Abstract

We speculate that the universe is filled with stars composed of electromagnetic and dilaton fields which are the sources of the powerful gamma-ray bursts impinging upon us from all directions of the universe. We calculate soliton-like solutions of these fields and show that their energy can be converted into a relativistic plasma in an explosive way. As in classical detonation theory the conversion proceeds by a relativistic self-similar solution for a spherical detonation wave which extracts the energy from the scalar field via a plasma in the wave front in the atmosphere of the star.

1 Introduction

Many modern theories of the universe assume the existence of various types of scalar fields. Such fields could explain the recently discovered acceleration of the expansion of the universe (see e.g. Ref. [1]), or the formation of clustered systems leading to
gravitational walls for galaxies and galaxy clusters. If such fields really exist, the universe could contain many compact star-like configurations of large total mass, called scalar stars.

Among the many possible scalar fields, the dilaton field has a special theoretical appeal. It couples in a unique minimal way to electromagnetism to make the Maxwell action dimensionless. This coupling leads to star-like objects which are composed of scalar and electromagnetic fields. That such objects can exist was pointed out by many authors [2, 3, 4]. We argue that such electro-dilaton stars may be responsible for the strong gamma-ray bursts observed in the universe.

At present there exists no simple conventional explanation for the origin of these events reaching us isotropically from all directions of the universe. For a comprehensive discussion of the subject, in particular for the failures of most theories, see the article by Ruffini [5]. At the same time, there exist models for the early universe where an initial state of large volume of relativistic plasma quickly expands as a spherical wave, like a critical bubble in an overheated liquid, causing a decay of a false vacuum, and creating the universe from such a bubble [6]. The electro-dilaton stars can supply such an explanation. We assume that the dark matter in the universe contains a multitude of such objects, whose total mass exceeds by far the total mass of luminous matter and is responsible for the large-scale structure of the universe. The luminous matter concentrates in the gravitational potential wells of the scalar fields.

Let us imagine that collision of relativistic particles produce a fireball of critical size. Such bubbles have been investigated as possible triggers of phase transitions in the early universe [7]. In the context of gamma-ray bursts, similar assumptions have been made in Ref. [8]. The critical bubble forms the seed for transferring effectively scalar fields into pairs of elementary particles and their antiparticles. The transfer may be initiated by fast oscillation of a field on the outer boundary of the fireball. The process causes a relativistic detonation. In a conventional detonation, chemical energy is converted into kinetic energy. In a relativistic detonation of an electro-dilaton star, it is the energy of the electric and scalar fields which is rapidly converted in particle-antiparticle pairs. The resulting fireball expands with
relativistic velocity and will therefore not depend on the weak gravitational fields of a Newtonian configuration. We may thus study the process within special relativity.

## 2 Relativistic Detonation

We begin by deducing the self-similar solutions for a spherical relativistic detonation which goes over to the well-known Zeldovich solution in the limit of small velocities [8]. The set of equations of relativistic hydrodynamics is conveniently described in a spherical coordinate system \( r, \Theta, \phi \). If \( v \) denotes the radial velocity of the plasma in three-dimensions [8, 9] and \( \varepsilon \) is energy density, \( p \) - pressure, the equation of motion reads

\[
\frac{1}{\gamma^2} \left( \frac{\partial v}{\partial \tau} + v \frac{\partial v}{\partial r} \right) + \frac{1}{W} \left( \frac{\partial p}{\partial r} + v \frac{\partial p}{\partial \tau} \right) = 0 \tag{1}
\]

while energy conservation requires that

\[
\frac{1}{W} \left[ \frac{\partial \varepsilon}{\partial \tau} + v \frac{\partial \varepsilon}{\partial r} \right] + \frac{1}{\gamma^2} \left( \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial \tau} \right) + \frac{2v}{r} = 0, \tag{2}
\]

where \( \gamma^2 \equiv 1 - v^2 \), \( W = \varepsilon + p \), and \( c = 1 \). As in the nonrelativistic case, the motion of the plasma behind the detonation front is considered as isentropic, such that (1) and (2) are the only relevant equations.

