IL NUOVO CIMENTO Vol. 12 A, N. 1 1 Novembre 1972

Compton Scattering and the Couplings of f, σ, η, A_2 and π to Photons and Nucleons (*).

J. Baacke, T. H. Chang and H. Kleinert

Institut für Theoretische Physik - Berlin

(ricevuto il 15 Marzo 1972)

Summary. — We use the technique of equating unsubtracted dispersion relations in the forward and backward directions at threshold in order to obtain sum rules for the invariant amplitudes of Compton scattering. Assuming f, σ, η and π, A_2 to dominate the absorptive part of the t-channel, and baryon resonances to saturate the integrals over the s- and u-channel cuts we are able to express the coupling constants of these mesons to photons and nucleons in terms of the electromagnetic multipole amplitudes of the baryon resonances. The results are compared with estimates obtained by other methods.

1. – Introduction.

Three hypotheses have recently given rise to a number of predictions concerning coupling constants of mesons:

1) The energy-momentum tensor is dominated by a single $\sigma(700, \Gamma \approx \approx 400)$ [1,2] and a single $f(1260, \Gamma \approx 150 \pm 25)$ [3] meson in its spin-zero and spin-two content, respectively.

(*) Supported in part by Deutsche Forschungsgemeinschaft under Grant No. KL 256.

(1) H. A. Kastrup: *Nucl. Phys.*, **15** B, 189 (1970); see however: H. Kleinert and P. H. Weisz: *Nucl. Phys.*, **27** B, 23 (1971); M. Dal-Cin and H. A. Kastrup: *Nucl. Phys.*, **15** B, 189 (1970); J. Ellis: *Nucl. Phys.*, **22** B, 478 (1970); M. Gell-Mann: *Lecture Notes Hawaii Summer School* (1969); B. Zumino: *Brandeis Lectures* (1970); H. Kleinert: *Lectures presented at the XI Institutwochen für Kernphysik* (Schladming, 1972).

(2) H. Kleinert, L. P. Staunton and P. H. Weizs: *Nucl. Phys.*, **38** B, 87 (1972).

(3) P. G. O. Freund: *Phys. Lett.*, **2**, 136 (1962); D. H. Sharp and W. G. Wagner: *Phys. Rev.*, **131**, 2226 (1963); B. Renner: *Phys. Rev. Lett.*, **33** B, 599 (1971); DESY preprint 71/14 (1971).

2) The pomeron trajectory is linked in some way with the f trajectory [3,4]. Involving the standard smoothness of reggeology one is able to conclude that properties of diffractive production are related to those of the f coupling.

3) Absence of exotic resonances in certain channels implies exchange degeneracy of f, ω and ρ, A_2 trajectories and equality of their Regge residues [5]. Assuming again the standard smoothness one can extrapolate these relations to the mass shells of these particles.

Many of these predictions fix coupling constants observable in Compton scattering. Thus assumption 1) has led to the estimates for the radiative decays of σ [2] and f [3] mesons

$$g_{\sigma\gamma\gamma} \approx 0 \, , \tag{1.1}$$

$$g^{(2)}_{f\gamma\gamma} \approx \frac{4}{\gamma_\rho^2} g_{f\pi\pi} \, , \qquad g^{(1-2)}_{f\gamma\gamma} \approx 0 \, , \tag{1.2}$$

while for the coupling of σ (*) and f [3] (**) to nucleons one obtains

$$g_{\sigma\mathcal{N}\mathcal{N}} \approx \frac{m}{m_\sigma} g_{\sigma\pi\pi} \, , \tag{1.3}$$

$$G^{(1)}_{f\mathcal{N}\mathcal{N}} \approx \frac{m}{m_f} g_{f\pi\pi} \, , \qquad G^{(2)}_{f\mathcal{N}\mathcal{N}} \approx 0 \, . \tag{1.4}$$

The pomeron trajectory has been found experimentally to decouple from helicity-flip amplitudes in $\pi\mathcal{N}$ and $\gamma\mathcal{N} \to \rho\mathcal{N}$ amplitudes [7]. Using assumption 2) one can then conclude that the f coupling to two photons is pre-

---

(4) P. Achutan, H. G. Schlaile and F. Steiner: *Nucl. Phys.*, **24** B, 398 (1970); P. G. O. Freund: Chicago preprint COO 264-572 (1971). R. Odorico, A. Garcia and C. A. Garcia Canal: *Phys. Lett.*, **32** B, 375 (1970); C. Michael and R. Odorico: *Phys. Lett.*, **34** B, 422 (1971).

(5) R. C. Arnold: *Phys. Rev. Lett.*, **14**, 657 (1965); C. Schmid: *Phys. Rev. Lett.*, **20**, 689 (1968).

(*) For a discussion see, for example, ref. [1] or the introduction of ref. [6]. Our σππ coupling is normalized by $\mathcal{L}_{\sigma\pi\pi} = (g_{\sigma\pi\pi}/2m_\sigma)\sigma\pi^2$ such that $|g_{\sigma\pi\pi}| = m_\sigma/\sqrt{2}F_\pi = 5.2$. In the model, $g_{\sigma\pi\pi} = -m_\sigma/F_\pi < 0$.

(6) H. Kleinert, L. P. Staunton and P. H. Weisz: *Nucl. Phys.*, **38** B, 104 (1972).

(**) Here $\mathcal{L}_{f\pi\pi} = g_{f\pi\pi}/m_f \, \partial^\mu\pi\partial^\nu\pi f_{\mu\nu}$ such that $\Gamma_{f\pi\pi} = \frac{2}{5}(g^2_{f\pi\pi}/4\pi)(k^5/m_f^4)$ and $|g_{f\pi\pi}| \approx 12$.

(7) G. Höhler and R. Strauss: *Zeits. Phys.*, **232**, 205 (1970); F. J. Gilman, J. Pumplin, A. Schwimmer and L. Stodolsky: *Phys. Lett.*, **31** B, 387 (1970); H. Harari and Y. Zarmi: *Phys. Lett.*, **32** B, 291 (1970).

dominantly helicity conserving, *i.e.*

(1.5) $$G^{(2)}_{f\mathcal{N}\mathcal{N}}/G^{(1+2)}_{f\mathcal{N}\mathcal{N}} \approx 0 ,$$

(1.6) $$g^{(1-2)}_{f\gamma\gamma}/g^{(2)}_{f\gamma\gamma} \approx 0 ,$$

in agreement with the predictions 2) and 4).

Finally, assumption 3) has led to the conclusion that the couplings of $f\mathcal{N}\mathcal{N}$ would have the same flip-to-nonflip ratio as $\omega\mathcal{N}\mathcal{N}$ which is known to vanish, in agreement with (1.5). Similarly the $A_2\mathcal{N}\mathcal{N}$ coupling should behave like the ρ coupling, *i.e.* should predominantly flip the nucleon spin (*).

Some of these predictions have been tested.

1) The coupling strengths $\sigma\mathcal{N}\mathcal{N}$, $f\mathcal{N}\mathcal{N}$ can be observed in backward dispersion relations of the isospin even amplitudes of $\pi\mathcal{N}$ scattering [8,9]. One finds the estimates

(1.7) $$g_{\sigma\mathcal{N}\mathcal{N}}\, g_{\sigma\pi\pi} = 69 \pm 4 ,$$

(1.8) $$G^{(1)}_{f\mathcal{N}\mathcal{N}}\, g_{f\pi\pi} = 301 \pm 28 , \qquad G^{(2)}_{f\mathcal{N}\mathcal{N}}\, g_{f\pi\pi} = -70 \pm 90$$

with an error which is hard to assess. Inserting $g_{\sigma\pi\pi} \approx -5$, $g_{f\pi\pi} \approx 12$ we only see that the orders of magnitude agree and that the vanishing of the helicity-flip coupling is in agreement with (1.4) and (1.5).

2) The coupling $A_2\mathcal{N}\mathcal{N}$ has been estimated by using a combination of backward and forward dispersion relations in the photoproduction amplitude. The coupling constants are calculated in terms of electromagnetic and pionic coupling constants of baryon resonances. One finds that the flip-to-nonflip ratio is related to the magnetic *vs.* electric ratio of the multipole couplings of the Δ-resonance and therefore turns out large, as predicted [10] (see Appendix A, 5)).

3) For the couplings $\sigma\gamma\gamma$ and $f\gamma\gamma$ there is an estimate based on finite-energy sum rules in pion Compton scattering [11]. Here one finds

(1.9) $$g_{\sigma\gamma\gamma} \approx 0.9 ,$$

(1.10) $$g^{(2)}_{f\gamma\gamma} \approx 1.3 , \qquad g^{(1-2)}_{f\gamma\gamma} \approx -0.04 .$$

(*) Both statements agree with what one would conclude from the flip-to-nonflip ratio at $t = m^2_\omega$ or m^2_ρ (which is $-(2m/\sqrt{-t})(f_{+-}/f_{++}) \to 2K^s \approx -0.12$ or $2K^v \approx 3.7$, respectively) if one assumes a smooth extrapolation to $t = 0$.

(8) H. Goldberg: *Phys. Rev.*, **171**, 1485 (1968); J. Engels and G. Höhler: Karlsruhe preprint (1970); J. Engels: *Nucl. Phys.*, **25** B, 141 (1970); J. L. Petersen and J. Pišut: CERN preprint TH 1375 (1971).

(9) H. G. Schlaile: Karlsruhe Thesis (1970); R. Strauss: Karlsruhe Thesis (1970).

(10) H. Kleinert and P. H. Weisz: *Lett. Nuovo Cimento*, **2**, 459 (1971).

