Deconfinement transition in three-dimensional compact $U(1)$ gauge theories coupled to matter fields

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It is shown that permanent confinement in three-dimensional compact U(1) gauge theory can be destroyed by matter fields in a deconfinement transition. This is a consequence of a non-trivial infrared fixed point caused by matter, and an anomalous scaling dimension of the gauge field. This leads to a logarithmic interaction between the defects of the gauge-fields, which form a gas of *magnetic* monopoles. In the presence of logarithmic interactions, the original *electric* charges are *unconfined*. The confined phase, which is permanent in the absence of matter fields, is reached at a critical electric charge, where the interaction between magnetic charges is screened by a pair unbinding transition in a Kosterlitz-Thouless type of phase-transition.

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In a seminal paper, Polyakov [1] has shown that QED in $d = 3$ dimensions with a compact $U(1)$ gauge field exhibits confinement of electric charges for all values of the coupling constant. The origin of this behavior lies in the fact that the defects of the gauge field defined by the boundaries of surfaces where the gauge field A_μ jumps by 2π form a gas of magnetic monopoles, whose initially long-range interaction is reduced to a short range interaction by Debye screening. This screening gives the initially unobservable jumping surfaces an energy leading to an area law for the Wilson integral and thus to permanent confinement between electric charges [2].

An important question is whether this behavior is changed by matter fields. The answer is particularly relevant for present-day condensed-matter physics, where the effective actions assumed to govern strongly correlated electrons contain a compact $U(1)$ gauge field coupled to matter [3–5]. This makes the confinement properties of three-dimensional euclidean gauge-theories relevant for the quantum properties of strongly correlated electrons at zero temperature in *two* spatial dimensions. The existence of a confinement-deconfinement (CD) transition in gauge-theories with matter has been suggested to offer an explanation for a spin-charge separation transition of slave particles in the electron system [3–8].

It has been argued that the presence of matter fields should *not* destroy the permanent confinement in compact $U(1)$ gauge theories when the matter field carries a fundamental charge [4, 9], but there is no universal agreement on this point [10, 11]. In this Letter we shall argue that the coupling to such matter fields induces an anomalous scaling dimension to the gauge field, which indeed may give rise to a CD transition in three dimensions.

We discuss first the case of bosonic matter in a Ginzburg-Landau (GL) model of superconductivity (denoted Higgs model in particle physics). We show how a CD transition arises at a certain Ginzburg parameter κ, which is the ratio between magnetic penetration depth and coherence length.

In the case of fermion matter, we consider QED with N four-component Dirac fermions. Such a system is believed to describe the low-energy behavior of a quantum Heisenberg antiferromagnet (QHA) around the mean-field flux phase [5, 12, 13]. For this model the situation is less clear due to spontaneous chiral symmetry breaking. In principle electric charges are permanently confined below a critical value of N.

For a noncompact gauge field, the GL Lagrangian reads

$$\mathcal{L}_b = \frac{1}{4e_0^2}F_{\mu\nu}^2 + |(\partial_\mu + iA_\mu)\phi|^2 + V(|\phi|^2), \qquad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\nu$ and $V(|\phi|^2) = -m_0^2|\phi|^2 + u_0|\phi|^4/2$. For the sake of the discussion to follow we assume the field ϕ to have $N/2$ complex components such that the theory is O(N)-symmetric. The superconductor has $N = 2$. The critical behavior of this model has been extensively discussed in the literature. Let us summarize the most important properties for the present discussion. Traditional RG calculations such as ϵ-expansion fail to show a non-trivial fixed point for $N = 2$ [14]. This result seems to be an artifact of the ϵ-expansion, since a non-trivial fixed point has been demonstrated to exist [15–18, 20]. The Ginzburg parameter κ where a fixed point appears first has been located by a duality transformation in Ref. [18] and confirmed in recent large-scale Monte Carlo simulations [19]. In the GL model itself, a non-trivial infrared stable fixed point has been found recently by working in $d = 3$ dimensions in the *ordered* phase [20].

