On the Helicity-Flip Property of the $A_2 N\bar{N}$ Coupling.

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(ricevuto 18 Luglio 1971)

Evidence is piling up in favour of the hypothesis that the more important meson Regge trajectories $P$, $f$, $\omega$ and $\rho$, $A_2$ couple to nucleons predominantly to $s$-channel helicity-nonflip (1-2) and helicity-flip amplitudes (3), respectively. The interesting point about this hypothesis is that the on-mass-shell coupling constants of the particles lying on these trajectories appear to possess the same properties, indicating a surprisingly smooth behaviour of the flip to nonflip ratio when continuing $t$ from the forward direction to the meson mass.

Consider $\pi N$ scattering. There the assumption of the absence of the $P$ and $f$ trajectory in the helicity-flip amplitude has the consequence that both invariant amplitudes $A^{(+)}$ and $B^{(+)}$ obey unsubtracted dispersion relations in the forward direction. Then using the additional information on the values of $A^{(+)}$ and $B^{(+)}$ at threshold supplied by an unsubtracted backward dispersion relation, Engel and Höhler (4) have derived the estimates for the coupling of $f$ to nucleons (5)

\[ \frac{G_{1NN}^{(+)}(t)}{4\pi} = 53 \pm 10 \ , \quad \frac{G_{1\bar{N}N}^{(+)}(t)}{4\pi} = 3 \pm 7 , \]

compatible with pure nonflip amplitude (6). It is interesting to note that this prop-

(8) Note also that H. Schäkel (Karlsruhe Thesis, 1970) using fixed-$u$ dispersion relations and R. Strauss (Karlsruhe Thesis, 1970) using fixed-angle dispersion relations have obtained similar values for these coupling constants; however, they quote larger errors.
(9) For $f$ as well as $A_2$ couplings to protons we use the Lagrangian

\[ \mathcal{L} = \frac{G_{1NN}^{(+)}(t)}{m} \gamma_\mu \left( \overline{\psi}_\mu \gamma_\nu \psi_\nu + \overline{\psi}_\mu \gamma_5 \gamma_\nu \psi_\nu \right) + \frac{G_{1\bar{N}N}^{(+)}(t)}{m^*} \overline{\psi}_\mu \gamma_\nu \psi_\nu \]

(\text{where } m = m_N \text{ is used here and in the rest of the paper). Then the helicity flip to nonflip ratio in } \pi N \text{ scattering is given for } v \rightarrow \infty \text{ close to the } f \text{ pole by}

\[ f_{++}/f_{+-} \approx \sqrt{2} \frac{G_{1NN}^{(1)}(t)}{G_{1\bar{N}N}^{(1)}(t)} \]
ergy allows the second of the gravitational form factors of the nucleon to be dominated by an $f$-meson via an untracted dispersion relation (').

Similarly, $\omega$ does not flip the nucleon spin on its mass shell since $x^\omega = -0.06$ and the flip to nonflip ratio is, for large $\nu$ and close to the $\omega$ pole,

$$I_{+} / I_{++} \approx \frac{\sqrt{-i} \{ A + mB \}}{2m} \frac{A + B}{A + \nu B} \approx -\frac{\sqrt{-i}}{2m} \frac{2x^\nu}{2x^\nu}.$$ 

For $\rho$ the same argument shows that on shell $\rho$ mostly flips the nucleon spin (since $2x^\rho = 3.7$).

No such direct on-shell argument has, until now, been presented for the coupling of $A_2$ to nucleons. We shall show in this note that, indeed, the $A_2$-meson couples on shell predominantly to the helicity-flip amplitude.

Consider the standard CGLN (') basis of photoproduction. The amplitudes $(1/s - u)A^{(1)}$ and $(1/s - u)D^{(1)}$ are even functions in $s-u$. They behave for large energy in the forward direction according to $s^\Delta A^{(1)} = 0$ and therefore certainly obey untracted dispersion relations (since $s_{A_2}(0) \approx 0.5$). At large energy in the backward direction they are dominated by

$$s^\Delta A^{(1)} \approx (-i)^{s_{A_2}(0) - 3}.$$ 

Since $s_{A_2}(0) \approx 0.2$, we can write also here an untracted dispersion relation. Equating both relations at threshold we obtain two sum rules. The nucleon Born term does not contribute to either one of them (**). Due to the strong fall-off for large energies these sum rules should saturate extremely quickly and we can be content with inserting only the lowest resonances which can contribute (**): A in the $s$-channel and $A_2$ in the $t$-channel. In this way we find from $(1/s - u)A^{(1)}$ and $(1/s - u)D^{(1)}$, respectively ($^{(*)}$) ($^{(*)}$),

$$g = \frac{m_e}{m_A} \left[ m^2 (C_4 + C_5) + mC_4 \right] = \frac{3}{2} \frac{m^2_{A_2}}{m^2_{A}} G_{A_2}^{(1)} \frac{m^2_A}{G_{A_2}}$$

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(*) If $\langle \rho \rho | p | p \rangle$ determines $F_i(0) = 1$ and $F_i(0) = 0$, respectively. $t$ dominance of $F_i$ gives $F_i(0) \propto G_{A_2}^{(1)}$, hence $G_{A_2}^{(1)} = 0$. Note, however, that a similar $t$-dominance assumption for $F_i$ gives $G_{A_2}^{(1)}$ about $1$ in magnitude of that given in (1), see ref. (').


(**) No does the Roper (or other $J^P = 1^+$ baryons) contribute.

(‘‘) The error arising by neglecting higher meson resonances is hard to estimate: the error introduced by leaving out higher baryon resonances can be shown to be small. A more complete treatment will be given in a future publication.