(11) B. Schrempp-Otto, F. Schrempp and T. F. Walsh: *Phys. Lett.*, **36** B, 463 (1971).

However, this work needs stronger assumptions than the determinations presented before. In particular it uses the smoothness of the couplings along the the Regge trajectory in order to determine the on-shell constants. Since this smoothness is part of the assumptions to be tested there is need for an estimate of these couplings via direct methods like 1) and 2).

It is the purpose of this paper to present a complete discussion of these coupling constants making combined use of forward and backward dispersion relations in Compton scattering. Apart from the above-mentioned determination of $\sigma\mathcal{N}\mathcal{N}$, $f\mathcal{N}\mathcal{N}$ and $A_2\mathcal{N}\mathcal{N}$ couplings, this method has been successful in the calculation of the coupling constants to nucleons ([12]) and baryon resonances ([13,14]). Therefore we believe it worth-while to investigate also the vertices $\sigma\gamma\gamma$, $f\gamma\gamma$, $A_2\gamma\gamma$ using this method. In addition we shall obtain once more results on $\sigma\mathcal{N}\mathcal{N}$, $f\mathcal{N}\mathcal{N}$ and $A_2\mathcal{N}\mathcal{N}$ couplings and also be able to give estimates for $\eta\gamma\gamma$ and $\pi\gamma\gamma$.

The method proceeds as follows. Among the invariant Compton amplitudes free of kinematic singularities and constraints we pick those which, by Regge arguments, obey an unsubtracted dispersion relation in the forward as well as in the backward direction. Via these dispersion relations we calculate the value at the point common to both directions, the threshold point $s = m^2$, $t = 0$ in two ways.

One way is by integrating along $t = 0$ and picking up only s- and u-channel absorptive parts and the other is by integrating along the curve $\theta = 0$ so that one obtains contributions from the t-channel cut as well as the s- and u-channel cuts. Since the resulting values have to be the same we obtain a sum rule relating the integrals over the absorptive parts of the t-channel to those of the s- and u-channel. These integrals are assumed to be dominated by σ, f, η, π, A_2 mesons in the t-channel and by baryon resonances in the s- and u-channel. The resonance couplings are known quite well from multipole analyses of photoproduction. Therefore we are able to express the meson couplings in terms of multipole moments of the resonances.

For amplitudes which in the forward direction need subtractions due to the exchange of a pomeron or the A_2 or π trajectory, we shall write a superconvergence relation in the backward direction which turns out to be possible in all cases.

([12]) J. ENGELS, G. HÖHLER and B. PETERSON: *Nucl. Phys.*, **15** B, 365 (1970); H. BANERJEE, B. DUTTA-ROY and S. MALLIK: *Lett. Nuovo Cimento*, **1**, 436 (1969); *Nuovo Cimento*, **66** A, 475 (1970); H. BANERJEE and B. DUTTA-ROY: *Phys. Rev. D*, **2**, 2414 (1970).

([13]) J. BAACKE and H. KLEINERT: *Lett. Nuovo Cimento*, **2**, 463 (1971); *Phys. Lett.*, **35** B, 159 (1971).

([14]) J. BAACKE and H. KLEINERT: *Nucl. Phys.*, **42** B, 301 (1972).

2. – Kinematics and resonance contribution.

A covariant basis for the Compton process

$$\mathcal{N}(p')\gamma(k') \leftarrow \mathcal{N}(p)\gamma(k) \tag{2.1}$$

has been given by PRANGE [15] (*). If one denotes the scattering amplitude (**) by

$$\langle p'k'|T|pk\rangle = \varepsilon_\mu^+(k')\bar{u}(p')T^{\mu\nu}u(p)\varepsilon_\nu(k)\,, \tag{2.2}$$

then $T^{\mu\nu}$ can be written as

$$T^{\mu\nu} = \sum_{i=1}^{6} B_i Q_i^{\mu\nu} \tag{2.3}$$

with the covariants

$$\left\{\begin{aligned} Q_1^{\mu\nu} &= \frac{P'^\mu P'^\nu}{P'^2}\,,\\ Q_2^{\mu\nu} &= \frac{N^\mu N^\nu}{N^2}\,,\\ Q_3^{\mu\nu} &= \frac{P'^\mu N^\nu - P'^\nu N^\mu}{\sqrt{P'^2 N^2}}\, i\gamma_5\,,\\ Q_4^{\mu\nu} &= \frac{P'^\mu P'^\nu}{P'^2}\,\gamma\cdot K - m\,\frac{PK}{P^2}\,Q_1^{\mu\nu}\,,\\ Q_5^{\mu\nu} &= \frac{N^\mu N^\nu}{N^2}\,\gamma\cdot K - m\,\frac{PK}{P^2}\,Q_2^{\mu\nu}\,,\\ Q_6^{\mu\nu} &= \frac{P'^\mu N^\nu + P'^\nu N^\mu}{\sqrt{P'^2 N^2}}\, i\gamma_5(\gamma\cdot K)\,. \end{aligned}\right. \tag{2.4}$$

Here K, P' and N are combinations of the particle momenta:

$$P'_\mu \equiv P_\mu - \frac{PK}{K^3}K_\mu\,,\qquad N_\mu \equiv -\varepsilon_{\mu\nu\varrho\sigma}P'^\nu K^\varrho \Delta^\sigma \qquad (\varepsilon_{0123}=1) \tag{2.5}$$

with

$$P \equiv \tfrac{1}{2}(p'+p)\,,\qquad K \equiv \tfrac{1}{2}(k'+k)\,,\qquad \Delta = p'-p\,. \tag{2.6}$$

([15]) R. E. PRANGE: *Phys. Rev.*, **110**, 240 (1958).
(*) We perform only a slight modification to have it orthogonal.
(**) Our convention is $S = 1-(2\pi)^4 i\delta^4(P_f-P_i)T = 1+i(2\pi)^4\delta^4(P_f-P_i)f$ and the states are normalized according to $\langle p'|p\rangle = (p_0/m)(2\pi)^3\delta^3(p'-p)$, $\langle k'|k\rangle = 2k_0(2\pi)^3\delta^3(k'-k)$. Hence $d\sigma/d\Omega_{c.m.} = |(m/4\pi W)f|^2$. Our γ-matrices are the same as those of S. GASIOROWICZ: *Elementary Particle Physics* (New York, 1966).

Obviously P', N and Δ can be used as an orthogonal set of independent vectors with square lengths

$$(2.7)\qquad \begin{cases} P'^2 = \dfrac{1}{K^2}\left(K^2P^2-(PK)^2\right) = -\dfrac{1}{t}(su-m^4)\,, \\[2ex] N^2 = 4K^2\left(K^2P^2-(PK)^2\right) = -\dfrac{t}{4}(su-m^4)\,, \\[1ex] \Delta^2 = -4K^2 = t\,. \end{cases}$$

As a consequence the covariants Q_i form an orthogonal basis with respect to the scalar product

$$(2.8)\qquad Q_i\cdot Q_j \equiv \tfrac{1}{4}\,\mathrm{Tr}\left(((\gamma\cdot p')+m)Q_i((\gamma\cdot p)+m)\bar{Q}_j\right).$$

Their squares are

$$(2.9)\qquad \begin{cases} Q_1^2 = Q_2^2 = 2P^2 = 2\left(m^2-\dfrac{t}{4}\right), \\ Q_3^2 = 4K^2 = -t\,, \\ Q_4^2 = Q_5^2 = -2\left(1-\dfrac{m^2}{P^2}\right)\left(P^2K^2-(PK)^2\right), \\ Q_6^2 = -4\left(P^2K^2-(PK)^2\right). \end{cases}$$

Given an arbitrary Feynman graph, its contribution to the amplitudes B_i is most easily calculated by using the scalar products (2.8) and (2.9). For a dispersion-theoretic analysis the amplitudes B_i are, however, not a convenient choice because of kinematic singularities and constraints. A set of amplitudes free of both is provided by the decomposition [16]

$$(2.10)\qquad T^{\mu\nu} = \sum A_i\,\mathscr{L}_i^{\mu\nu}$$

with

$$(2.11)\qquad \begin{cases} \mathscr{L}_1^{\mu\nu} = K^2g^{\mu\nu}-2K^\mu K^\nu\,, \\[1ex] \mathscr{L}_2^{\mu\nu} = \tfrac{1}{2}K^2[\gamma^\mu(\gamma\cdot K)\gamma^\nu-\gamma^\nu(\gamma\cdot K)\gamma^\mu]-PK(K^\mu\gamma^\nu+K^\nu\gamma^\mu)+ \\ \qquad\qquad +(\gamma\cdot K)(K^\mu P^\nu+P^\mu K^\nu)\,, \\[1ex] \mathscr{L}_3^{\mu\nu} = m(\gamma\cdot K)g^{\mu\nu}-(PK)g^{\mu\nu}-\tfrac{1}{2}K^2(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)+ \\ \quad +\tfrac{1}{2}K^\mu[(\gamma\cdot K),\gamma^\nu]+\tfrac{1}{2}[\gamma^\mu,(\gamma\cdot K)]K^\nu-m(K^\mu\gamma^\nu+\gamma^\mu K^\nu)+(K^\mu P^\nu+P^\mu K^\nu)\,, \end{cases}$$

---

[16] W. A. BARDEEN and WU-KI TUNG: *Phys. Rev.*, **173**, 1423 (1968).

