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On the Helicity-Flip Property of the $A_2\mathcal{N}\mathcal{N}$ Coupling.

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Evidence is piling up in favour of the hypothesis that the more important meson Regge trajectories P, f, ω and ρ, A_2 couple to nucleons predominantly to s-channel helicity-nonflip ([1-3]) and helicity-flip amplitudes ([2]), respectively. The interesting point about this hypothesis is that the on-mass-shell coupling constants of the particles lying on these trajectories appear to possess the same properties, indicating a surprisingly smooth behaviour of the flip to nonflip ratio when continuing t from the forward direction to the meson mass.

Consider $\pi\mathcal{N}$ scattering. There the assumption of the absence of the P and f trajectory in the helicity-flip amplitude has the consequence that both invariant amplitudes $A^{(+)}$ and $B^{(+)}$ obey unsubtracted dispersion relations in the forward direction. Then using the additional information on the values of $A^{(+)}$ and $B^{(+)}$ at threshold supplied by an unsubtracted backward dispersion relation, ENGELS and HÖHLER ([4]) have derived the estimates for the coupling of f to nucleons (*)

$$\frac{G^{(1)2}_{f\mathcal{N}\mathcal{N}}}{4\pi} = 53 \pm 10\,, \qquad \frac{G^{(2)2}_{f\mathcal{N}\mathcal{N}}}{4\pi} = 3 \pm 7\,, \tag{1}$$

compatible with pure nonflip amplitude (**). It is interesting to note that this prop-

(1) G. HÖHLER and R. STRAUSS: *Fortschr. Phys.*, **232**, 205 (1970).
(2) F. J. GILMAN, J. PUMPLIN, A. SCHWIMMER and L. STODOLSKY: *Phys. Lett.*, **31** B, 387 (1970); H. HARARI and Y. ZARMI: *Phys. Lett.*, **32** B, 291 (1970).
(3) R. ODORICO, A. GARCIA and C. A. GARCIA CANAL: *Phys. Lett.*, **32** B, 375 (1970); C. MICHAEL and R. ODORICO: *Phys. Lett.*, **34** B, 422 (1971).
(4) J. ENGELS and G. HÖHLER: Karlsruhe preprint (1970).
(*) Note also that H. SCHAILE (Karlsruhe Thesis, 1970) using fixed-u dispersion relations and R. STRAUSS (Karlsruhe Thesis, 1970) using fixed-angle dispersion relations have obtained similar values for these coupling constants; however, they quote larger errors.
(**) For f as well as A_2 couplings to protons we use the Lagrangian

$$\mathscr{L} = \left[\frac{G^{(1)}_{f\mathcal{N}\mathcal{N}}}{m}\,\frac{i}{4}\,\bar{\psi}(\overleftrightarrow{\partial}_\mu\gamma_\nu + \overleftrightarrow{\partial}_\nu\gamma_\mu)\psi + \frac{G^{(2)}_{f\mathcal{N}\mathcal{N}}}{m^2}\,\partial_\mu\bar{\psi}\partial_\nu\psi\right] f^{\mu\nu}$$

(where $m = m_{\mathcal{N}}$ is used here and in the rest of the paper). Then the helicity flip to nonflip ratio in $\pi\mathcal{N}$ scattering is given for $\nu \to \infty$ close to the f pole by

$$f_{+-}/f_{++} \approx \frac{\sqrt{-t}}{2m}\,G^{(2)}_{f\mathcal{N}\mathcal{N}}/G^{(1+2)}_{f\mathcal{N}\mathcal{N}}\,.$$

erty allows the second of the gravitational form factors of the nucleon to be dominated by an f-meson via an unsubtracted dispersion relation (*).

Similarly, ω does not flip the nucleon spin on its mass shell since $x^s = -0.06$ and the flip to nonflip ratio is, for large ν and close to the ω pole,

$$f_{+-}/f_{++} \approx \frac{\sqrt{-t}}{2m}\left(\frac{A+mB}{A+\nu B}\right) \approx -\frac{\sqrt{-t}}{2m}\,2x^s\,.$$

For ρ the same argument shows that on shell ρ mostly flips the nucleon spin (since $2x^v = 3.7$).

No such direct on-shell argument has, until now, been presented for the coupling of A_2 to nucleons. We shall show in this note that, indeed, the A_2-meson couples on shell predominantly to the helicity-flip amplitude.

