rewritten as a sum over non-self-backtracking loops of superflow. Under a duality transformation, these go over into the magnetic vortex lines of the superconductor. The parameter β which is the inverse temperature in the XY-model grows with the temperature in the superconductor. The loops of superflow can have strengths 1, 2, 3, ... on the lattice. They are dual representation of the quantized flux strengths of the magnetic vortex lines in the superconductor. In the second-order regime, the critical properties of the model have been shown to be the same as for a simplified model which can contain only loops of unit strength [14, 12].

For a single loop, the partition function of this simplified model can easily be written down. If n is the length of the loop in lattice units, we have

$$Z = \sum_n N_n e^{-\beta_V \epsilon_n}, \quad (2)$$

where β_V is a function β, δ which plays the role of an inverse temperature for this one-loop model, ϵ_n is the loop energy, and N_n is the number of different loops of length n . For large n , the energy ϵ_n is proportional to n , say $\epsilon_n \approx \epsilon n$. The notation ϵ_n is really an approximation, since it neglects a slight dependence on the loop shape. This, however, is very weak for lines which are much longer than the length scale n^{st} over which the lines show stiffness. This stiffness is a result of the non-self-backtracking property and the fact that if two (or more) portions of a loop merge into a line of strength two (or larger), the energy of this portion is much larger than the sum of the energies of the constituent lines, causing a strong Boltzmann suppression. Writing the number N_n as e^{s_n} , we define the configurational entropy s_n of loops of length n . Also s_n grows linearly for large n , say like $s n$. As β_V becomes smaller than a critical value $\beta_V^c \equiv s/\epsilon$, the free energy of the loops

$$f_n = \epsilon_n - \beta_V^{-1} s_n \quad (3)$$

goes to negative infinity for large n , so that the sum over n in (2) diverges. The loop length diverges and the loop fills the entire system with superflow, a characteristic feature of the phase transition into the superfluid state. A large energy of a loop will always be canceled by the configurational entropy if the temperature is sufficiently large.

A decrease of the parameter δ in (2) brings the phase transition into the first-order regime. In the loop picture, this change the n -dependence of the energy ϵ_n . In the partition function (2), The entropy s_n of the loops in the small- β expansion of (2) depends on n as shown in Fig. 1 [13]. After an initial rise it flattens out somewhat around $n \approx 10$, where it merges into the asymptotic linear behavior σn . The energy may depend on n in different characteristic ways, also indicated in Fig. 1. The region $n \approx 10$ where the linear behavior is reached is determined by the effective stiffness of the vortex lines.

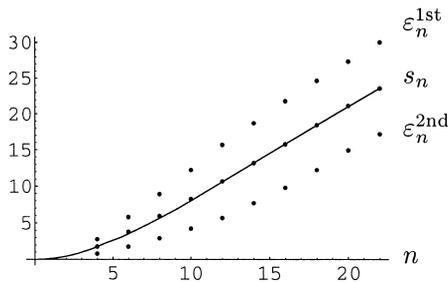


Fig. 1. Dependence of the entropy of lines of length n in the partition function (2) on the length n of the loops. The curves above and below are possible energies ϵ_n leading to second- or first-order phase transitions.

The associated free energies f_n have the typical shapes displayed in Fig. 2. The left-hand plot shows the free energy for $\epsilon^{2\text{nd}}$ in an ordinary XY-model. For sufficiently large temperatures, it possesses a minimum at a nonzero value of n , say at n^m . This value moves continuously from zero to infinity as β_V is raised above the critical value β_V^c . The transition is of second order. Even before the critical value is reached, there are loops of size n^m in the system. Such *precritical loops* are found in Monte Carlo simulations of the model (2). They are plotted as 3D-figures in Ref. [16].

The free energy in the right-hand plot of Fig. 2 corresponds to the energy $\epsilon^{1\text{st}}$, and shows a completely different behavior. As the critical value β_V^c is reached, the free energy has a barrier at n^m which prevents the lines from growing infinitely long. Thermal fluctuations have to create a loop of length n^m , which can then expand and fill the entire system with superflow, thereby converting the normal state of the XY-model into a superfluid one, or the ordered state of a superconductor with magnetic vortex lines into the normal state. The size of n^m is of the order of the length scale of stiffness n^{st} .