$$
(2.11)\quad \left\{\begin{aligned}
\mathscr{L}_4^{\mu\nu} &= K^2(\gamma^\mu P^\nu + P^\mu\gamma^\nu) - (PK)(K^\mu\gamma^\nu + \gamma^\mu K^\nu) - \\
&\quad - (\gamma\cdot K)(K^\mu P^\nu + P^\mu K^\nu) + (PK)g^{\mu\nu}(\gamma\cdot K) - mK^2 g^{\mu\nu} + 2mK^\mu K^\nu,\\
\mathscr{L}_5^{\mu\nu} &= K^2 P^\mu P^\nu - (PK)(K^\mu P^\nu + P^\mu K^\nu) - \\
&\quad - \tfrac{1}{2}(P^2K^2 - (PK)^2)\, g^{\mu\nu} + P^2 K^\mu K^\nu,\\
\mathscr{L}_6^{\mu\nu} &= P^\mu P^\nu(\gamma\cdot K) - \tfrac{1}{2}(PK)(\gamma^\mu P^\nu + P^\mu\gamma^\nu) + \\
&\quad + \tfrac{1}{4}(PK)\,[\gamma^\mu(\gamma\cdot K)\gamma^\nu - \gamma^\nu(\gamma\cdot K)\gamma^\mu] + \frac{m}{4}K^2(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) + \\
&\quad + \frac{m}{2}(PK)\,g^{\mu\nu} - \tfrac{1}{2}P^2 g^{\mu\nu}(\gamma\cdot K) + K^\mu K^\nu(\gamma\cdot K) + \\
&\quad + \frac{m^2}{2}(K^\mu\gamma^\nu + \gamma^\mu K^\nu) - \frac{m}{2}(K^\mu P^\nu + P^\mu K^\nu) - \\
&\quad - \frac{m}{4}K^\mu[(\gamma\cdot K), \gamma^\nu] - \frac{m}{4}[\gamma^\mu, (\gamma\cdot K)]K^\nu .
\end{aligned}\right.
$$

Notice that due to crossing symmetry $k \leftrightarrow -k'$, $A_{1,2,4,5}$ are symmetric, while $A_{3,6}$ are antisymmetric under the exchange $s \leftrightarrow u$. The connection of B_i and A_i is given by (*) $B_i = A_i M_{ij}$ with

$$
(2.12)\quad M_{ij} = \begin{pmatrix}
-\frac{t}{4} & -\frac{t}{4} & 0 & 0 & 0 & 0\\
0 & 0 & m\frac{t}{4} & 0 & 0 & 0\\
\frac{s-u}{4m^2-t}\frac{t}{4} & \frac{s-u}{4m^2-t}\frac{t}{4} & -\frac{s-u}{4} & m & m & 0\\
+m\frac{su-m^4}{4m^2-t} & -m\frac{su-m^4}{4m^2-t} & 0 & -\frac{s-u}{4} & \frac{s-u}{4} & -\frac{t}{4}\\
\frac{su-m^4}{8} & -\frac{su-m^4}{8} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{1}{8}(4m^2-t) & -\frac{1}{8}(4m^2-t) & -\frac{s-u}{8}
\end{pmatrix}
$$

(*) Notice that these relations are consistent with those of ref. [15] if one changes the overall sign in their eqs. (4.6), (4.7). The reason for the overall sign is that ref. [15] defines $M_{\mu\nu}$ like our $T_{\mu\nu}$, but compares it with the amplitude $F_{\mu\nu}$ of HEARN and LEADER [16], which is defined opposite to our $T_{\mu\nu}$. For A_3' there is no sign change. The authors claim in a footnote [20] to have found an error in ref. [16], but they must have overlooked the fact that HEARN and LEADER [17] contract the photons as $F = \varepsilon_\nu^+(k')\bar{u}(p')F^{\mu\nu}u(p)\varepsilon_\mu(k)$, *i.e.* opposite to the natural order of the indices $\mu\nu$. Note that Bardeen and Tung's helicity amplitudes eq. (51) and (52) contain some printing errors. Compare our eqs. (2.14).

([17]) A. C. HEARN and E. LEADER: *Phys. Rev.*, **126**, 789 (1962).

with an inverse

$$(2.13)\quad M_{ij}^{-1} =$$

$$= \begin{pmatrix} -\frac{2}{t} & 0 & 0 & 0 & \frac{4}{su-m^4} & 0 \\ -\frac{2}{t} & 0 & 0 & 0 & -\frac{4}{su-m^4} & 0 \\ 0 & \frac{4}{mt} & 0 & 0 & 0 & 0 \\ \frac{1}{2m}\frac{s-u}{4m^2-t} & \frac{s-u}{2m^2t} & \frac{1}{2m} & \frac{s-u}{2(su-m^4)} & -\frac{4m(s-u)}{(4m^2-t)(su-m^4)} & -\frac{t}{su-m^4} \\ \frac{1}{2m}\frac{s-u}{4m^2-t} & \frac{s-u}{2m^2t} & \frac{1}{2m} & -\frac{s-u}{2(su-m^4)} & \frac{4m(s-u)}{(4m^2-t)(su-m^4)} & \frac{t}{su-m^4} \\ 0 & 0 & 0 & \frac{4m^2-t}{su-m^4} & -\frac{2m}{su-m^4} & 2\frac{s-u}{su-m^4} \end{pmatrix}$$

By making use of these transformation rules it is an easy, albeit laborious, task to calculate the contributions of the simplest Feynman graphs.

a) For the exchange of a nucleon one has an amplitude

$$(2.14)\quad T^{\mu\nu} = (s-m^2)^{-1}\left(e\gamma^\mu - \frac{\boldsymbol{K}}{2m}i\sigma^{\mu\lambda}k'_\lambda\right)((\gamma\cdot p)+(\gamma\cdot k)+m)\cdot \cdot\left(e\gamma^\nu + \frac{\boldsymbol{K}}{2m}i\sigma^{\nu\varkappa}k_\varkappa\right) + (u-m^2)^{-1}\left(e\gamma^\nu + \frac{\boldsymbol{K}}{2m}i\sigma^{\nu\varkappa}k_\varkappa\right)\cdot \cdot((\gamma\cdot p)-(\gamma\cdot k')+m)\left(e\gamma^\mu - \frac{\boldsymbol{K}}{2m}i\sigma^{\mu\lambda}k'_\lambda\right),$$

where (*)

$$(2.15)\qquad e \equiv \tfrac{1}{2}(1+\tau_3)\,,\qquad \boldsymbol{K} = K_s 1 + K_v\tau_3\,.$$

Projecting out the amplitudes B_i and transforming to A_i via (2.13) one finds that A_i can be written as a sum of three terms

$$(2.16)\quad A_i = R_i^+[(m^2-s)^{-1}+(m^2-u)^{-1}] + R_i^-[(m^2-s)^{-1}-(m^2-u)^{-1}] + R_i^{su}[(m^2-s)(m^2-u)]^{-1}$$

with the residues given in Table I ([16]).

(*) $K_s = (K_p + K_n)/2 = -0.068$, $K_v = (K_p - K_n)/2 = 1.858$.

TABLE I. – *Residues of nucleon exchange.*

	R_i^+	R_i^-	R_i^{su}
A_1	$-(2Ke+K^2)/2m$	0	$4me^2$
A_2	$(2Ke+K^2)/2m^2$	0	$4e(e+K)$
A_3	0	$(2Ke+K^2)/2m$	0
A_4	$K^2/2m^2$	0	$4e(e+K)$
A_5	0	0	$-8Ke/m$
A_6	0	$-K^2/m^2$	0

b) For the exchange of a π^0-meson in the t-channel one has

$$T^{\mu\nu}=\frac{1}{m_\pi}g_{\pi\mathcal{N}\mathcal{N}}g_{\pi\gamma\gamma}\frac{1}{m_\pi^2-t}i\gamma_5\tau_3\varepsilon^{\mu\nu\lambda\varkappa}k'_\lambda k_\varkappa \tag{2.17}$$

and finds (*)

$$A_2^v=-\frac{2}{mm_\pi}g_{\pi\mathcal{N}\mathcal{N}}g_{\pi\gamma\gamma}\frac{1}{m_\pi^2-t}\,. \tag{2.18}$$

For η exchange one has to replace $\pi\to\eta$ in this formula and $A_2^v\to A_2^s$.

c) The exchange of σ gives

$$T^{\mu\nu}=-g_{\sigma\gamma\gamma}g_{\sigma\mathcal{N}\mathcal{N}}\frac{2}{m_\sigma}\frac{1}{m_\sigma^2-t}(k'kg_{\mu\nu}-k'_\nu k_\mu)\,, \tag{2.19}$$

hence

$$A_1^s=-\frac{4}{m_\sigma(m_\sigma^2-t)}g_{\sigma\gamma\gamma}g_{\sigma\mathcal{N}\mathcal{N}}\,. \tag{2.20}$$

d) The exchange of an f-meson yields after some tedious algebra (**)

$$\left\{\begin{aligned}A_1^s&=\left\{2\frac{g_{f\gamma\gamma}^{(1-2)}}{m_f^3}G_{f\mathcal{N}\mathcal{N}}^{(1+2)}\left(\nu^2+\frac{t}{12}\right)+\right.\\&\qquad\left.+\frac{t^2}{24m^2m_f^3}\left[\left(\frac{t}{m_f^2}-2\right)g_{f\gamma\gamma}^{(1)}+\left(\frac{2t}{m_f^2}-1\right)g_{f\gamma\gamma}^{(2)}\right]G_{f\mathcal{N}\mathcal{N}}^{(2)}\right\}\frac{1}{t-m_f^2}\,,\\A_2^s&=-\frac{2\nu^2}{mm_f^3}G_{f\mathcal{N}\mathcal{N}}^{(1)}g_{f\gamma\gamma}^{(1-2)}\frac{1}{t-m_f^2}\,,\end{aligned}\right. \tag{2.21}$$

(*) We use the standard isospin decomposition.
(**) Here $\nu=(s-u)/4m=pq/m$.

$$
(2.21)\quad \begin{cases}
A_3^s = -\dfrac{\nu}{2}\dfrac{t}{mm_f^3}\,G^{(1)}_{f\mathcal{N}\mathcal{N}}\,g^{(1-2)}_{f\gamma\gamma}\,\dfrac{1}{t-m_f^2}\,,\\[2ex]
A_4^s = -\dfrac{1}{2}\dfrac{t}{mm_f^3}\,g^{(2)}_{f\gamma\gamma}\,G^{(1)}_{f\mathcal{N}\mathcal{N}}\,\dfrac{1}{t-m_f^2}\,,\\[2ex]
A_5^s = -\dfrac{t}{m^2m_f^3}\,g^{(2)}_{f\gamma\gamma}\,G^{(2)}_{f\mathcal{N}\mathcal{N}}\,\dfrac{1}{t-m_f^2}\,,\\[2ex]
A_6^s = 0\,.
\end{cases}
$$

Finally, in order to obtain the contribution of A_2 exchange one just has to replace $f\to A_2$ and $A^s\to A^v$.