Consider the standard CGLN (6) basis of photoproduction. The amplitudes $(1/s-u)A^{(-)}$ and $(1/s-u)D^{(-)}$ are even functions in s-u. They behave for large energy in the forward direction according to $s^{\alpha_{A_2}(0)-2}$ and therefore certainly obey unsubtracted dispersion relations (since $\alpha_{A_2}(0) \approx 0.5$). At large energy in the backward direction they are dominated by

$$s^{\alpha_\Delta(0)-\frac{3}{2}} \approx (-t)^{\alpha_\Delta(0)-\frac{3}{2}}\,.$$

Since $\alpha_\Delta(0) \approx 0.2$, we can write also here an unsubtracted dispersion relation. Equating both relations at threshold we obtain two sum rules. The nucleon Born term does not contribute to either one of them (**). Due to the strong fall-off for large energies these sum rules should saturate extremely quickly and we can be content with inserting only the lowest resonances which can contribute (***): Δ in the s-channel and A_2 in the t-channel. In this way we find from $(1/s-u)A^{(-)}$ and $(1/s-u)D^{(-)}$, respectively $({}_{*}{}^{*}{}_{*})\,({}^{*}{}_{*}{}^{*})$,

$$g^*\frac{m}{m_\pi}\left[m^2(C_4+C_5)+mC_3\right] = \frac{3}{2}\,G^{(2)}_{A_2\mathcal{N}\mathcal{N}}\,g_{A_2\pi\gamma}\,\frac{m_\Delta^2}{m_{A_2}^2}\,, \tag{2}$$

$$g^*\frac{m}{m_\pi}\,m^2(C_4+C_5) = 6G^{(1+2)}_{A_2\mathcal{N}\mathcal{N}}\,g_{A_2\pi\gamma}\,\frac{m_\Delta^2 m^2}{m_{A_2}^4}\,. \tag{3}$$

(*) If $\langle p'|\theta_{\mu\nu}|p\rangle \equiv \bar u(p')[(\gamma_\mu P_\nu+\gamma_\nu P_\mu)F_1(t)/4 + P_\mu P_\nu F_2(t)/4m + (q_\mu q_\nu - t g_{\mu\nu})F_3(t)]\,u(p)$, mass and spin normalization determine $F_1(0)=1$ and $F_2(0)=0$, respectively. f dominance of F_2 gives $F_2(t) \propto G^{(2)}_{f\mathcal{N}\mathcal{N}} \cdot (m_f^2-t)^{-1}$, hence $G^{(2)}_{f\mathcal{N}\mathcal{N}}=0$. Note, however, that a similar f-dominance assumption for F_1 gives $G^{(1)}_{f\mathcal{N}\mathcal{N}}$ about $\frac{1}{3}$ in magnitude of that given in (1), see ref. (5).

(5) G. F. CHEW, M. L. GOLDBERGER, F. E. LOW and Y. NAMBU: *Phys. Rev.*, **106**, 1345 (1957).

(6) J. ENGELS, G. HÖHLER and B. PETERSSON: *Nucl. Phys.*, **15** B, 365 (1970).

(**) Nor does the Roper (or other $J^p = \frac{1}{2}^+$ baryons) contribute.

(***) The error arising by neglecting higher meson resonances is hard to estimate; the error introduced by leaving out higher baryon resonances can be shown to be small. A more complete treatment will be given in a future publication.

$({}_{*}{}^{*}{}_{*})$ Here g^* is defined by

$$\mathcal{L}_{\Delta\mathcal{N}\pi} = \frac{g^*}{m_\pi}\,\bar\Delta_{\mu,a}N\,\partial^\mu\pi_a\,,$$

such that from

$$\Gamma_{\Delta\mathcal{N}\pi} = \frac{1}{2}\frac{g^{*2}}{4\pi}\,p^3\,\frac{E^*+m}{M^*m_\pi^2}\,,\qquad g^{*2}/4\pi \approx 0.37\,.$$

Taking finite width into account, one estimates $g^{*2}/4\pi \approx 0.26$ (7). The Gourdin-Salin (8,9) coupling con-