We introduce the dimensionless renormalized couplings $\alpha = e^2\mu^{d-4}$ and $g = u\mu^{d-4}$, where μ is the running mass scale. Gauge invariance implies that $e^2 = Z_A e_0^2$ [21], where Z_A is the gauge field wave function renormalization. Thus, we obtain the β-function for α:

$$\beta_\alpha(\alpha, g) = [\gamma_A(\alpha, g) + d - 4]\alpha, \tag{2}$$

where we have defined the RG function $\gamma_A \equiv \mu\partial \ln Z_A/\partial\mu$. If a non-trivial infrared stable fixed point exists, it must satisfy the equations $\gamma_A(\alpha_*, g_*) = 4 - d$ and $\beta_g(\alpha_*, g_*) = 0$, where β_g is the β-function for the g-coupling. The anomalous dimension of the gauge field is such that the critical correlation function $\langle A_\mu(p)A_\nu(-p)\rangle = D(p)(\delta_{\mu\nu} - p_\mu p_\nu/p^2)$ with $D(p) \sim 1/p^{2-\eta_A}$ at large distances. The existence of the infrared stable fixed point implies that η_A is determined exactly: $\eta_A = \gamma_A(\alpha_*, g_*) = 4 - d$, for all dimensions $d \in (2, 4)$ [15]. This exact result was confirmed by Monte Carlo simulations in conjunction with duality arguments [16], providing further evidence for the existence of the non-trivial fixed point. The exact result $\eta_A = 4-d$ is of great importance for the scaling behavior of physical quantities in the superconductor [15, 22]. We now consider how it affects the physics of the CD transition.

The result $\eta_A = 4-d$ implies that $D(p) \sim 1/|p|^{d-2}$, or $D(x) \sim 1/|x|^2$ in real space, for all $d \in (2, 4)$. Hence, the scaling dimension of the gauge field is one for this range of dimensionality. Consequently, the Maxwell term is irrelevant in the RG sense if $d \in (2, 4)$. We emphasize that this corresponds to an *exact* behavior of the theory which is *independent* of perturbation theory. The result amounts to an effective Lagrangian for the gauge field

$$\mathcal{L}_A = \frac{1}{4\alpha_*} F_{\mu\nu} \frac{1}{(-\partial^2)^{\eta_A/2}} F_{\mu\nu}. \tag{3}$$

The anomalous scaling leads to a potential between two test charges at equal times given by

$$V(R) \sim \frac{1}{R^{d-3+\eta_A}} \sim \frac{1}{R}, \tag{4}$$

for all $d \in (2, 4)$. If the anomalous dimension were zero, we would obtain a behavior $V(R) \sim \ln R$ in $d = 3$. The behavior (4) corresponds to an unconfined Coulomb gas in the theory. There exists a similar phase in supersymmetric QCD in four dimensions, where the β-function is known exactly [23]. It has been argued by Seiberg [24] that this theory has a non-trivial infrared stable fixed point for all $N_f \in (3N_c/2, 3N_c)$, where N_f and N_c are the number of flavors and colors, respectively. In this range of N_f, the quarks and gluons are interacting *massless* particles which are not confined.

We next account for the compact nature of the gauge field which gives rise to magnetic monopoles producing confinement. In general, a coupling to matter fields weakens confinement. The competition between the two in principle could lead to a CD transition. The compact nature of the gauge fields is most easily accounted for by the introduction of so-called *plastic gauge fields* [2, 26] $n_{\mu\nu}$ which are superpositions of δ-functions on surfaces over which the angular vector field components A_μ jump by 2π:

$$\mathcal{L}'_A = \frac{1}{4\alpha_*}(F_{\mu\nu} - 2\pi n_{\mu\nu})\frac{1}{\sqrt{-\partial^2}}(F_{\mu\nu} - 2\pi n_{\mu\nu}). \tag{5}$$

Here, we have specialized to $d = 3$ and thus $\eta_A = 1$. From the $n_{\nu\mu}$, we obtain the monopole density:

$$\epsilon_{\mu\nu\lambda}\partial_\mu n_{\nu\lambda} \equiv m(x) = \sum_a q_a \delta^3(x - x_a), \tag{6}$$

where $q_a = \pm$integer are the monopole charges. By a duality transformation, the partition function associated with (5) can be brought to the equivalent form

$$Z = \sum_{\{m(x)\}} \int \mathcal{D}\chi \exp\left\{\int d^3x \left[-\frac{\alpha_*}{2}(\partial_\mu\chi)\sqrt{-\partial^2}(\partial_\mu\chi) - 2\pi i m(x)\chi(x)\right]\right\}, \tag{7}$$

