DO PRESENT MESON BARYON SCATTERING DATA REALLY SUPPORT $(\bar{3},3)+(3,\bar{3})$ DOMINANCE IN $SU(3)\times SU(3)$ SYMMETRY BREAKING ?

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A general analysis is presented of the hypothesis that $SU(3)\times SU(3)$ is broken by terms in the hamiltonian transforming according to $(\bar{3},3)+(3,\bar{3})$ and $(8,1)+(1,8)$. We show that the popular assumption of dominance of $(\bar{3},3)+(3,\bar{3})$ cannot be confirmed as yet because of the scarceness of experimental data and the uncertainties in the off mass shell continuation involved.

Some time ago, it was conjectured that the local hamiltonian density of hadrons would reflect the $SU(3)\times SU(3)$ structure of the observable vector and axial vector currents of electromagnetic and weak interactions by being a sum of the simplest possible tensor operators transforming like $(1,1)$, $(\bar{3},3)+(3,\bar{3})$, and $(8,1)+(1,8)$ with respect to the algebra of charges [1]. Explicitly, one assumes that $\theta_{00}(x)$ can be written as

$$\theta_{00}(x) = \theta^{*}_{00}(x) + u_0(x) + cu_8(x) + g_8(x) \tag{1}$$

where $\theta^{*}_{00}(x)$ is an $SU(3)\times SU(3)$ singlet, while $u_0(x), u_8(x)$ and $g_8(x)$ are members of the set of local scalar operators $u_{i'}(x), v_{i'}(x)$ $(i'=0,\dots,8)$ and $g_i(x), h_i(x)$ $(i=1,\dots,8)$, respectively. The parameter c is an unknown constant.

As a direct consequence one obtains expressions for the divergence of the vector and axial currents such as

$$\partial^{\mu} A^{i}_{\mu}(x) = -(d^{ii0}+cd^{ii8})v_i - \delta^{i8}\sqrt{\tfrac{2}{3}}\,cv^0 + f^{i8j}h_j \tag{2}$$

and from this one can calculate directly the charge divergence commutators

$$\Sigma^{ij} \equiv i[\int d^3x A^{i}_{0}(x,0), \partial^{\mu}A^{j}_{\mu}(0)] = (\tfrac{2}{3}\delta^{ij} + \sqrt{\tfrac{2}{3}}cd^{ij8})u_0(0) + (\sqrt{\tfrac{2}{3}}d^{ijk} + cd^{j8l}d^{lik} + \tfrac{2}{3}cd^{j8}f^{ik})u_k(0) + f^{j8l}f^{lik}g_k(0). \tag{3}$$

Relation (3) offers the possibility of an experimental test in meson baryon scattering. If one assumes that the divergence of the axial current $\partial^{\mu}A^{a}_{\mu}$ satisfies the PCAC hypothesis, one can derive a low energy theorem for the scattering amplitude $\mathcal{M}^{ba}_{\beta\alpha}$ of pseudoscalar mesons $P_{b,a}$ on baryons $B_{\beta,\alpha}$ † at a point where all spatial momenta are zero [2]. If the initial and final (off mass shell) mesons have energies ω_a, ω_b (such that $\omega_a - \omega_b = \Delta M \equiv M_{\beta} - M_{\alpha}$) we find the theorems

† We normalize states according to $\langle p'|p\rangle = N(2\pi)^3\,\delta^3(\boldsymbol{p}\cdot\boldsymbol{p})$ with $N = p_0/m$ for baryons and $N = 2p_0$ for mesons and define the amplitude for the processes $P_b(q_b)B_{\beta}(p_{\beta}) \leftarrow P_a(p_a)\,B_{\alpha}(p_{\alpha})$ by $S = 1 - i(2\pi)^4\delta^4(P_f - P_i)\mathcal{M}^{ba}_{\beta\alpha}$.

Table 1

The experimental values of the scattering length $T = -(1/4\pi)\{M_\alpha/(M_\alpha+\mu_a)\}\mathcal{M}^{ba}_{\rho\alpha}$ are shown together with the contribution T_Q coming from $\mathcal{M}_Q$ [eq. (5)] and the cut correction T_{cut} of ref. [3]. The discrepancy $\Delta \mathrm{Re}\ T = \mathrm{Re}\,(T_{exp} - T_Q - T_{cut})$ has to be extrapolated to the low energy point giving Σ_{exp}. This Σ_{exp} is compared with the predictions of pure (8,1) + (1,8) breaking in column 9 and with pure $(\bar{3},3) + (3,\bar{3})$ breaking in figs. 1 - 3.