The s-channel helicity amplitudes can be calculated with the Jacob-Wick phase convention (photon $=$ particle 2) yielding

$$
(2.22)\quad \begin{cases}
\hat f_1 \equiv \left(\dfrac{1}{4}\cos\dfrac{\theta}{2}\right)^{-1} f_1 =\\
\qquad = 2[(s-m^2)^2+m^2t]A_4 - m(su-m^4)A_5 - [(s-m^2)^2-m^2t]A_6\,,\\[1ex]
\hat f_2 \equiv \left(\dfrac{1}{4W}\sin\dfrac{\theta}{2}\right)^{-1} f_2 =\\
\qquad = t(s+m^2)A_1 - mt(s-m^2)A_2 + 2[(s-m^2)^2-(su-m^4)]A_3\,,\\[1ex]
\hat f_3 \equiv \left(mp^2\sin^2\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right)^{-1} f_3 = 2A_1+2A_3\,,\\[1ex]
\hat f_4 \equiv \left(Wp^2\sin\dfrac{\theta}{2}\cos^2\dfrac{\theta}{2}\right)^{-1} f_4 = -2mA_4 - \dfrac{1}{2}(s+m^2)A_5 - mA_6\,,\\[1ex]
\hat f_5 \equiv \left(sp^2\cos^3\dfrac{\theta}{2}\right)^{-1} f_5 = 2A_4 + mA_5 + A_6\,,\\[1ex]
\hat f_6 \equiv \left(\dfrac{1}{W}p^2\sin^3\dfrac{\theta}{2}\right)^{-1} f_6 = (s+m^2)A_1 + m(s-m^2)A_2 + 2m^2A_3\,.
\end{cases}
$$

Conversely the invariant amplitudes $\boldsymbol{A}\equiv(A_1,\ldots,A_6)$ can be expressed in term of the amplitudes $\hat{\boldsymbol{f}}\equiv(\hat f_1,\ldots,\hat f_6)$ by $\boldsymbol{A}=\boldsymbol{M}\hat{\boldsymbol{f}}$ with a matrix

$$
(2.23)\quad \boldsymbol{M} = \frac{1}{4(s-m^2)^2}\cdot
\begin{Bmatrix}
0 & -1 & t(s+m^2)+2(s-m^2)^2 & 0 & 0 & -t\\[1ex]
0 & \dfrac{1}{m} & -\dfrac{(s+m^2)(s-u)}{m} & 0 & 0 & \dfrac{4(s-m^2)+t}{m}\\[1ex]
0 & 1 & -(s+m^2)t & 0 & 0 & t\\
1 & 0 & 0 & 2m(s-u) & m^2(s-u)+(s^2-m^4) & 0\\
0 & 0 & 0 & -8(s-m^2) & -8m(s-m^2) & 0\\
-2 & 0 & 0 & -4mt & 2[(s-m^2)^2-m^2t] & 0
\end{Bmatrix}.
$$

This procedure allows for a simple calculation of the contribution of an arbitrary s- and u-channel resonance to A. If s is close to a resonance $\mathcal{N}$ of mass m_n^2, T behaves as

$$(2.24)\qquad T_{\lambda'_{\mathcal{N}}\lambda'_{\gamma},\lambda_{\mathcal{N}}\lambda_{\gamma}} = \frac{2mm_n}{s-m_n^2}\,\varepsilon_{\mu}^{\lambda'_{\gamma}}(k')\,\varepsilon_{\nu}^{\lambda_{\gamma}}(k)\,\langle N(p')\lambda'_{\mathcal{N}}|j^{\mu}|N^*(p+q)\lambda'_{\mathcal{N}}\rangle\cdot$$
$$\cdot\langle N^*(p+q)\,\lambda_{\mathcal{N}}|j^{\nu}|N(p)\,\lambda_{\mathcal{N}}\rangle\, d^J_{\lambda_{\mathcal{N}}-\lambda_{\gamma},\lambda'_{\mathcal{N}}-\lambda'_{\gamma}}(\theta)$$

with the intermediate states at rest, the nucleon momenta running in the Z-positive direction and the photon momenta in the negative direction. Let us define the current vertices (*)

$$(2.25)\qquad \begin{cases} \Gamma_{\frac{3}{2};\frac{1}{2},-1} \equiv \langle N^*(p+q)\,\frac{3}{2}\,|j^{\nu}|N(p)\,\tfrac{1}{2}\rangle\,\varepsilon_{\nu}^{-1}(k) \equiv +G^+\,, \\ \Gamma_{-\frac{1}{2};\frac{1}{2},1} \equiv \langle N^*(p+q)-\tfrac{1}{2}\,|j^{\nu}|N(p)\,\tfrac{1}{2}\rangle\,\varepsilon_{\nu}^{1}(k) = +G^-\,, \end{cases}$$

with the parity property

$$(2.26)\qquad \Gamma_{\lambda_{\mathcal{N}}-\lambda_{\gamma};\lambda,\lambda_{\gamma}} = -\eta^*\eta(-)^{J-J'-\lambda_{\gamma}}\,\Gamma_{-(\lambda_{\mathcal{N}}-\lambda_{\gamma});-\lambda,-\lambda_{\gamma}} = n\Gamma_{-(\lambda_{\mathcal{N}}-\lambda_{\gamma});-\lambda,-\lambda_{\gamma}}$$
$$(\text{for } \lambda_{\gamma}=\pm 1\,,\; J=\tfrac{1}{2}\,,\; \eta=1)\,,$$

where $n \equiv \eta^*(-)^{J'-\frac{1}{2}}$ is the normality of the resonance state. Then we can write for the six helicity amplitudes

$$(2.27)\qquad f = \frac{2mm_n}{m_n^2-s}\begin{cases} (G^-)^2 & d_{-\frac{1}{2},-\frac{1}{2}}(\theta)\,, \\ n(G^-)^2 & d_{-\frac{1}{2},\frac{1}{2}}(\theta)\,, \\ G^+G^- & d_{-\frac{1}{2},\frac{3}{2}}(\theta)\,, \\ nG^+G^- & d_{-\frac{1}{2},-\frac{3}{2}}(\theta)\,, \\ (G^+)^2 & d_{-\frac{3}{2},-\frac{3}{2}}(\theta)\,, \\ n(G^{+2}) & d_{-\frac{3}{2},\frac{3}{2}}(\theta)\,. \end{cases}$$

(*) The third independent current vertex $\Gamma_{\frac{1}{2};\frac{1}{2},0} \equiv \langle N^*(p+q)\,\frac{1}{2}|j^{\nu}|N(p)\,\tfrac{1}{2}\rangle\,\varepsilon_{\nu}^{k}(0) = -G^0$ does not contribute for real photons with $q^2=0$. For the connection of these G's with the definitions of form factors see ref. [18].

[18] H. KLEINERT: *Springer Tracts in Modern Physics*, Vol. **49** (1969), see Appendix D.

This leads to a vector $\hat{f}$ in the forward direction (*)

$$\hat{f}=\frac{4mm_n}{m_n^2-s}\begin{cases} 4(G^-)^2\,, \\ 4W\,n\,(G^-)^2(J+\tfrac{1}{2})\,, \\ \dfrac{1}{mp^2}\,G^+G^-\sqrt{(J-\tfrac{1}{2})(J+\tfrac{3}{2})}\,\dfrac{J+\tfrac{1}{2}}{2}\,, \\ -\dfrac{1}{Wp^2}\,G^+G^-\sqrt{(J-\tfrac{1}{2})(J+\tfrac{3}{2})}\,, \\ \dfrac{1}{sp^2}\,(G^+)^2\,, \\ \dfrac{W}{p^2}\,n\,(G^+)^2\,\dfrac{1}{3!}\,(J-\tfrac{1}{2})(J+\tfrac{1}{2})(J+\tfrac{3}{2})\,, \end{cases} \tag{2.28}$$

and in the backward direction

$$\hat{f}=(-)^{J-\frac{1}{2}}\frac{4mm_n}{m_n^2-s}\begin{cases} 4\,(G^-)^2(J+\tfrac{1}{2})\,, \\ 4W\,n\,(G^-)^2\,, \\ -\dfrac{1}{mp^2}\,G^+G^-\sqrt{(J-\tfrac{1}{2})(J+\tfrac{3}{2})}\,, \\ \dfrac{1}{Wp^2}\,n\,G^+G^-\sqrt{(J-\tfrac{1}{2})(J+\tfrac{3}{2})}\,\dfrac{J+\tfrac{1}{2}}{2}\,, \\ -\dfrac{1}{sp^2}\,(G^+)^2(J-\tfrac{1}{2})(J+\tfrac{1}{2})(J+\tfrac{3}{2})/6\,, \\ -\dfrac{W}{p^2}\,n\,(G^+)^2\,. \end{cases} \tag{2.29}$$