where $\chi(x)$ is the dual electromagnetic potential which is a scalar in three dimensions. It can be integrated out to yield a monopole gas with a partition function

$$Z = \sum_{\text{mon.configs.}} \exp\left[-\frac{2\pi^2}{\alpha_*}\sum_{a,b} q_a q_b V(x_a - x_b)\right], \tag{8}$$

with the potential $V(x) = \int d^3k\, e^{ikx}/(2\pi)^3|k|^3$. This is a logarithmic potential in three dimensions. The monopoles have a large self-energy and thus a low fugacity ζ. We may therefore restrict the sum to $q_a = \pm 1$, where (7) reduces to the following sine-Gordon-like partition function in three dimensions

$$Z \approx \int \mathcal{D}\chi e^{-\frac{1}{2t}\int d^3x\left[\chi(-\partial^2)^{3/2}\chi - z\cos\chi\right]}, \qquad (9)$$

where $z = 8\pi^2\zeta/\alpha_*$, $t = 4\pi^2/\alpha_*$. The above treatment closely parallells that of Polyakov for pure compact QED [1], the novel result being the appearance of the anomalous gradient term due to the presence of matter fields. This anomalous gradient term is in contrast to the usual U(1) gauge theory where it has the standard $\chi(-\partial^2)\chi$ and receives a mass from the $\cos\chi$ term causing permanent confinement of electric charges.

Remarkably, the logarithmic behavior caused by the anomalous gradient term gives rise to a CD phase transition in three dimensions driven by a magnetic monopole-antimonopole unbinding transition, very similar to a Kosterlitz-Thouless vortex-antivortex unbinding transition in two dimensions. Its position is governed by the precise value of α_* which depends on N. By calculating the classical expectation value of the dipole moment of a single pair $\langle r^2\rangle$ it is easy to see that the KT-like pair separation transition occurs at $t = t_c = 12\pi^2$. For $t < t_c$, the field χ is massive and electric charges are confined.

In the ordered phase the system has two length scales whose ratio gives the Ginzburg parameter κ, which in turn can be written as $\kappa = \sqrt{g/2\alpha}$. Thus, we see that the theory can be parametrized in terms of α and κ, instead of α and g. In such a situation it is more convenient to use the Higgs mass as the running scale, i.e. $\mu = m$. At one-loop level, the RG function γ_A in the ordered phase for $d = 3$ is given by [20]

$$\gamma_A = \frac{\sqrt{2}C(\kappa)\alpha}{24\pi(2\kappa^2-1)^3}, \qquad (10)$$

where $C(\kappa) = 4\kappa^6 + 10\kappa^4 - 24\sqrt{2}\kappa^3 + 27\kappa^2 + 4\sqrt{2}\kappa - 1/2$. The non-polynomial form in κ of γ_A comes from the fact that $\kappa = m/m_A$, where m_A is the gauge field mass. Details of the derivation can be found in Ref. [20].

An effective gauge coupling $\bar{\alpha}(\kappa)$ can be defined by the solution of the equation $\gamma_a(\bar{\alpha},\kappa) = 1$, which gives a critical line. This critical line makes sense only for $\kappa > 1/\sqrt{2}$, that is, in the type II regime, or in the interval $0 \leq \kappa < 0.096/\sqrt{2}$ [20] deep in the type I regime. In the interval $0.096/\sqrt{2} < \kappa < 1/\sqrt{2}$ the RG function γ_A is negative, which means that *the theory is asymptotically free in this interval.* This is a remarkable result for an Abelian theory. It cannot be obtained with standard perturbation theory using the ϵ-expansion, but is easily obtained by performing a one-loop calculation in the ordered phase and $d = 3$. Typically, ordinary perturbation theory can access only the deep type I regime. Note that $\bar{\alpha}(\kappa) \to \infty$ as $\kappa \to 0.096/\sqrt{2}$ from the left. This means that near $\kappa = 0.096/\sqrt{2}$ perturbation theory breaks down. Remarkably, perturbation theory can be trusted in the type II regime sufficiently close to $\kappa = 1/\sqrt{2}$ where $\bar{\alpha}$ is small [20]. We stress that all these results are made possible only because there exist two mass scales in the ordered phase.