Since the superconductor on a lattice can be represented exactly in terms of loops, there is no place for bubble nucleation in such a system. But there are also simple energy-entropy arguments to justify this conclusion.

3. Consider now, in contrast, a hypothetical bubble nucleation of the transition [1,2,17]. Such bubbles may be calculated in a continuous approximation to the partition function (2) derived by standard

field theoretic techniques [15,11]. In this approximation, the partition function (2) becomes a functional integral over a complex disorder field $\phi(x)$ with quartic and sextic self-interactions [11,9]. When cooling the disordered phase slightly below the transition point, such a field theory possesses spherically-symmetric solutions whose inside contains the ordered phase whose energy is slightly lower than that of the disordered phase. Let ϵ be the difference in energy density and σ the surface energy density. The total energy of the bubble is then

$$E^{\text{bubble}} = S_D R^{D-1} \sigma - \frac{S_D}{D} R^D \epsilon, \quad (4)$$

where $S_D = 2\pi^{D/2}/\Gamma(D/2)$ is the surface of a unit sphere in D dimensions. This energy is maximal at $R_c = (D-1)\sigma/\epsilon$, where it is equal to

$$E_c = \frac{S_D}{D} (D-1)^{D-1} \frac{\sigma^D}{\epsilon^{D-1}}. \quad (5)$$

The important point is now that for temperatures which lie only very little beyond the transition temperature, the energy difference ϵ between the two phases is very small, corresponding a huge bubble radius and energy. The probability of nucleating such a bubble is therefore suppressed by an infinitesimally small factor $\exp(-\text{const}/\epsilon^{D-1})$. Only after consid-

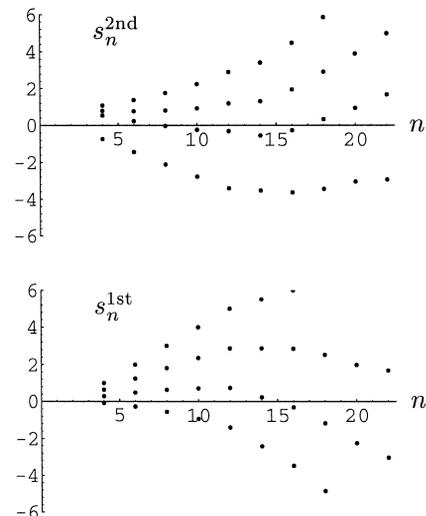


Fig. 2. Free energy as a function of loop length n for second- and first-order phase transitions at different inverse model temperatures β_V .

erable overheating (or overcooling) does the bubble energy become small enough to nucleate spontaneously (in the absence of other condensation nuclei such as dirt). In freezing transition of water, the radius r_c is about 50 Å.

In a superconductor, however, the phase transition proceeds without overheating, and the reason for this is the vortex nucleation discussed above. The energy of a critical vortex may be estimated by imagining a planar phase boundary rolled up to a thin line whose radius is the coherence length ξ_0 of the disorder theory. This, in turn, is bent into a doughnut of radius $n^{\text{st}}\xi_0$. Neglecting the bending energy, we estimate the critical vortex energy to be of the order

$$E_{\text{vort}}^c \approx 2\pi\xi_0 \times n^{\text{st}}\xi_0 \sigma. \quad (6)$$

This energy does not depend on the energy difference ϵ between the two phases, so that the rate is practically independent of the degree of overheating (or undercooling), this being in contrast to the energy of the critical bubble which is extremely large slightly beyond the transition point.

Note that in contrast to vortex nucleation, bubble nucleation is not enhanced significantly by the configurational entropy of fluctuations of the bubble surface. The reason is that apart from translations, all surface fluctuations are massive [17], Configurational distortions of a long vortex line, on the other hand, require practically no energy as long as they happen on length scales longer than the finite stiffness length.

4. The above discussion shows that bubble nucleation is of negligible relevance to the phase transitions in superconductors of second as well as first order, and for that matter, to the first-order transitions in the early universe, as long as we believe the theory describing the latter to be of a generalized Ginzburg-Landau type, allowing for line-like topological excitations. These drive the transition with much greater efficiency than bubbles due to their larger configurational entropy.

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