Therefore, we find for the ratio of flip to nonflip couplings

$$G^{(1+2)}_{A_2\mathcal{N}\mathcal{N}}/G^{(2)}_{A_2\mathcal{N}\mathcal{N}} = \frac{1}{4}\frac{m^2_{A_2}}{m^2}\frac{x}{1+x} \approx 0.44\frac{x}{1+x} \tag{4}$$

with

$$x \equiv m(C_4 + C_5)/C_3 . \tag{5}$$

The ratio x can be taken from experiment by relating it to the ratio of electric-quadrupole and magnetic-dipole amplitudes E_{1+}/M_{1+} of CGLN at the Δ-resonance. One finds

$$-E_{1+}/M_{1+} = \left(1 - \frac{m_\Delta}{m}x\right)\Bigg/\left(\frac{3m_\Delta + m}{m_\Delta - m} - \frac{m_\Delta}{m}x\right) \approx 6.5(1 - 1.32x)\% . \tag{6}$$

Experimentally, one has

$$-E_{1+}/M_{1+} \approx \begin{cases} 4.6\% \;(^{10}), \\ 3.1\% \;(^{11}) , \\ 5.9\% \;(^{12}), \end{cases}$$

and we take $C_3 m \approx 2$ [13], giving $x \approx 0.2$ close to the original value of GOURDIN and SALIN [8] of $x \approx 0.16$. This corresponds to [*]

$$G^{(1+2)}_{A_2\mathcal{N}\mathcal{N}}/G^{(2)}_{A_2\mathcal{N}\mathcal{N}} \approx 0.07 , \tag{7}$$

such that A_2 indeed couples more strongly to nucleon s-channel helicity-flip than to nonflip amplitudes. We would here also like to remark that by applying similar

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stants C_3, C_4, C_5 are defined at $t = 0$ by

$$\langle \Delta(p')|j^\mu|N(p)\rangle \equiv \bar{u}_\nu(p')\gamma_5\big[C_3\big((\gamma\cdot k)g^{\mu\nu} - k^\nu\gamma^\mu\big) + C_4(kp'g^{\mu\nu} - k^\nu p'^\mu) + C_5(kpg^{\mu\nu} - k^\nu p^\mu)\big]u(p) .$$

The $A_2\pi\gamma$ coupling we use is

$$\langle \pi(q)|j^{A_2}_{\mu\tau}|\gamma(k)\rangle \equiv ie\frac{g_{A_2\pi\gamma}}{2m^2_{A_2}}[\varepsilon_{\mu\nu\lambda\varkappa}q_\tau q^\nu(\varepsilon^\varkappa k^\lambda - \varepsilon^\lambda k^\varkappa) + (\mu \leftrightarrow \tau)] .$$

(7) B. RENNER: *Phys. Lett.*, **33** B, 599 (1971).
(8) M. GOURDIN and PH. SALIN: *Nuovo Cimento*, **27**, 193, 309 (1963); **32**, 521 (1964).
(9) J. BAACKE and H. KLEINERT: *Phys. Lett.*, **35** B, 159 (1970).
(**) We neglect corrections to the equations of order m_π/m.
(10) R. L. WALKER: *Phys. Rev.*, **182**, 1729 (1969).
(11) KIM-KONG TAILE: Diplomarbeit, Bonn (1968).
(12) W. PFEIL and D. SCHWELA: *Springer Tracts of Modern Physics*, Vol. **55** (1970), p. 213.
(13) J. MATHEWS: *Phys. Rev.*, **137**, B 444 (1965); A. J. DUFNER and Y. S. TSAI: *Phys. Rev.*, **168**, 1801 (1967).
(*) Note that this corresponds to a ratio $(A'/\nu B)_{t=m^2_{A_2}} \approx -0.1$ for the t-channel $I = 1$ on amplitudes of kaon-nucleon scattering at the A_2 pole at high energies. This ratio is of the same magnitude but differing in sign to the ratio of the corresponding A_2-meson Regge residue functions at $t = 0$ obtained in Regge fits [14]. We also remark that the same ratio at the ρ pole is estimated by

$$(A'/\nu B)_{t=m^2_\rho} \approx \frac{1}{1+2\varkappa^\nu} \approx 0.2 .$$

(14) G. V. DASS and C. MICHAEL: *Phys. Rev.*, **175**, 1774 (1968).

techniques to the amplitudes of Compton scattering on nucleons, one arrives at the same conclusion ([15]). For completeness we use eq. (2) to estimate

$$G^{(2)}_{A_2\mathcal{N}\mathcal{N}}\, g_{A_2\mathcal{N}\mathcal{N}} \approx 20 . \tag{8}$$