If we make use of the matrix (2.19) we arrive at contributions to $\boldsymbol{A}$ in the

---

(*) Here we use the fact that if we define a modified rotation function

$$\hat{d}^J_{\lambda\lambda'}(\theta)\equiv\left(\cos\frac{\theta_s}{2}\right)^{-|\lambda+\lambda'|}\left(\sin\frac{\theta_s}{2}\right)^{-|\lambda-\lambda'|}d^J_{\lambda\lambda'}(\theta)$$

one finds for

$$\lambda\geqslant\lambda':\ \hat{d}^J_{\lambda\lambda'}(0)=(-)^{\lambda-\lambda'}\hat{d}^J_{\lambda\lambda'}(0)=(-)^{\lambda-\lambda'}\frac{1}{(\lambda-\lambda')!}\sqrt{\frac{(J+\lambda)!\,(J-\lambda')!}{(J+\lambda')!\,(J-\lambda)!}}$$

and

$$\hat{d}^J_{\lambda\lambda'}(\pi)=(-)^{J-\lambda'}\,d^J_{-\lambda'\lambda}(0)=(-)^{J+\lambda}d^J_{\lambda,-\lambda'}(0)\,.$$

forward direction

$$(2.30)\qquad A_F = \frac{m}{m_n p_n^2(m_n^2 - s)} \cdot \left\{ \begin{array}{l} (J+\frac{1}{2})W\left\{-n\,(G^-)^2 + \frac{W}{m}G^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})}\right\}, \\ (J+\frac{1}{2})\frac{W}{m}\left\{n\,(G^-)^2 - \frac{W}{m}\frac{E}{p}G^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} + \frac{1}{3}\frac{W}{p}(G^+)^2(J-\frac{1}{2})(J+\frac{3}{2})\right\}, \\ (J+\frac{1}{2})\,Wn(G^-)^2, \\ (G^-)^2 - 2\frac{m}{p}\,nG^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} + \frac{W^2+3m^2}{2Wp}(G^+)^2, \\ \frac{4}{p}\left\{nG^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} - \frac{m}{W}(G^+)^2\right\}, \\ -2\left((G^-)^2 - (G^+)^2\right), \end{array} \right.$$

and in the backward direction

$$(2.31)\qquad A_B = \frac{m(-)^{J-\frac{1}{2}}}{m_n p_n^2(m_n^2 - s)} \cdot \left\{ \begin{array}{l} -W\left\{n(G^-)^2 + 2\frac{p}{m}G^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} + n(G^{+2})\right\}, \\ \frac{W}{m}\left\{n(G^-)^2 + 2\frac{E}{m}\frac{E}{p}G^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} - \frac{3W^2+m^2}{2pW}n(G^+)^2\right\}, \\ W\left\{n(G^-)^2 - 2\frac{E}{m}G^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} + n(G^+)^2\right\}, \\ (J+\frac{1}{2})\left\{(G^-)^2 + \frac{m}{W}\frac{E}{p}nG^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} - \frac{E}{Wp}(G^+)^2\frac{1}{3}(J-\frac{1}{2})(J+\frac{3}{2})\right\}, \\ -(J+\frac{1}{2})\frac{2}{p}\left\{nG^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} - \frac{m}{W}(G^+)^2\frac{1}{2}(J-\frac{1}{2})(J+\frac{3}{2})\right\}, \\ -2(J+\frac{1}{2})\left\{(G^-)^2 - \frac{m}{W}nG^+G^-\sqrt{(J-\frac{1}{2})(J+\frac{3}{2})} + \frac{E}{W}(G^+)^2\frac{1}{3}(J-\frac{1}{2})(J+\frac{3}{2})\right\}. \end{array} \right.$$

For the special cases of Roper and Δ resonances these expressions simplify somewhat and are listed in Appendix B.

Notice that since A_i are amplitudes free of kinematic singularities and constraints with definite symmetry, the contributions close to the resonance poles in the s- or u-channel can be written in the form

$$\text{(2.32)}\quad \frac{1}{s-u}A_i\left(\theta=\begin{Bmatrix}0\\ \pi\end{Bmatrix}\right)=\left(\frac{1}{s-m_n^2}\pm\frac{1}{u-m_n^2}\right)\operatorname*{res}_{s=m_n^2}A_i\left(\theta=\begin{Bmatrix}0\\ \pi\end{Bmatrix}\right),\qquad i=\begin{cases}1,2,4,5,\\ 3,6.\end{cases}$$

With these preparations we are now ready to present the derivation of the sum rules.

3. – The sum rules.

From crossing symmetry it follows that A_1, A_2, $(1/(s-u))A_3$, A_4 and $(1/(s-u))A_5$ are functions of $\nu^2=(s-u)^2/16m^2$ and t only. If the asymptotic behaviour is sufficiently good we can write a forward dispersion relation for the value of any of these amplitudes at the threshold point $(\nu^2, t)=(0, 0)$:

$$\text{(3.1)}\qquad A(0,0)=\frac{1}{\pi}\int_0^\infty\frac{\operatorname{Im}A(\nu'^2,0)}{\nu'^2-\nu^2}\,\mathrm{d}\nu'^2\,.$$

In the backward direction we observe that ν^2 is a good function of t:

$$\nu_B^2(t)\equiv\frac{1}{16m^2}\,t(t-4m^2)\,.$$

Hence, if $A\to 0$ for $t\to\infty$, $A(0,0)$ can be calculated once more via a dispersion relation

$$\text{(3.2)}\qquad A(0,0)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\operatorname{Im}A(\nu_B^2(t'),t')}{t'-t}\,\mathrm{d}t'.$$

Equating both integrals we obtain our sum rule

$$\text{(3.3)}\quad \frac{1}{\pi}\int_0^\infty\frac{\operatorname{Im}A(\nu_B^2(t'),t')}{t'-t}\,\mathrm{d}t'=\frac{1}{\pi}\int_0^\infty\frac{\operatorname{Im}A(\nu'^2,0)}{\nu'^2-\nu^2}\,\mathrm{d}\nu'^2-\frac{1}{\pi}\int_{-\infty}^0\frac{\operatorname{Im}A(\nu_B'(t'),t')}{t'-t}\,\mathrm{d}t',$$

which expresses pure meson contributions on the left-hand side in terms of the difference of forward and backward contributions of s- and u-channel resonances on the right-hand side. For this reason we shall call (3.3) a « forward-backward sum rule ».

Let us see which of the amplitudes allow for such a sum rule. The asymptotic behaviour can be estimated in forward direction by making the standard assumption that the t-channel helicity amplitudes are controlled by the meson Regge trajectories exchanged in the t-channel, *i.e.*

$$f^t_{\lambda'_{\mathcal{N}}\lambda_{\mathcal{N}},\lambda'_\gamma\lambda_\gamma} \simeq s^{\alpha_t}\,.$$

One finds that for $s \to \infty$, the invariant amplitudes

$$\left(A_1, A_2, \frac{1}{s-u} A_3, A_4, A_5, \frac{1}{s-u} A_6\right) \tag{3.4}$$

behave according to

$$(s^{\alpha_t}, s^{\alpha_t}, s^{\alpha_t-2}, s^{\alpha_t-2}, s^{\alpha_t-2}, s^{\alpha_t-3})\,. \tag{3.5}$$

For the isosinglet amplitudes A^s the pomeron trajectory can be exchanged such that $\alpha_t = 1$, and A_1, A_2 are no good candidates for an unsubtracted dispersion relation. For isovector amplitudes A^v, the leading trajectory is A_2 with $\alpha_{A_2} \approx 0.5$. Thus also here A_1 and A_2 need a subtraction.

In the backward direction the baryon trajectory with $\alpha_B \approx 0.2$ dominates the high-energy behaviour. From the s-channel helicity amplitudes (2.14) we see that

$$\left(A_1 + mA_2, A_2, \frac{1}{s-u} A_3, A_4 - \frac{1}{2} A_6, A_5, \frac{1}{s-u} A_6\right) \tag{3.6}$$

behave as

$$(t^{\alpha_B-\frac{3}{2}}, t^{\alpha_B-\frac{1}{2}}, t^{\alpha_B-\frac{5}{2}}, t^{\alpha_B-\frac{3}{2}}, t^{\alpha_B-\frac{3}{2}}, t^{\alpha_B-\frac{5}{2}})\,. \tag{3.7}$$

For the purpose of dispersing in t, $A_4 - \frac{1}{2} A_6$ is not yet a good combination since it is of mixed symmetry in $s \leftrightarrow u$. Therefore we form the symmetric combination

$$A_4 + \frac{t}{2(s-u)} A_6\,, \tag{3.8}$$

which obviously has the same asymptotic behaviour in the backward direction but is a function of ν^2 and t only.

As a result the following amplitudes can be used for our sum rule (3.3):

$$\frac{1}{s-u} A_6, A_5, \frac{1}{s-u} A_3, A_4 + \frac{t}{2(s-u)} A_6, A_4\,, \tag{3.9}$$

with decreasing degree of confidence. If we use a Michelin type of notation for characterizing the expected quality of the sum rules we can grant the first rule three stars, the next three rules two stars and the rule for A_4 one star. The corresponding rapidity of convergence of the dispersion integrals is shown in Table II.