$(\text{Reaction})_1$	Re T_{exp} (fm)	Re T_Q (fm) for $f_K/f_\pi = 1$	Re T_Q (fm) for $f_K/f_\pi = 1.25$	Re T_{cut} (fm)	ΔRe T (fm) for $f_K = f_\pi$	ΔRe T (fm) for $f_K/f_\pi = 1.25$	Σ (MeV) for $f_K/f_\pi = 1$	Σ (MeV) for $f_K/f_\pi = 1.85$	Σ (18)+(81)
$(\hat{\Lambda}N)_1$	0.27 ± 0.03	0.21	0.21	0.02 ± 0.01	0.04 ± 0.04	0.04 ± 0.04	-25 ± 25	-25 ± 25	0
$(\hat{\Lambda}N)_3$	-0.14 ± 0.01	-0.10	-0.10	0.01 ± 0.01	0.05 ± 0.02	-0.05 ± 0.02	35 ± 15	35 ± 15	0
$(KN)_0$	-0.15... + 0.08	0	0	0.02 ± 0.01	-0.18... + 0.07	-0.18... + 0.07	-60... + 155	-95... + 245	-60
$(KN)_1$	-0.30 ± 0.03	-0.57	-0.37	0.20 ± 0.01	0.07 ± 0.04	-0.13 ± 0.04	-60 ± 35	180 ± 55	-190
$(\pi\Sigma)_1$	-0.61... + 0.46	0.21	0.21	0.00 ± 0.10	-0.92... + 0.35	-0.92... + 0.35	-225... + 590	-225...590	0
$(\pi\Sigma)_0$	-0.34... + 0.45	0.43	0.43	0.45 ± 0.14	-1.36... - 0.29	-1.36... - 0.29	185...870	185...870	0
$(\bar{K}N)_1$	-0.15... - 0.06	0.29	0.18	-0.23 ± 0.04	-0.25... - 0.08	-0.14... + 0.03	70...220	-40...190	-125
$(\bar{K}N)_0$	-1.67 ± 0.11	0.86	0.55	-2.80 ± 0.18	0.27 ± 0.29	-0.58 ± 0.29	-235 ± 255	-790 ± 395	-255

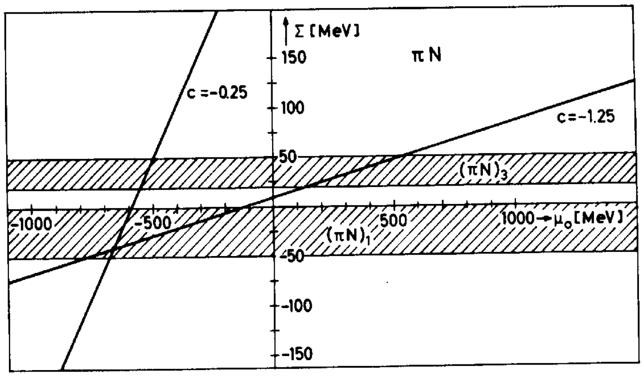

Fig. 1.

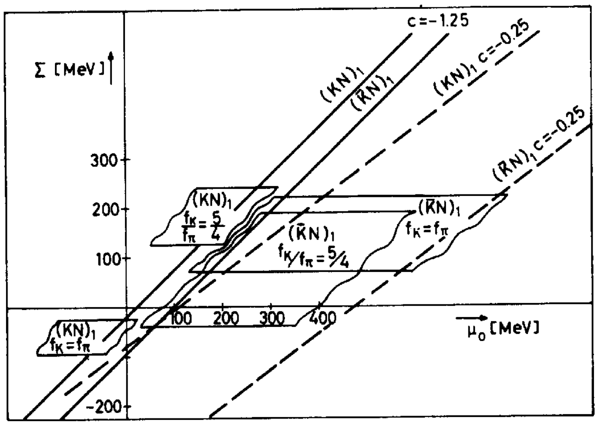

Fig. 2.