(‘‘) Here $g$ is defined by

$$\Gamma_{A_2} = \frac{g^2}{m_\pi} \gamma_{A_2} \gamma_{A_2}^\mu \gamma_5 \gamma_5,$$

such that from

$$\Gamma_{A_2} = \frac{g^2}{4 \pi} \frac{m^2}{M^4} \frac{F^4}{m^2} \approx 0.37.$$ 

Taking finite width into account, one estimates $g^2 / 4 \pi \approx 0.26$ (‘). The Gourdin-Sain (‘‘) coupling con-
ON THE HELICITY-FLIP PROPERTY OF THE $A_2N\pi$ COUPLING

Therefore, we find for the ratio of flip to nonflip couplings

$$G_A^{(1+2)}(G_A^{(2)}) = \frac{1}{4} \frac{m_\Delta}{m^2} \frac{x}{1 + x} \approx 0.44 \frac{x}{1 + x}$$

with

$$x = m(C_4 + C_6)/C_2.$$ 

The ratio $x$ can be taken from experiment by relating it to the ratio of electric-quadrupole and magnetic-dipole amplitudes $E_{1+}/M_{1+}$ of CGLN at the $\Delta$-resonance. One finds

$$-E_{1+}/M_{1+} = \left( 1 - \frac{m_\Delta}{m} x \right) \left( \frac{3m_\Delta + m}{m_\Delta - m} - \frac{m_\Delta}{m} x \right) \approx 6.5(1 - 1.32x)\%.$$ 

Experimentally, one has

$$-E_{1+}/M_{1+} \approx \begin{cases} 4.6\% & (10), \\ 3.1\% & (11), \\ 5.9\% & (12), \end{cases}$$

and we take $C_6/m \approx 2$ (12), giving $x \approx 0.2$ close to the original value of Gourdin and Salin (10) of $x \approx 0.16$. This corresponds to (10)

$$G_A^{(1+2)}(G_A^{(2)}) \approx 0.07,$$

such that $A_2$ indeed couples more strongly to nucleon s-channel helicity-flip than to nonflip amplitudes. We would here also like to remark that by applying similar

\[\text{References}\]

\[\text{Footnotes}\]

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techniques to the amplitudes of Compton scattering on nucleons, one arrives at the same conclusion \([15]\). For completeness we use eq. (2) to estimate

\[
G_{A^2 N N}^{(2)} g_{A^2 N N} \approx 20 .
\]

If we take an estimate on the \(A_2 \pi \gamma\) coupling coming from vector-meson dominance and \(A_2 \rightarrow \pi \rho\) decay \([4]\)

\[
g_{A_2 \pi \gamma}^2 \approx 10.6 ,
\]

we find

\[
\frac{G_{A^2 N N}^{(1)} g_{A^2 N N}}{4\pi} \approx \frac{G_{A^2 N N}^{(2)} g_{A^2 N N}}{4\pi} = 3 .
\]

In conclusion we see that the simple technique of subtracting forward and backward dispersion relations from each other at threshold provides a powerful tool \([8,15,21]\) for the determination of on-shell coupling constants of particles exchanged in the \(t\)-channel. We hope that a combination of this method with Regge fits of high-energy scattering will supply us with direct information on the structure of Regge residues when continued from the forward direction to the particle poles.

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We are grateful to G. DASS, M. JACOB, C. MICHAEL, R. ODORICO and V. RUUSKANEN for helpful discussions.

\([4]\) J. BAACKE, T. Y. CHANG and H. KLEINERT: to be published.

\([4]\) We use

\[
g_{\rho}^2/4\pi = 2.5 \quad \text{and} \quad \Gamma_{A_2 \pi \gamma} = \frac{4}{5} \frac{g_{A_2 \pi \gamma}^2}{m_{A_2}} = 85 \text{ MeV} ,
\]

where we have used the total \(A_2\) width \(\approx 100 \text{ MeV}\) and branching ratio to \(\rho\pi \approx 85 \% \) \([16]\) (note that a slightly larger total \(A_2\) width \(\approx 125 \text{ MeV}\) is obtained from ref. \([17]\)). \(g_{A_2 \pi \rho}\) is defined analogously to \(g_{A_2 \gamma\gamma}\) in footnote \((\star\star)\) on p. 460. Equation (9) corresponds to

\[
\Gamma_{A_2 \pi \gamma} = \frac{2}{3} \frac{g_{A_2 \pi \gamma}^2}{m_{A_2}} \approx 0.115 \gamma_{A_2 \pi \gamma} \approx 1.2 \text{ MeV} ,
\]

agreeing with \(\star\) old \(\star\) estimates using vector-meson dominance \([16]\). This value of the \(A_2 \pi \gamma\) width is much larger than estimates obtained from pion Compton-scattering sum rules \([16]\), combined with the Cabibbo-Radicati \([4]\) sum rule: in this way, \(\text{HARAH}\) \([4]\) estimated \(\Gamma_{A_2 \rightarrow \pi \gamma} = (0.3 \pm 0.3) \text{ MeV}\); \(\text{SINGH}\) \([4]\) estimated \(\Gamma_{A_2 \rightarrow \pi \gamma} = 0.4 \text{ MeV}\) and \(\text{SARKER}\) \([4]\) estimated \(\Gamma_{A_2 \rightarrow \pi \gamma} = (0.5 \pm 0.1) \text{ MeV}\). These values of the widths would of course increase the estimates of the coupling constants \(G_{A^2 N N}^{(1)}/4\pi\) in eq. (10) by a factor \(\approx 3\).