TABLE II. – *Amplitudes used for our sum rule, the number of stars grading their quality, and the corresponding speed of convergence of isoscalar* (s) *and isovector* (v) *sum rules as expected from Regge exchange.*

Number of stars	Amplitudes	Forward behaviour A^s	Forward behaviour A^v	Backward behaviour $A^{s,v}$
★★★	$(1/(s-u))A_6$	s^{-2}	$s^{-2.5}$	$t^{-1.3}$
★★	A_5, $(1/(s-u))A_3$, $A_4+(t/2(s-u))A_6$	s^{-1}	$s^{-1.5}$	$t^{-1.3}$
★	A_4	s^{-1}	$s^{-1.5}$	$t^{-0.3}$
★	tA_5, $(t/(s-u))A_6$, $t(A_1+mA_2)$	0	0	$t^{-0.3}$

The amplitudes (3.9) do not yet, however, exhaust all possible candidates for which we can write our sum rule (3.3).

If an amplitude A behaves according to $t^{-1.3}$, it can be multiplied by an additional factor of t, and tA is still expected to obey an unsubtracted dispersion relation in the backward direction. In the forward direction, on the other hand, tA will vanish identically. Thus also tA should satisfy (3.3).

Obviously the resulting sum rules are exactly the same as would be obtained by using the fact that tA has a well-determined low-energy value at $t=0$ (since it receives contributions only from the nucleon Born term) and inserting this value directly in the backward dispersion relation (3.2). From the list of asymptotic behaviour given in (3.7) we see that

$$tA_5,\ \frac{t}{s-u}A_6,\ t(A_1+mA_2) \tag{3.10}$$

qualify for these sum rules. Since the asymptotic behaviour is only $t^{-0.3}$ we should not expect them to saturate rapidly. Thus they do not deserve more than one star (see Table II). The contributions of the different resonances to the unsubtracted dispersion relation are calculated in the following way:

a) *t-channel resonances.* Here we simply use our Feynman graphs (2.17)-(2.21), set $t=0$ in the pole denominator and $t=m_r^2$, $\nu=\nu_B(m_r^2)$ in the numerators, where m_r is the mass of the t-channel resonance. Thus, if A denotes any of the crossing-symmetric amplitudes (3.9) or (3.10), we can write for the left-hand side of (3.3)

$$\text{l.h.s.} = \sum_r -\frac{1}{m_r} \operatorname*{res}_{t=m_r^2} A \,.$$

b) s-channel resonances. There are two type of pole terms

$$\begin{cases} P_n^+ \equiv \dfrac{1}{s-m_n^2} + \dfrac{1}{u-m_n^2}\,, \\ P_n^- \equiv \dfrac{1}{s-u}\left(\dfrac{1}{s-m_n^2} - \dfrac{1}{u-m_n^2}\right). \end{cases}$$

Both have an unsubtracted form in the forward direction. Thus at threshold, $s = m^2$, $u = m^2$, they contribute

$$\text{(3.11)} \qquad \begin{cases} P_n^+|_{F,\text{th}} = -\dfrac{2}{m_n^2 - m^2}\,, \\ P_n^-|_{F,\text{th}} = -\dfrac{1}{m_n^2 - m^2}\,. \end{cases}$$

In order to disperse in the backward direction we rewrite

$$\text{(3.12)} \qquad \begin{cases} P_n^+ = -\dfrac{2(m_n^2 - m^2) + t}{(s-m_n^2)(u-m_n^2)}\,, \\ P_n^- = -\dfrac{1}{(s-m_n^2)(u-m_n^2)}\,. \end{cases}$$

Now for $\theta = \pi$, s and u satisfy the equation

$$\text{(3.13)} \qquad \begin{Bmatrix} s \\ u \end{Bmatrix} = -\frac{t}{2} \mp \frac{1}{2}\sqrt{t(t-4m^2)}\,.$$

As a consequence the denominators give rise to a simple pole at

$$t = t_n \equiv -\frac{1}{m_n^2}(m_n^2 - m^2)^2\,,$$

i.e.

$$\text{(3.14)} \qquad \begin{cases} P^+ = -\dfrac{2(m_n^2 - m^2) + t}{(t-t_n)\,m_n^2}\,, \\ P^- = -\dfrac{1}{(t-t_n)\,m_n^2}\,. \end{cases}$$

Obviously P^- has an unsubtracted form such that at threshold

$$\text{(3.15)} \qquad P_n^-|_{B,\text{th}} = \frac{1}{t_n m_n^2} = -\frac{1}{(m_n^2 - m^2)^2}\,.$$

The symmetric pole term P^+, on the other hand, has an unsubtracted form if one replaces t by t_n in the numerator. Thus one obtains at threshold

$$P_n^+|_{B,\mathrm{th}} = -\frac{1}{m_n^2}\frac{m_n^2+m^2}{m_n^2-m^2}\,. \tag{3.16}$$

Therefore the integral over the s-channel resonances at the right-hand side of our sum rule (3.3) is given by

$$\text{r.h.s.} = \sum_n P_n|_{F,\mathrm{th}} \operatorname*{res}_{s=m_n^2} A_F - \sum_n P_n|_{B,\mathrm{th}} \operatorname*{res}_{s=m_n^2} A_B \equiv \sum_n A_F^n - \sum_n A_B^n \equiv \sum_n A_{F-B}^n\,, \tag{3.17}$$

where we have denoted the forward and backward contributions of the resonance m_n as

$$A^n_{\substack{F\\B}} \equiv P_n|_{\substack{F,\mathrm{th}\\B}} \operatorname*{res}_{s=m_n^2} A_{\substack{F\\B}} \tag{3.18}$$

and their difference by

$$A^n_{F-B} \equiv A^n_F - A^n_B\,. \tag{3.19}$$

Here A stands again for any of the crossing-symmetric combinations of amplitudes.

c) The nucleon pole. The calculation of contributions of this form needs special care since it has poles and double poles right at threshold due to the presence of massless photons. Therefore we calculate our sum rule along a line of very small negative $t = -\varepsilon^2$ instead of the forward direction and equate with the backward dispersion integral at the point (*):

$$t = -\varepsilon^2\,, \qquad s = m^2 - \frac{t}{2} - \frac{1}{2}\sqrt{t(t-4m^2)}\,. \tag{3.20}$$

Consider for example

$$P^+ = \frac{1}{s-m^2} + \frac{1}{u-m^2}\,. \tag{3.21}$$

Dispersing at fixed $t = -\varepsilon^2$ we find at the point (*)

$$P^+|_{F,*} = -\frac{1}{m^2}\,. \tag{3.22}$$

In the backward direction, P^+ is identically equal to

$$P^+|_B \equiv \frac{1}{m^2} \tag{3.23}$$

and therefore does not contribute to an unsubtracted dispersion relation at all. Hence

$$P^+|_{B,*} = 0\,. \tag{3.24}$$

The odd pole term as well as the double pole contribution

$$P^- = \frac{1}{s-u}\left(\frac{1}{s-m^2} - \frac{1}{u-m^2}\right) \equiv \frac{1}{(s-m^2)(u-m^2)}\,,$$

on the other hand, yield due to the identity

$$\frac{1}{(s-m^2)(u-m^2)} \equiv -\frac{1}{t}\left(\frac{1}{s-m^2} + \frac{1}{u-m^2}\right) \tag{3.25}$$

a factor $1/\varepsilon^2$ in (3.22), *i.e.*

$$P^-|_{F,*} = -\frac{1}{\varepsilon^2 m^2}\,.$$

In the backward direction, P^- now has a pole at $t = 0$:

$$P^-|_B \equiv \frac{1}{tm^2}$$

giving in an unsubtracted dispersion relation at the point (*):

$$P^-|_{B,*} = -\frac{1}{\varepsilon^2 m^2}\,,$$

i.e. the same contribution as $P^-_{F,*}$. As a result, the contribution of the nucleon Born term to the right-hand side of our sum rule (3.3) is for the amplitudes A_i

$$A_i|_{B-F,\text{th}} = R_i^+ \frac{1}{m^2}\,, \qquad i = 1, 2, 4, 5,$$

$$\frac{1}{s-u} A_i|_{F-B,\text{th}} = 0\,, \qquad i = 3, 6\,.$$

If we multiply all amplitudes A_i by t we find

$$tA_i|_{F-B,\text{th}} = R_i^{su} \frac{1}{m^2}\,, \qquad i = 1, 2, 4, 5,$$

$$\frac{t}{s-u} A_{F-B,\text{th}}| = R_i^- \frac{1}{m^2}\,, \qquad i = 3, 6\,.$$

The explicit contributions are listed in Table III.

TABLE III. – *Contributions of the nucleon Born terms to the right-hand side of our forward-backward sum rules.*

		$A^s_{iF-B,\text{th}}$	$A^v_{iF-B,\text{th}}$	$tA^s_{iF-B,\text{th}}$	$tA^v_{iF-B,\text{th}}$
	1	$-\frac{1}{2m^3}(K_s^2+K_v^2+K_p)$	$-\frac{1}{m^3}(K_sK_v+K_p/2)$	$\frac{2}{m}$	$\frac{2}{m}$
	2	$\frac{1}{2m^4}(K_s^2+K_v^2+K_p)$	$\frac{1}{m^4}(K_sK_v+K_p/2)$	$\frac{2}{m^2}(1+K_p)$	$\frac{2}{m^2}(1+K_p)$
$\frac{1}{s-u}$	3	0	0	$\frac{1}{2m^3}(K_s^3+K_v^2+K_p)$	$\frac{1}{m^3}(K_sK_v+K_p/2)$
	4	$\frac{1}{2m^4}(K_s^2+K_v^2)$	$\frac{1}{m^4}K_sK_v$	$\frac{2}{m^2}(1+K_p)$	$\frac{2}{m^2}(1+K_p)$
	5	0	0	$-\frac{4K_p}{m^3}$	$-\frac{4}{m^3}K_p$
$\frac{1}{s-u}$	6	0	0	$-\frac{1}{m^4}(K_s^2+K_v^2)$	$-\frac{2}{m^4}K_sK_v$

It is amusing to note that nucleon and Roper poles do not contribute to the amplitudes (3.9). The amplitudes multiplied by t, on the other hand, receive a large nucleon contribution.