If we now use the critical coupling t_c of our sine-Gordon-like theory $4\pi^2/\bar{\alpha}(\kappa)$, we find that the Kosterlitz-Thouless-like phase transition takes place at $\kappa_c = 1.17/\sqrt{2}$, *which is precisely the fixed-point value of* κ *obtained from the zero of the* β*-function* $\beta_\kappa \equiv m\partial\kappa/\partial m$ *in the non-compact theory* [20]. This result is consistent with the scenario that there is no phase boundary between the Higgs and the confining phase when the matter field carries the fundamental charge [9, 27].

The coupling to fermionic matter fields will now be considered. The Lagrangian is given by

$$\mathcal{L}_f = \frac{1}{4e_0^2}F_{\mu\nu}^2 + \sum_{i=1}^{N}\bar{\psi}_i\gamma_\mu(\partial_\mu + iA_\mu)\psi_i. \qquad (11)$$

The above Lagrangian corresponds to an effective theory for the QHA, obtained by taking into account the fluctuations around the so-called flux phase [5, 13]. The situation differs considerably from the case of bosonic matter fields because we have only one coupling constant. However, the β-function of the α-coupling has the same form as in Eq. (2), except for a different expression for γ_A, which is here a function of α alone. All the preceeding results for the bosonic theory apply, but there is no critical line and the fixed point α_* is a function of N only. Hence, we expect a critical value $N = N_c$ at which a CD transition takes place. By a one-loop renormalization group calculation, we obtain $\gamma_a = N\alpha/8$, giving therefore the approximate value $\alpha_* = 8/N$. Inserting this fixed point value into the sine-Gordon-like Lagrangian and using the fact that $t_c = 12\pi^2$, we find a critical value $N = N_c \equiv 24$ separating the confined from the deconfined regime. This agrees with the rather crude value obtained in Ref [29] but we can in fact expect a true $N_c << 24$. For instance, Marston computed the effective action for the monopoles approximately, obtaining the much lower value $N_c = 0.9$ [12]. This shows that presently, there is considerable uncertainty in determining N_c. A precise determination of N_c is not the topic of this paper, but we shall now point out to the reader some subtleties concerning the fermionic case.

In principle, the above results indicate that in the presence of massless Dirac fermions, a compact U(1) gauge field would confine electric charges for $N < N_c$ and deconfine them for $N > N_c$. However, the above considerations are valid only in the absence of spontaneous chiral symmetry breaking. If such symmetry breaking occurs, the fermions become massive and no anomalous dimension is generated for the gauge field. Chiral symmetry breaking is believed to occur for $N < N_{\rm ch}$, where typically $N_{\rm ch} \in (3,4)$ [28]. The dynamical mass generation in Eq. (11) is usually shown by using a Schwinger-Dyson approach controlled by a $1/N$ expansion [25] and, therefore, is inherently non-perturbative. If the true critical value of N is such that $N_c < N_{\rm ch}$, then the value of N_c should be considered as a calculational artifact. This is because in our picture, this value is a direct consequence of the existence of an anomalous scaling behavior for the gauge field which, as discussed above, does not exist if a fermion mass is spontaneously generated. A closely related argument is that the screening properties of the theory weaken the logarithmic interaction, thus leading to a $1/R$ behavior of the interaction between monopoles [12]. If such a scenario were to hold, the monopoles would never be confined, and as a consequence no CD-transition would take place in the case of three-dimensional QED defined by Eq. (11).

Summarizing, we have studied the influence of the gauge field anomalous dimension induced by the coupling to matter fields to the confinement-deconfinement transition. Our analysis reveals that the anomalous scaling of the gauge field plays an essential role for bosonic matter which possesses two relevant couplings. There, the electric charges deconfine for a Ginzburg parameter $\kappa > \kappa_c = 1.17/\sqrt{2}$, inside the type II regime of a superconductor. For the fermion theory, on the other hand, a deconfinement transition seems to take place only as a function of the number N of fermion components. However, due to the possibility of spontaneous chiral symmetry breaking and/or strong screening effects, further study is necessary in order to firmly establish that a deconfinement transition really takes place in the fermion theory. Such a study will be in part numerical, with the use of Monte Carlo simulations. Detailed large-scale Monte Carlo simulations for the bosonic case are currently in progress [30].

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