The pairs of relativistic particles and antiparticles created behind the wave front generate a high-temperature plasma with the equation of state:

\[
p = c_s^2 \varepsilon; \quad c_s^2 = \left( \frac{\partial p}{\partial \varepsilon} \right)_S = \frac{1}{3}, \tag{3}
\]

where \( c_s \) is the sound velocity. As in an ordinary spherical detonation problem, we search for a solution depending on the self-similar variable

\[
\xi = r/\tau, \tag{4}
\]

in which the differential equations (1) and (2) reduce to ordinary differential equations, which can be combined to the equation for \( v \):

\[
\frac{dv}{d\xi} \left[ \frac{1}{c_s^2} \left( \frac{v - \xi}{1 - v\xi} \right)^2 - 1 \right] = \frac{2v}{\xi^2} \frac{\gamma^2}{1 - v\xi}. \tag{5}
\]

In the nonrelativistic limit where \( v(\xi), \xi \ll 1 \), this equation reduces to a textbook equation [8]. The qualitative analysis of the relativistic equation (3) is similar to
the one in the textbook: The solutions for \( v \) and \( \varepsilon \) have a diverging derivative at the wave front. Such singularity of a derivative is defined by the following: the expression in parentheses is velocity of current of plasma in relation to the wave front. According to the theory of detonation, this velocity is equal to sound velocity \( c_s \). Therefore, at approaching to the wave front the expression in brackets tends to zero. The coordinate of the wave front \( \xi = v_d \) (where \( v_d \) is a velocity of detonation wave in central frame) and the velocity of gas behind the wave in laboratory frame \( v_g \) is uniquely determined from conservation laws at the wave front.

![Image of graph showing \( \varepsilon \) and \( v \) dependence on \( \xi \)](image)

**Figure 1**: Dependence of the energy density \( \varepsilon \) and velocity \( v \) of plasma on the self-similar variable \( \xi \) behind the front of detonation wave.

The discussion of possible mechanisms of “recycling” of the field behind the wave front into relativistic plasma is considered below. Let us specify here the following estimates of the value of \( v_d \) and energy density behind the detonation wave. Consider a scalar field in the simplest form, with an energy-momentum tensor

\[
T^k_i = \varphi,^i \varphi^k - \delta^k_i \left[ \frac{1}{2} \varphi,^\mu \varphi^{\mu} - V(\varphi) \right], \quad V(\varphi) = m^2 \varphi^2 / 2.
\]

In the “scalaron” regime where it undergoes fast oscillations of frequency \( m \), i.e. where \( \varphi(r, t) = a(r) \sin mt \), the spatial gradients of the field can be ignored and the energy density in the laboratory frame (7) is

\[
\varepsilon_f \approx m^2 a^2 / 2.
\]
The expression for \( v_d \) and the energy density of the plasma behind the wave front are determined from the conservation laws \( T^0_0(\text{field}) = T^0_0(\text{plasma}) \) and \( T^1_1(\text{field}) = T^1_1(\text{plasma}) \) for an observer moving with the wave front. Recall that the plasma is emitted from the wave front with the velocity \( c_s \). Hence

\[
\begin{align*}
    v_g &= c_s; \\
    v_d &= \frac{2c_s}{1 + c_s^2}; \\
    \varepsilon_p &= \frac{2}{1 - c_s^2} \varepsilon_f.
\end{align*}
\]

(8)

Since a relativistic plasma has \( c_s = 1/\sqrt{3} \), we find \( v_d = \sqrt{3}/2 \) and \( \varepsilon_p = 3 \varepsilon_f \). The self-similar solutions for this case are presented in Fig. 4.

We now turn to the mechanism of transition of the field energy into a relativistic plasma.

### 3 Electro-Dilaton Wave

So far, the description of the conversion of the energy of a scalar field into relativistic plasma at the front of “detonation” wave is purely phenomenological. The physical properties of the front of the detonation wave are completely determined by the conservation laws for a general energy-momentum tensor \( T^k_i \).