If we take an estimate on the $A_2\pi\gamma$ coupling coming from vector-meson dominance and $A_2 \to \pi\rho$ decay (*)

$$g^2_{A_2\pi\gamma} \approx 10.6 , \tag{9}$$

we find

$$\frac{G^{(1)2}_{A_2\mathcal{N}\mathcal{N}}}{4\pi} \approx \frac{G^{(2)2}_{A_2\mathcal{N}\mathcal{N}}}{4\pi} = 3 . \tag{10}$$

In conclusion we see that the simple technique of subtracting forward and backward dispersion relations from each other at threshold provides a powerful tool ([8,15,21]) for the determination of on-shell coupling constants of particles exchanged in the t-channel. We hope that a combination of this method with Regge fits of high-energy scattering will supply us with direct information on the structure of Regge residues when continued from the forward direction to the particle poles.

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([15]) J. BAACKE, T. Y. CHANG and H. KLEINERT: to be published.

(*) We use

$$\gamma_\rho^2/4\pi = 2.5 \qquad \text{and} \qquad \Gamma_{A_2\pi\rho} = \frac{4}{5}\,\frac{p^2_{\text{c.m.}}}{m^4_{A_2}}\,\frac{g^2_{A_2\pi\rho}}{4\pi} = 85\ \text{MeV} ,$$

where we have used the total A_2 width ≈ 100 MeV and branching ratio to $\rho\pi \approx 85\,\%$ ([16]) (note that a slightly larger total A_2 width ≈ 125 MeV is obtained from ref. ([17])). $g_{A_2\pi\rho}$ is defined analogously to $g_{A_2\pi\gamma}$ in footnote (**) on p. 460. Equation (9) corresponds to

$$\Gamma_{A_2\pi\gamma} = \frac{2}{5}\,\alpha\,\frac{p^5_{\text{c.m.}}}{m^4_{A_2}}\,g^2_{A_2\pi\gamma} \approx 0.113\,g^2_{A_2\pi\gamma}\ \text{MeV} = 1.2\ \text{MeV} ,$$

agreeing with « old » estimates using vector-meson dominance ([18]). This value of the $A_2\pi\gamma$ width is much larger than estimates obtained from pion Compton-scattering sum rules ([18]), combined with the Cabibbo-Radicati ([19]) sum rule; in this way, HARARI ([18]) estimated $\Gamma_{A_2\to\pi\gamma} = (0.3 \pm 0.3)$ MeV; SINGH ([19]) estimated $\Gamma_{A_2\to\pi\gamma} = 0.4$ MeV and SARKER ([20]) estimated $\Gamma_{A_2\to\pi\gamma} = (0.3 \pm 0.1)$ MeV. These values of the widths would of course increase the estimates of the coupling constants $G^{(i)2}_{A_2\mathcal{N}\mathcal{N}}/4\pi$ in eq. (10) by a factor ≈ 3.

([16]) M. ALSTON-GARNJOST, A. BARBARO-GALTIERI, W. F. BUHL, S. E. DERENZO, L. D. EPPERSON, S. M. FLATTÉ, J. H. FRIEDMAN, G. R. LYNCH, R. L. OTT, S. D. PROTOPOPESCU, M. S. RABIN and F. T. SOLMITZ. *Phys. Lett.*, **33** B, 607 (1970).

([17]) G. GRAYER, B. HYAMS, C. J. JONES, P. SCHELIN, W. BLUM, H. DIETL, W. KOCH, H. LIPPMANN, E. LORENZ, G. LÜTJENS, W. MÄNNER, J. MEISSBURGER, U. STIERLIN and P. WEILHAMMER: *Phys. Lett.*, **34** B, 333 (1970).

([18]) H. PAGELS: *Phys. Rev. Lett.*, **18**, 316 (1967); H. HARARI: *Phys. Rev. Lett.*, **18**, 319 (1967).

([19]) N. CABIBBO and L. A. RADICATI: *Phys. Lett.*, **19**, 697 (1966); V. SINGH: *Phys. Rev. Lett.*, **19**, 730 (1967).

([20]) A. Q. SARKER: *Phys. Rev. Lett.*, **25**, 1527 (1970).

([21]) J. BAACKE and H. KLEINERT: Berlin preprint (1971).