4. – The multipole couplings.

Let us prepare our input data for the baryon resonances. We have to determine the electromagnetic couplings G^+, G^- from the current multipole analyses of photoproduction of pions. For this purpose we first note that the helicity amplitudes of the photoproduction process $\mathcal{N}\pi \leftarrow \mathcal{N}\gamma$ (*) $\mathcal{F}_{\lambda'_{\mathcal{N}},\lambda_{\mathcal{N}}\lambda_\gamma}$ can be expanded in terms of amplitudes of total angular momentum according to

$$\mathcal{F}_{\lambda'_{\mathcal{N}},\lambda_{\mathcal{N}}\lambda_\gamma} = \frac{1}{2\sqrt{k^*q}}\sum_J (2J+1)\langle\lambda'_{\mathcal{N}}|F^J|\lambda_{\mathcal{N}}\,\lambda_\gamma\rangle\, d^J_{\lambda_{\mathcal{N}}-\lambda_\gamma,\lambda'_{\mathcal{N}}}(\theta)\,. \tag{4.1}$$

If one passes to states of definite parity $(-)^{J\pm\frac{1}{2}}$, *i.e.* on the photon side to

$$|JM;\tfrac{1}{2},1^\pm\rangle \equiv [|JM\tfrac{1}{2}1\rangle \mp |JM-\tfrac{1}{2}-1\rangle]/\sqrt{2}\,, \tag{4.2}$$

$$|JM;-\tfrac{1}{2},1^\pm\rangle \equiv [|JM-\tfrac{1}{2}1\rangle \mp |JM\tfrac{1}{2}-1\rangle]/\sqrt{2}\,, \tag{4.3}$$

(*) We use here the CGLN conventions, *i.e.* $d\sigma_{\text{c.m.}}/d\Omega = (q/k^*)|\mathcal{F}|^2$, where k^*, q are the c.m. momenta of the photon and pion, respectively. Thus with our definition of T: $S = 1-(2\pi)^4 i\delta^4(P_f-P_i)\,T$, $\mathcal{F} = -\sqrt{k^*/q}\,(m/4\pi W)\,T$.

and on the pion side to

$$|l^\pm M, \tfrac{1}{2}\rangle \equiv [|JM\tfrac{1}{2}\rangle \pm |JM-\tfrac{1}{2}\rangle]/\sqrt{2} \tag{4.4}$$

of angular momentum $J = l \pm \frac{1}{2}$ and parity $(-)^{l\pm1} = (-)^{J\pm\frac{1}{2}}$, we can form the combinations

$$\left\{\begin{aligned} F^{l\pm}_{\frac{1}{2}} &\equiv \langle l^\pm|F^J(W)|\tfrac{1}{2}1^\pm\rangle\,, \\ F^{l\pm}_{\frac{3}{2}} &\equiv \langle l^\pm|F^J(W)|-\tfrac{1}{2}, 1^\pm\rangle\,. \end{aligned}\right. \tag{4.5}$$

The CGLN multipole amplitudes are then related to these $F^{l\pm}$ by

$$\left\{\begin{aligned} 2\sqrt{k^*q}\,(l+1)\,M_{l+} &= \frac{1}{\sqrt{2}}\left[F^{l+}_{\frac{1}{2}} + \sqrt{\frac{l+2}{l}}\,F^{l+}_{\frac{3}{2}}\right], \\ 2\sqrt{k^*q}\,(l+1)\,E_{l+} &= \frac{1}{\sqrt{2}}\left[F^{l+}_{\frac{1}{2}} - \sqrt{\frac{l}{l+2}}\,F^{l+}_{\frac{3}{2}}\right], \\ 2\sqrt{k^*q}\,lM_{l-} &= \frac{1}{\sqrt{2}}\left[F^{l-}_{\frac{1}{2}} - \sqrt{\frac{l-1}{l+1}}\,F^{l-}_{\frac{3}{2}}\right], \\ 2\sqrt{k^*q}\,lE_{l-} &= -\frac{1}{\sqrt{2}}\left[F^{l-}_{\frac{1}{2}} + \sqrt{\frac{l+1}{l-1}}\,F^{l-}_{\frac{3}{2}}\right]. \end{aligned}\right. \tag{4.6}$$

The combinations of electric and magnetic multipoles appearing on the right-hand side are almost directly the amplitudes $A_{l\pm}$, $B_{l\pm}$ listed in the analysis of photoproduction of WALKER ([19]). We find

$$\left\{\begin{aligned} \frac{1}{2\sqrt{k^*q}}F^{l+}_{\frac{1}{2}} &= \sqrt{2}\,A_{l+}\,, \\ \frac{1}{2\sqrt{k^*q}}F^{(l+1)-}_{\frac{1}{2}} &= \sqrt{2}\,A_{(l+1)-}\,, \\ \frac{1}{2\sqrt{k^*q}}F^{l+}_{\frac{3}{2}} &= -\frac{1}{\sqrt{2}}\sqrt{l(l+2)}\,B_{l+}\,, \\ \frac{1}{2\sqrt{k^*q}}F^{(l+1)-}_{\frac{3}{2}} &= -\frac{1}{\sqrt{2}}\sqrt{l(l+2)}\,B_{(l+1)-}\,. \end{aligned}\right. \tag{4.7}$$

On the other hand, close to a resonance m_n, $F^{l\pm}$ can be expressed in terms of the collinear matrix elements of the pionic current between nucleon m and

([19]) R. L. WALKER: *Phys. Rev.*, **182**, 1729 (1969).

resonance m_n, both running in the z-direction with helicity λ,

$$j^{\pi_i}_{mm_n}(\tfrac{1}{2}) = \langle p'\, m\, \tfrac{1}{2}\, \tfrac{1}{2} | j^i_\pi(0) | p m_n J\, \tfrac{1}{2}\rangle$$

and the vertices Γ introduced before. One finds for $W \approx m_n$

$$(4.8)\quad \begin{cases} -\dfrac{4\pi W}{m}(2J+1)\dfrac{1}{2\sqrt{k^* q}} F^{l\pm}_{\frac{1}{2}} \simeq \dfrac{e}{W - m_n + i(\Gamma_n/2)}\, j_{mm_n}(\tfrac{1}{2})\, 2G^-, \\[2ex] -\dfrac{4\pi W}{m}(2J+1)\dfrac{1}{2\sqrt{k^* q}} F^{l\pm}_{\frac{3}{2}} \simeq \dfrac{e}{W - m_n + i(\Gamma_n/2)}\, j_{mm_n}(\tfrac{1}{2})(\mp 2G^+)\,. \end{cases}$$

Comparing with (4.7) we obtain at the resonance position the simple relation (*)

$$\operatorname{Im} A_{\substack{l+\\(l+1)-}}(m_n) = \frac{e}{\sqrt{2}}\,\frac{m}{4\pi m_n}\,\frac{1}{2J+1}\,\frac{2}{\Gamma_n}\, j_{mm_n}(\tfrac{1}{2})\, 2G^-,$$

$$\operatorname{Im} B_{\substack{l+\\(l+1-)}}(m_n) = e\,\frac{\sqrt{2}}{\sqrt{l(l+2)}}\,\frac{m}{4\pi m_n}\,\frac{1}{2J+1}\,\frac{2}{\Gamma_n}\, j_{mm_n}(\tfrac{1}{2})(-n 2G^+)\,.$$

Up to a phase η, the pionic coupling $j_{mm_n}(\frac{1}{2})$ can be expressed in terms of the partial width $\Gamma^{\pi\mathcal{N}}$. Thus

$$j^{\pi_i}_{mm_n}(\tfrac{1}{2}) = n\,\mathrm{CG}\sqrt{\frac{\pi m_n}{qm}(2J+1)}\,\sqrt{\Gamma_{\pi\mathcal{N}}}\,,$$

where CG denotes the Clebsch-Gordan coefficient for the decay channel under consideration. Hence we find

$$\operatorname{Im} A_{\substack{l+\\(l+1)-}}(m_n) = \eta\,\mathrm{CG}\,\frac{\sqrt{2}}{\varkappa}\, eG^-,$$

$$\operatorname{Im} B_{\substack{l+\\(l+1)-}}(m_n) = \eta\,\mathrm{CG}\,\frac{2\sqrt{2}}{\sqrt{l(l+2)}\,\varkappa}(-enG^-)\,,$$

where $\varkappa$ stands for (*)

$$\varkappa = \sqrt{4\pi\frac{m_n}{m}\,q(2J+1)\Gamma}\Big/\sqrt{\frac{\Gamma_{\pi\mathcal{N}}}{\Gamma}}\,.$$

Our input values of $A_{l\pm}$, $B_{l\pm}$ are taken from Walker's original analysis [19]. For amplitudes which have not been determined by WALKER we use the values

(*) We have cross-checked our calculation by going through the elastic unitarity relation of HEARN and LEADER [17] (their eqs. (4.14)-(4.18)) and inserting $E_{l\pm}$, $M_{l\pm}$ according to the formulae (4.8). We find agreement except that the last of their equations (4.18) needs a sign change. Due to our narrow-resonance approximation, our formulae carry a correction factor $\Gamma/\Gamma_{\pi\mathcal{N}}$ with respect to their elastic unitarity.

found by TAKASAKI [20] in his fits of pion production in the backward direction. In addition there exists a recent fit of photoproduction of neutrons [21]. These authors find a large coupling of the neutral Roper resonance. Since there is independent evidence for such a large coupling [13,22] we prefer to use this value rather than the small Walker value. Our choice of input is listed in Table IV, together with the resulting values of $G^{\pm}$ (up to the unknown phase factor η).