One specific mechanism of such a conversion was considered in [12] based on an analogy with a laser. Here we shall consider an alternative possibility where the relativistic plasma and radiation are obtained from the energy of a dilaton field.

The star-like configurations for such field with strong and weak (Newtonian) gravitational field have been considered before [13, 14, 15, 16]. As in Section 2, we shall discuss only the case of a weak gravitational field and treat the problem within special relativity.

The Lagrangian density of a system of dilaton and electromagnetic fields is [14, 15]:

\[
    L = 2\Phi,^i \Phi^i - \zeta (F_{lm} F^{lm}) e^{-2\alpha \Phi}.
\]

(9)

The parameter \( \zeta \) can have the values \( \pm 1 \) as will be explained below. The normalization of the fields is the same as in [14, 15]. The unique interaction of the scalar field \( \Phi \) with an electromagnetic field required by scale invariance allows for a nontrivial

\footnote{Our choice is \( \zeta = -1 \)}
combined electro-dilaton configuration. In the front of the detonation wave, the electromagnetic field reduces the dilaton field strength by dissipation. Depending on the intensity of the electric field, the dissipation may be due to the creation of pairs of particles and antiparticles and to a heating of the plasma. The combined process—generation of an electromagnetic field and its subsequent dissipation—supplies the energy to the front of the detonation wave. Let us study the process in a simple plane-wave configuration. The equations for $\Phi$ and $F_{ik}$ following from Lagrangian (9) are

$$
\left[ e^{-2\alpha}\Phi F^{ik} \right]_{;k} = 0,
$$

$$
\Phi_{;i}^i = -\frac{\alpha \zeta}{2} e^{-2\alpha}\Phi (F_{lm} F^{lm}).
$$

The system is supplemented by the missing Maxwell equations (electromagnetic versions of the Bianchi identities):

$$
e^{iklm} F_{kl,m} = 0.
$$

The total energy-momentum tensor associated with the Lagrange density (9) is

$$
T^k_i = 2\Phi_{;i} \Phi_i^k - 2\zeta e^{-2\alpha}\Phi F_{il} F^{lk} - \frac{1}{2} \delta^k_i [2(\Phi_{;j} \Phi^j) - \zeta e^{-2\alpha}\Phi (F_{lm} F^{lm})].
$$

4 Longitudinal Electro-Dilaton Wave

We now show that there exist plane electro-dilaton waves travelling along the $x$-axis in which the electric field has a component $F^{10} = E_x$ with all other components vanishing. Whereas Eq. (12) is fulfilled trivially, Eq. (11) yields

$$
(e^{-2\alpha}\Phi E_x)_{,\tau} = 0; \quad (e^{-2\alpha}\Phi E_x)_{,x} = 0.
$$

Thus we find a constant of motion

$$
e^{-2\alpha}\Phi E_x = E_0 = \text{const}.
$$

The quadratic field combination

$$
I = F_{lm} F^{lm} = -2E_x^2
$$
has the same negative value in all systems of special relativity. For this reason we follow Ref. [14] in choosing the parameter $\zeta = -1$ in (9).

The other field equation (11) becomes

$$\frac{\partial^2 \Phi}{\partial \tau^2} - \frac{\partial^2 \Phi}{\partial x^2} = \alpha E_0^2 e^{2\alpha \Phi}. \quad (17)$$

or, introducing new variables $2\alpha E_0 x \rightarrow x$, $2\alpha E_0 \tau \rightarrow \tau$ and function $\psi = 2\alpha \Phi$, we have

$$\frac{\partial \psi}{\partial \tau^2} - \frac{\partial \psi}{\partial x^2} = e^\psi/2. \quad (18)$$