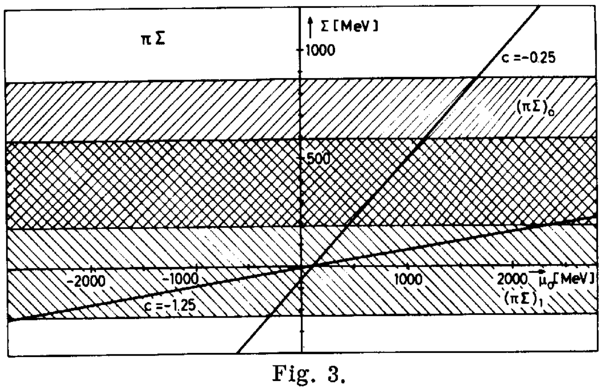

Fig. 3.

Figs. 1 - 3. The ranges of the free parameter $\mu_0 = \langle p|u_0|p\rangle$ for $(\bar{3},3) + (3,\bar{3})$ breaking can be read off the intersection of the straight lines with the experimental Σ-values. The figures speak for themselves. Van Hippel and Kim's value comes from neglecting the πN and $\pi\Sigma$ data and assuming $c = -1.25$ and $f_K/f_\pi = 5/4$). We see that by this method we find $\mu_0 \in [0.150, 0.280]$. For $f_K = f_\pi$ the $(KN)_1$ and $(\bar{K}N)_1$ data have no intersection.

$$\lim_{\omega_{\mathrm{a}\atop \mathrm{b}}\to 0} \mathcal{M}^{\mathrm{ba}}_{\beta\alpha} \equiv \mathcal{M}_{\mathrm{a}\atop \mathrm{b}} = [1 - (\Delta M/\mu_{\mathrm{b}\atop \mathrm{a}})^2][\mathcal{M}^{\mathrm{ba}}_{\sigma_{\beta\alpha}} \pm \tfrac{1}{2}\Delta M \mathcal{M}^{\mathrm{ba}}_{\mathrm{Q}_{\beta\alpha}}] \tag{4}$$

where $\mathcal{M}_{\mathrm{Q}}$ are the matrix elements of the vector current

$$\mathcal{M}^{\mathrm{ba}}_{\mathrm{Q}_{\beta\alpha}} \equiv \frac{1}{f_{\mathrm{b}} f_{\mathrm{a}}} \langle B_\beta | [Q^5_{\mathrm{b}}(0), A_{\mathrm{a}}(0)] | B_\alpha \rangle = \frac{i}{f_{\mathrm{b}} f_{\mathrm{a}}} f_{\mathrm{bac}} \langle B_\beta | V_{\mathrm{c}}(0) | B_\alpha \rangle \tag{5}$$

and $\mathcal{M}^{\mathrm{ba}}_{\sigma_{\beta\alpha}}$ are those of the symmetrized commutator Σ^{ba}:

$$\mathcal{M}^{\mathrm{ba}}_{\sigma_{\beta\alpha}} = \frac{1}{2 f_{\mathrm{b}} f_{\mathrm{a}}} \{\langle B_\beta | \Sigma^{\mathrm{ba}} | B_\alpha \rangle + (\mathrm{a} \leftrightarrow \mathrm{b})\} \equiv \frac{1}{f_{\mathrm{b}} f_{\mathrm{a}}} \Sigma_{\mathrm{b}\beta,\mathrm{a}\alpha} \tag{6}$$

The symmetrized matrix elements $\Sigma_{\mathrm{b}\beta,\mathrm{a}\alpha}$ are customarily called Σ-terms.

It is the purpose of this letter to compare Σ-terms of the scattering processes $(\pi_{\mathrm{b}} B_\beta \leftarrow \pi_{\mathrm{a}} B_\alpha)$, $(\mathrm{K_b}\, B_\beta \leftarrow \mathrm{K_a}\, B_{\mathrm{a}})$ and $(\mathrm{K_b}\, B_\beta \leftarrow \mathrm{K_a}\, B_\alpha)$ with experiment. Such a comparison has been made by von Hippel and Kim [2]. The authors came to the conclusion that in the SU(3) × SU(3) breaking of eq. (1) the term $u_0 + c u_8$ dominates. In the following we shall critically analyze the evidence leading to this conclusion.