This is one of form of Liouville equation [17]. Its possesses a steady-state wave solution which, after the redefinition of the variables

$$\xi = x - u\tau, \quad \eta = \xi/\sqrt{1 - u^2}, \quad (19)$$

takes the form

$$\frac{d^2 \psi}{d\eta^2} = -\frac{1}{2} e^\psi. \quad (20)$$

The first integral of this equation leads to the differential equation

$$\left( \frac{d\psi}{d\eta} \right)^2 = 1 - e^\psi$$

whose solution

$$\psi = -2 \ln \cosh(\eta/2), \quad E_x = E_0/\cosh^2(\eta/2) \quad (22)$$

describes a soliton. Though the potential of dilaton field $\psi$ diverges at $|\eta| \rightarrow \infty$, where $\Phi \approx -\eta$, the derivatives $\Phi_i$ remain finite, thus ensuring a finite energy density of the field for all $\eta$. Note that in contrast to a charged plane in electrostatics, the electric field of this solution has a zero flux $E_x$ at infinity. Asymptotically, no electric field is detectable.

There exists a general class of solutions of the Liouville equation (18) containing two arbitrary functions $f_1(x-\tau)$, $f_2(x+\tau)$. Setting $\psi^* \equiv 2\alpha \Phi + \ln \alpha E_0^2$, the solution of eq. (17) is

$$\psi^* = \ln \left[ \frac{16f_1(x-\tau)f_2(x+\tau)}{\cosh^2[f_1(x-\tau)+f_2(x+\tau)]} \right] \quad (23)$$
The primes denote derivatives. Especially simple solutions are obtained for the linear functions

\[
\begin{align*}
  f_1(x - \tau) &= \gamma(x - \tau), \quad \gamma \equiv \frac{1 + u}{4 \sqrt{1 - u^2}}, \\
  f_2(x + \tau) &= \beta(x + \tau), \quad \beta \equiv \frac{1 - u}{4 \sqrt{1 - u^2}}.
\end{align*}
\]  

(24)

(25)

where \(u\) is velocity of soliton in Eq.(19).

The following solutions are of special interest:

a) Localized solution for \(E_x\) with a fixed asymptotic energy density \(\varepsilon(|x| \to \infty) = \varepsilon_0\), where \(E_x\) goes over into the previous soliton solution (22). In these solutions, the functions \(f_1\) and \(f_2\) in (23) and their derivatives are regular. Singular solutions of Liouville equation are also known [18]. They may lead to a local catastrophic growth of electric field, and require special attention.

b) The main purpose of this section is to show that at different values of the energy of electro-dilaton wave before and behind the electric layer the electromagnetic energy may increase in time. A similar distributions of the energy may be created by gravitational configuration.

Let us specify the arbitrary functions in Liouville solution as

\[
\begin{align*}
  f_1 &= \gamma(x - \tau) + F, \quad F = -\ln \tanh \mu(x - \tau), \\
  f_2 &= \beta(x + \tau),
\end{align*}
\]  

(26)

(27)

where \(\gamma, \beta\) are determined by the initial velocity of the wave from (24) and (25), and \(\mu\) is an arbitrary constant. Now the profile of the electric field looks like

\[
\frac{E_x}{E_0} = \frac{16\beta[\gamma + \mu \tanh(\mu - x)]}{\cosh^2[(\gamma + \beta) x - (\gamma - \beta) \tau + \ln \cosh \mu(\tau - x)]}
\]  

(28)

The profile of such a wave is shown on Fig. 2. Let us assume that the numerator of Eq. (28) slowly changes in comparison with the denominator. In that case, at given \(\tau\) the maximum of electric soliton concentrates about a zero point of argument of \(\cosh\). As we see from Eq. (24), this region moves with velocity
Figure 2: Growth of electrical soliton in wave with different asymptotic values of energy both before and behind wave. Solutions (28) with \( u = 1/\sqrt{3} \) and \( \mu = \gamma \) are shown for increasing times \( \tau \). The energy density in units of \( E_0^2 \) is equal to 7.4 behind and 0.2 in front of the wave front. For large \( \tau \) the solution gives stationary soliton with smaller value of energy.

\[ u < 1, \] so that the argument of \( \tanh \) is always positive in this region. The numerator and thus the electric field increase with time. This increase comes to an end as \( \tanh[\mu(\tau - x)] \) reaches its asymptotical value 1. This is illustrated in Fig. 2. If instead of \( \tanh[\mu(\tau - x)] \) in (26) a function is chosen which grows without bounds, then also the electric field will keep growing.

The growth of the energy of the electric field will be consumed by dissipation. Its influence on the electro-dilaton wave will be considered in Section 6.

5 Wave of Transverse-Magnetic Type

Let us use the above results to study a wave with a transverse magnetic wave. In this case the Maxwell equations (12) are not fulfilled identically but yield a relation between transverse components \( F_{ik} \). Let us select for consideration the following nonzero components \( F^{20} = E_y, F^{21} = H_z \). Then the complete set of the equations is

\[
\frac{\partial}{\partial \tau}(e^{-2\alpha \Phi} E_y) + \frac{\partial}{\partial x}(e^{-2\alpha \Phi} H_z) = 0,
\] (29)
\[
\frac{\partial E_y}{\partial x} + \frac{\partial H_z}{\partial \tau} = 0. \tag{30}
\]

A travelling wave has the field components

\[
E_y = uH_z(\xi); \quad H_z = H_0 e^{2\alpha \Phi}; \quad I = 2H_0^2(1 - u^2)e^{4\alpha \Phi}. \tag{31}
\]

Since \( u < 1 \) and \( I > 0 \), the field is transverse magnetic. It dictates in (10) the choice of the sign \( \zeta = 1 \). Further, using similar variables as in (19)

\[
\xi = x - u\tau, \quad \psi = 2\alpha \Phi, \quad \eta = 2\alpha \xi |H_0|, \tag{32}
\]

we obtain the solutions for dilaton and electromagnetic fields similar to (22)

\[
\psi = -2 \ln \cosh(\eta/2); \quad E_y = uH_z = uH_0/\cosh^2(\eta/2). \tag{33}
\]

The principal difference with respect to (22) is the absence of a relativistic factor \( \sqrt{1 - u^2} \) in the argument \( \eta \).

The transverse magnetic wave is focused in a band of width \( \Delta x \sim 1/\alpha |H_0| \). Outside of this, the energy density lies mainly in the gradient part of the dilaton field. From Eq. (33) it follows that energy flux density is independent of \( \xi \):

\[
T_0^0 = H_0^2(1 + u^2); \quad T_1^0 = 2H_0^2u. \tag{34}
\]

This can be interpreted as follows: the conversion of the energy of the dilaton field from the gradient to the potential part implies a conversion of the gradient energy of the dilaton field to the energy of electromagnetic field. Certainly, this conversion is completely reversible.

6 Dissipation

The above electro-dilaton soliton appears at the front of the detonation wave. If we use the plasma behind the front as a frame of reference, then this soliton will be move with a velocity \( u = c_s = 1/\sqrt{3} \) in the positive \( x \)-direction. Such a movement generates heat in the plasma and creates of pairs of particles and antiparticles in high concentration. This reduces the \( E \) and \( H \) fields. In the simplest description,
the decay wave can be taken into account in a travelling-wave approximation by replacing Eqs. (29) by

\[(1 - u^2) \frac{d}{d\xi}(e^{-2\alpha \Phi} H_z) = fe^{-2\alpha \Phi} H_z; \quad (e^{-2\alpha \Phi} H_z) = H_0 \exp[f \xi/(1 - u^2)],\] (35)

where \(f\) is the friction constant with the dimension of a reverse length. While moving though the plasma with \(\xi < 0\), the electromagnetic wave decays. The equation for the dilaton field with \(\zeta = 1\) becomes

\[\frac{d^2 \Phi}{d\xi^2} = -\frac{\alpha}{2} H_z^2 e^{-2\alpha \Phi}.\] (36)

Substitution here (35) and using the redefinitions (32), we obtain

\[H_z = H_0 h(\eta)e^{\beta \eta}, \quad \text{with} \quad \beta = f/2\alpha H_0,\] (37)

\[\frac{d^2 \psi}{d\eta^2} = -\frac{1}{2} h^2(\eta)e^{-\psi} = -\frac{1}{2} e^{\psi+2\beta \eta}.\] (38)