Our analysis will proceed in the following way. First we develop theorems for the Σ-terms which do not depend on any model of symmetry breaking and follow alone from the symmetry of $\Sigma_{\mathrm{b}\beta,\mathrm{a}\alpha}$ in a and b. Any comparison of experimental Σ-values with predictions of a specific breaking assumption should proceed only once these relations are satisfied. The set of possible independent predictions is then considerably reduced. If one does not properly take care of these relations one may erroneously take some agreements with experiment as successes of the model.

For the experimental test one had to tackle the difficult problem of continuing the experimental data to the off mass shell point where the low energy theorems (4) are valid. This problem has been discussed in the literature [3]. The extrapolation procedure involves a dispersion integral over a cut whose imaginary part is not given by physical processes. In addition, for scattering involving kaons, there is a strong dependence of the result on the value one assumes for f_{k}/f_π (which is uncertain by ≈ 30%). We tentatively accept the cut corrections given in ref. [2] and find for typical values $f_{\mathrm{k}}/f_\pi = 1.25$ and $f_{\mathrm{k}}/f_\pi = 1$ the experimental Σ-terms displayed in columns 7 and 8 of table 1. Our numbers are based on scattering length taken from the compilation of Ebel et al. [4].

Let us now compare these numbers with the general symmetry theorems for the Σ-term [‡]

$$\Sigma^{1/2}_{\pi\mathrm{N}} = \Sigma^{3/2}_{\pi\mathrm{N}} \equiv \Sigma_{\pi\mathrm{N}} \tag{ST 1}$$

$$2\Sigma^{0}_{\pi\Sigma} + 3\Sigma^{1}_{\pi\Sigma} - 5\Sigma^{2}_{\pi\Sigma} = 0 \tag{ST 2}$$

$$\Sigma^{1}_{\pi\Lambda\pi\Sigma} = 0 \tag{ST 3}$$

$$\Sigma^{\pm}_{\mathrm{KN}} = \pm\Sigma^{\pm}_{\overline{\mathrm{K}}\mathrm{N}} \tag{ST 4}$$

$$\Sigma^{\pm}_{\mathrm{K}\Sigma} = \pm\Sigma^{\pm}_{\overline{\mathrm{K}}\Sigma} \tag{ST 5}$$

$$\Sigma_{\mathrm{K}\Lambda} = \Sigma_{\overline{\mathrm{K}}\Lambda} \tag{ST 6}$$

$$\Sigma_{\mathrm{K}\Lambda\mathrm{K}\Sigma} = -\Sigma_{\overline{\mathrm{K}}\Lambda\overline{\mathrm{K}}\Sigma} \tag{ST 7}$$

‡ It is useful to go into s-channels of definite isospin which is shown by the superscript of Σ^{I_s}. For elastic reactions the particle indices are written only once. Furthermore we shall use the standard abbreviations $\Sigma^{+}_{\mathrm{K}\Sigma} = \frac{1}{3}(2\Sigma^{3/2}_{\mathrm{K}\Sigma} + \Sigma^{1/2}_{\mathrm{K}\Sigma})$; $\Sigma^{-}_{\mathrm{K}\Sigma} = \frac{1}{3}(\Sigma^{1/2}_{\mathrm{K}\Sigma} - \Sigma^{3/2}_{\mathrm{K}\Sigma})$; $\Sigma^{+}_{\mathrm{KN}} = \frac{1}{4}(3\Sigma^{1}_{\mathrm{KN}} + \Sigma^{0}_{\mathrm{KN}})$; $\Sigma^{-}_{\mathrm{KN}} = \frac{1}{4}(\Sigma^{1}_{\mathrm{KN}} - \Sigma^{0}_{\mathrm{KN}})$.

We find 1) Disagreement for (ST 1) ($\Sigma_{\pi N}^{1/2} = \Sigma_{\pi N}^{3/2}$) 2) Comptability of (ST 4) ($\Sigma_{KN}^{\pm} = \pm\Sigma_{\bar{K}N}^{\pm}$) for $f_K = f_\pi$, disagreement for $f_K/f_\pi = 1.25$.