After a change of the variable \(\psi + 2\beta \eta \rightarrow \psi\), this equation reduces to the Liouville form (21). Then subject to dissipation we have the following solution:

\[\psi_d = -2[\beta \eta + \ln \cosh(\eta/2)]; \quad H_d(\eta) = H_0 e^{-\beta \eta} / \cosh^2(\eta/2).\] (39)

Here, the subscript \(d\) denotes a solution with dissipation. Thus electro-dilaton solitons with dissipation become asymmetric with a steeper front part. The creation of particles and heat of plasma happens at decreasing of the energy density of dilaton field. For \(|\eta| \rightarrow \infty\) the energy of the system is concentrated only in the gradient part of dilaton field (see (13), (19))

\[T_0^0 = (1 + u^2) H_0^2 (\psi_{d,\eta})^2.\]

In the limit \(|\eta| \rightarrow \infty\), the equations (39) becomes

\[T_0^0(\infty) = (1 + u^2) H_0^2 (1 + 2\beta)^2; \quad T_0^0(-\infty) = (1 + u^2) H_0^2 (1 - 2\beta)^2.\]

It is clear from here that the energy density of dilaton field before the wave is more than the energy density behind the wave (\(\beta > 0\)). The limiting value for \(\beta\) in this example is \(\beta \rightarrow 0.5\). This implies that the whole energy of the dilaton field transforms to the heat energy of plasma.
7 Induced Current

Consider now the dissipation of energy by the induced fields at the wave front. For this purpose we substitute a current on the right hand side of Eq. (29)

\[ j_y = \sigma E_y = \sigma u H_z(\xi), \]

where \( \sigma \) is specific conductivity of the medium. Then we find the following system of dimensionless equations:

\[ 2 \frac{d^2 \psi}{d\eta^2} = -h^2 e^{-\psi}, \quad \frac{d}{d\eta}(he^{-\psi}) = -\beta h, \]

(40)

with the parameter \( \beta \sim \sigma u \). At \( \beta = 0 \), this reduces to the previous electro-dilaton soliton (33).

To analyze Eqs. (40) it is convenient to introduce the new variable

\[ z = he^{-\psi}, \]

(41)

which is unity in the absence of dissipation. The first integral of the differential equations (40) leads to the solution:

\[ z^2 = 4\beta \left( \frac{\partial \psi}{\partial \eta} \right) + C; \quad C = 1 - 4\beta. \]

The integration constant is selected so that in the limit \( \eta \to -\infty \) the solution tends to the dissipation-free soliton with \( (d\psi/d\eta)_- \to 1 \) for \( z^2 \to 1 \). In the opposite limit \( \eta \to \infty \), the variable \( z^2 \) tends to zero and

\[ \left( \frac{d\psi}{d\eta} \right)_+ \to \frac{4\beta - 1}{4\beta}. \]

(42)

Thus for \( z^2 \to 0 \), the solution represents a kink moving in the positive \( x \)-direction. By virtue of \( \beta > 0 \), the asymptotic value of the dilaton energy density before the front is less than behind it. Physically this means that the pressure of the dilaton field creates a plasma at the wave front pushing it ahead. It follows from Eq. (42) that the limiting value of \( \beta \) is now \( \beta_c = 1/4 \) implying that the electro-dilaton energy goes completely over into an energy flux of moving plasma. The solutions are plotted in Fig. 3.
Figure 3: Dependence of the energy density $\varepsilon$ and velocity $v$ of plasma on the self-similar variable $\xi$ behind the front of detonation wave for various parameters $\beta$.

8 Conclusion

From our discussion it appears perfectly plausible that dark matter consists of electro-dilaton stars and is not dark at all, but has been showing its presence quite dramatically all along via the powerful gamma-ray bursts observed since.

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References


For experimental data see


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See also

According to these works, gamma-ray bursts are correlated with supernovas. The discussion in the present paper attempts to give an alternative interpretation.