Hippel and Kim like the value $f_K/f_\pi = 1.25$ and argues that Σ_{KN}^{0} is bad due to a large cut correction (and thus large possible systematic errors). If one believes this one can eliminate Σ_{KN}^{0} from (ST 4) and obtains one relation

$$\Sigma_{KN}^{1} + \Sigma_{KN}^{0} = 2\,\Sigma_{\bar{K}N}^{1} \qquad \text{(ST 4)'}$$

which is compatible with experiment. Obviously only two independent KN and $\bar{K}N$ data remain for comparison with a model, say Σ_{KN}^{1} and $\Sigma_{\bar{K}N}^{1}$. All other relations cannot be tested due to the lack of information on the $(\pi\Sigma)^2$, KΣ, and KΛ channels.

We are now ready to discuss consequences of the breaking assumption (1). In order to obtain results we have to parametrize the matrix elements of u and g. Taking into account some first order Gell-Mann - Okubo type SU(3) corrections suggested by models [5], we parametrize

$$\langle B_\beta|\overset{0}{u}{}^{8}|B_\alpha\rangle = \overset{0}{\mu}{}^{8}\,\delta_{\beta\alpha} - \mathrm{i}\overset{0}{F}{}^{8}f_{8\beta\alpha} + \overset{0}{D}{}^{8}d_{8\beta\alpha}\,; \qquad \langle B_\beta|u^{i\neq 0,8}|B_\alpha\rangle = -\mathrm{i}Ff_{i\beta\alpha} + Dd_{i\beta\alpha}\,;$$

$$\langle B_\beta|g^{8}|B_\alpha\rangle = k^{8}\,\delta_{\beta\alpha} - \mathrm{i}f^{8}f_{i\beta\alpha} + d^{8}d_{8\beta\alpha}\,; \qquad \langle B_\beta|g^{i\neq 8}|B_\alpha\rangle = -\mathrm{i}ff_{i\beta\alpha} + dd_{i\beta\alpha}\,, \qquad (7)$$

where μ^8, F^0, D^0, k^8, F^8-F, D^8-D, f^8-f, and d^8-d are first order ($\approx 30\%$) corrections to the corresponding values of μ^0, F, D, f and d. This parametrization has one general property: The t-channel is nonexotic, i.e., no states of isospin $I_t = 2$ or 3/2 are exchanged. For the processes under consideration this implies the nonexoticity theorem

$$2\Sigma_{\pi\Sigma}^{0} - 3\Sigma_{\pi\Sigma}^{1} + \Sigma_{\pi\Sigma}^{2} = 0 \qquad \text{(NET)}$$

Due to the absence of an analysis of the $(\pi\Sigma)_2$ channel we can test this theorem only in conjunction with ST 2, leading to

$$\Sigma_{\pi\Sigma}^{0} = \Sigma_{\pi\Sigma}^{1} \equiv \Sigma_{\pi\Sigma}\ (=\Sigma_{\pi\Sigma}^{2}) \qquad (8)$$

compatible with the extremely rough experimental results. In spite of the many parameters in (9) one can derive one non-trivial theorem

$$2\Sigma_{KN}^{-} = \Sigma_{K\Lambda K\Sigma} - \Sigma_{K\Sigma}^{-} \qquad \text{(T)}$$

which cannot be tested as yet due to the absence of information on the KΣ, KΛ channels. This exhausts all theorems independent of the 13 parameters *.

If one wants to get more results one needs more assumptions. One possibility is to suppose all D/F ratios to be the same as the one found in the SU(3) mass difference. Such a parametrization occurs naturally in some lagrangian models [5]. Because of lack of space we shall not list these theorems here ** and proceed directly to a much rougher approximation: we assume that SU(3) breaking can be neglected in the parametrization (7) and that either $(\bar{3},3)+(3,\bar{3})$ or $(8,1)+(1,8)$ dominate the SU(3) $\times$ SU(3) breaking. In these cases one can determine immediately

$$cF = f = (\tfrac{1}{3}\sqrt{3})\,(m_N - m_\Xi)\,, \quad cD = d = \tfrac{1}{2}\sqrt{3}(m_\Sigma - m_\Lambda) \qquad (9)$$

and one finds in the case a) Pure lowest order $(\bar{3}3)+(3\bar{3})$ breaking

$$\Sigma_{\pi\Sigma} + \Sigma_{\pi\Lambda} = 2W_\pi(c)\sqrt{\tfrac{2}{3}}\,\mu_0 \qquad \text{(T 1)}$$

$$\Sigma_{\pi\Sigma} - \Sigma_{\pi\Lambda} = \tfrac{2}{3}W_\pi(c)\,D \qquad \text{(T 2)}$$

$$4\Sigma_{\pi N} - \Sigma_{\pi\Sigma} - 3\Sigma_{\pi\Lambda} = 2W_\pi(c)\,F \qquad \text{(T 3)}$$

* For the processes where π, K or $\bar{K}$ are unchanged in the scattering process.

** A more complete discussion of the properties of the Σ-terms can be found in ref. [6].

$$\Sigma_{K\Lambda K\Sigma} = \tfrac{1}{2} W_K D \tag{T 4}$$

$$\Sigma^-_{K\Sigma} = -\tfrac{1}{2} W_K F \tag{T 5}$$

$$\Sigma^+_{K\Sigma} + \Sigma_{K\Lambda} = 2 W_K \sqrt{\tfrac{2}{3}}\, \mu_0 \tag{T 6}$$

$$\Sigma^+_{K\Sigma} - \Sigma_{K\Lambda} = -\tfrac{1}{3} W_K D \tag{T 7}$$

$$4\Sigma^+_{KN} - \Sigma^+_{K\Sigma} - 3\Sigma_{K\Lambda} = -W_K F, \tag{T 8}$$

where $W_\pi(c) = (\sqrt{2}+c)/\sqrt{3}$ and $W_K(c) = (\sqrt{2}-\frac{1}{2}c)/\sqrt{3}$.

In order to obtain theorems for the case b) Pure lowest order (81) + (18) breaking we simply set all $\Sigma_{\pi\beta\pi\alpha} = 0$ and replace the right hand sides in (T 4 - 8) by $\frac{1}{4}\sqrt{3}d$, $-\frac{1}{4}\sqrt{3}f$, 0, $+\frac{1}{2}\sqrt{3}d$, $\frac{3}{2}\sqrt{3}f$, respectively.

Part of the theorems a) can be tested by eliminating the unknown $\Sigma_{\pi\Lambda}$ in (T 2) and (T 3) in terms of μ_0 and substituting (T 6) and (T 7) into (T 8) and (T). The resulting four equations for μ_0 are shown in the figs. 1-3 for the two most popular values of c♯. We see that experiments give large non-overlapping ranges of μ_0 and no common value can be found.

How did Hippel and Kim find their value $\mu_0 = 0.215$? They discard the πN data on the grounds that the slope of the straight line for $c = -1.25$ is too small to give a sensitive determination of μ_0. But even with this insensitivity the $(\pi N)_1$ data give large negative values of μ_0. Such values are also obtained from the $\pi\Sigma$ data of Hippel and Kim. The authors neglect these data expecting large systematic uncertainties. Indeed, plotting only the data of other authors (quoted in ref. [4]) in fig. 3, we find large positive values of μ_0 in the range where $(\pi\Sigma)_0$ and $(\pi\Sigma)_1$ overlap (as they should from eq. (8)). Finally they discard the $(\bar{K}N)_0$ data and give a good two parameter fit to $(KN)_1$ and $(\bar{K}N)_1$ assuming the value $c = -1.25$ and $\mu_0 = 0.215$ (they also claim to fit $(KN)_0$, but do not notice that this follows generally from (ST 4'). For $f_K = f_\pi$ there is no μ_0 to fit both values. It is not clear why $(KN)_1$ and $(\bar{K}N)_1$ data should be any more significant $(\pi N)_1$ data.

The hypothesis that (8, 1) + (1, 8) is dominant in the symmetry breaking can clearly be eliminated in spite of the roughness of data (see column 9 of table 1). We conclude:

1) In testing models one should take care in counting only the model dependent agreements with experiment.

2) As long as the probably best known πN data violate the general theorems there should be a thorough re-investigation of the extrapolation procedure.

3) The dominating presence of $(\bar{3}, 3) + (3, \bar{3})$ cannot be confirmed as yet.

4) Dominance of (8, 1) + (1, 8) is outruled by the data.

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♯ We display the curve for the philosophy dependent values $c = -1.25$ of ref. [1] and $c = -0.25$ of ref. [7].

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