TABLE IV. – *Multipoles of s-channel resonances used for the evaluation of s-channel contributions to* F-B *sum rules.* The author marks W, T and C refer to the analyses of WALKER [19], TAKASAHI [20] and CARBONARA *et al.* [21], respectively.

Reso-nance	J^P	I	Mass (MeV)	Width (MeV) total	Width (MeV) elastic	Multipole amplitude	$\gamma p \to \pi^+ n$	$\gamma n \to \pi^- p$
P_{33}	$\frac{3}{2}^+$	$\frac{3}{2}$	1236	120	120	A_{1+}	1.00 W	1.00 W
						B_{1+}	−2.43 W	−2.43 W
P_{11}	$\frac{1}{2}^+$	$\frac{1}{2}$	1471	204	122	A_1	−0.25 W	−1.06 C
D_{13}	$\frac{3}{2}^-$	$\frac{1}{2}$	1520	102	73	A_{2-}	−0.20 W	−0.25 T
						B_{2-}	−1.32 W	−1.15 W
S_{11}	$\frac{1}{2}^-$	$\frac{1}{2}$	1561	180	72	A_{0+}	−0.65 W	−0.80 W
S_{31}	$\frac{1}{2}^-$	$\frac{3}{2}$	1630	160	43	A_{0+}	−0.45 T	−0.32 T
D_{15}	$\frac{5}{2}^-$	$\frac{1}{2}$	1652	134	54	A_{3-}	0	0
						B_{2+}	0.141 W	0.141 W
F_{15}	$\frac{5}{2}^+$	$\frac{1}{2}$	1672	104	69	A_{3-}	−0.20 T	−0.22 T
						B_{3-}	−0.60 W	−0.50 W

5. – Quantitative discussion of the sum rules.

We are now ready to present the final form of our forward-backward sum rules. The nucleon contributions are given in Table III. For the contributions of the s-channel resonances we take the multipole moments shown in Table IV and insert them into the formulae for the residues of the amplitudes A_i of eqs. (2.30) and (2.31). These residues are combined with the factors P^n of Sect. **3** and summed up in eq. (3.17). The resulting contributions are listed for forward and backward directions and their difference, separately, in Tables V and VI. In these Tables we have used the nucleon mass as a unit mass.

(20) F. TAKASAKI: Tokyo preprint INS-J-126 (January 1971).

(21) F. CARBONARA, L. FIORE, G. GIALANELLA, E. LODI-RIZZINI, G. MANTOVANI, M. NAPOLITANO, A. PIAZZA, A. PIAZZOLI, R. RINZIVILLO, V. ROSSI, G. SUSINNO and L. VOTANO: *Lett. Nuovo Cimento*, **3**, 697 (1970).

(22) A. PROIA and F. SEBASTIANI: *Lett. Nuovo Cimento*, **2**, 560 (1971); **3**, 483 (1970); Università di Roma, preprint No. 363.

TABLE V. – *The contributions of the baryon resonances to forward-backward sum rules for the scalar*

Amplitude	Asymptotic behaviour F	Asymptotic behaviour B		N		R
$\frac{1}{s-u}A_6^s$	s^{-2}	$t^{-1.3}$	F		10.8223	−0.404
			B		10.8798	−0.404
			$F-B$	0	− 0.0575	0
A_5^s	s^{-1}	$t^{-1.3}$	F		9.4523	0
			B		7.4481	0
			$F-B$	0	2.0042	0
$\frac{1}{s-u}A_3^s$	s^{-1}	$t^{-1.3}$	F		− 4.1592	0.317
			B		− 4.4513	0.317
			$F-B$	0	0.2922	0
$A_4^s+\frac{t}{2(s-u)}A_6^s$	s^{-1}	$t^{-1.3}$	F		7.8908	0.590
			B		4.4856	0.590
			$F-B$	0	3.4052	0
A_4^s	s^{-1}	$t^{-0.3}$	F		7.8908	0.590
			B		6.1843	0.414
			$F-B$	1.71	1.7065	0.175
$t\frac{A_6^s}{s-u}$	0	$t^{-0.3}$	F		0	0
			B	3.42	− 3.3973	0.350
			$F-B$	−3.42	3.3973	−0.350
tA_5^s	0	$t^{-0.3}$	F	0	0	0
			B	7.16	− 2.3257	0
			$F-B$	−7.16	2.3257	0
$t\frac{A_3^s}{s-u}$	0	$t^{-0.3}$	F	0	0	0
			B	−2.61	1.3900	−0.274
			$F-B$	2.61	− 1.3900	−0.274
$t(A_1^s+mA_2^s)$	0	$t^{-0.3}$	F	0	0	0
			B	−7.78	11.8161	0
			$F-B$	7.78	−11.8161	0

amplitudes. For comparison the forward and backward contributions are listed separately.

520	1561	1630	1652	1672	Total ($m_p = 1$)
0.2337	−0.2844	−0.1509	0.0037	0.0198	
0.4682	−0.2844	−0.1509	−0.0197	−0.1940	
-0.2345	0	0	−0.0234	0.2138	−0.0548
-3.0978	0	0	−0.0295	−0.6268	
-2.1388	0	0	+0.0791	0.9405	
-0.9590	0	0	−0.1086	−1.5673	−0.6307
0.0177	−0.23665	−0.1311	0.0000	0.1268	
-0.3666	−0.23665	−0.1311	−0.0033	0.08916	
0.3843	0	0	0.0033	−0.0765	0.6033
1.9643	0.5032	0.3048	0.0227	0.3771	
0.8572	0.5032	0.3048	−0.0394	0.3288	
1.1071	0	0.0000	0.0621	0.7058	5.2802
1.9643	0.5032	0.3048	0.0227	0.3771	
1.0929	0.3424	0.2029	−0.0535	−0.4734	
0.8714	0.1608	0.1019	0.0762	0.8505	5.6523
0	0	0	0	0	
0.4713	0.3215	0.2039	+0.0281	0.2895	
0.4713	−0.3215	−0.2039	−0.0281	−0.2895	−0.7445
0	0	0	0	0	
2.1531	0	0	−0.1122	−1.4033	
−2.1531	0	0	0.1122	1.4033	−5.4719
0	0	0	0	0	
0.3691	0.2675	0.1772	0.0047	−0.1330	
−0.3691	−0.2675	−0.1772	−0.0047	0.1330	0.8090
0	0	0	0	0	
−3.5362	0	0	−0.0767	0.9214	
3.5362	0	0	0.0767	−0.9214	−1.3446

Numerically the sum rules become for the isoscalar amplitudes:

1) ★★★ $\frac{A_6^s}{s-u}$, $0=-0.06$,

2) ★★ A_5^s, $0.40\,g_{f\gamma\gamma}^{(2)}G_{f\mathcal{N}\mathcal{N}}^{(2)}=-0.63$,

3) ★★ $\frac{A_3^s}{s-u}$, $0.05\,g_{f\gamma\gamma}^{(1-2)}G_{f\mathcal{N}\mathcal{N}}^{(1)}=0.60$,

4) ★★ $A_4^s+\frac{t}{2(s-u)}A_6^s$, $0.20\,g_{f\gamma\gamma}^{(2)}G_{f\mathcal{N}\mathcal{N}}^{(1)}=5.3$,

5) ★ A_4^s, $0.20\,g_{f\gamma\gamma}^{(2)}G_{f\mathcal{N}\mathcal{N}}^{(1)}=5.6$,

6) ★ $t\frac{A_6^s}{s-u}$, $0=-0.74$,

7) ★ tA_5^s, $0.74\,g_{f\gamma\gamma}^{(2)}G_{f\mathcal{N}\mathcal{N}}^{(2)}=-5.5$,

8) ★ $t\frac{A_3^s}{s-u}$, $0.09\,g_{f\gamma\gamma}^{(1-2)}G_{f\mathcal{N}\mathcal{N}}^{(1)}=+0.81$,

9) ★ $t(A_1+mA_2)^s$, $0.13\,g_{f\gamma\gamma}^{(1-2)}G_{f\mathcal{N}\mathcal{N}}^{(1+2)}-0.26\,g_{f\gamma\gamma}^{(1-2)}G_{f\mathcal{N}\mathcal{N}}^{(1)}-$

$$-2g_{\eta\mathcal{N}\mathcal{N}}g_{\eta\gamma\gamma}\frac{m}{m_\eta}-4\frac{m}{m_\sigma}g_{\sigma\gamma\gamma}g_{\sigma\mathcal{N}\mathcal{N}}=-1.34\,.$$

Sum rule 1) is excellently fulfilled but empty.

Sum rules 2) and 4) together give for the ratio

$$G_{f\mathcal{N}\mathcal{N}}^{(2)}/G_{f\mathcal{N}\mathcal{N}}^{(1)}=-0.059\,, \tag{5.1}$$

which is in excellent agreement with the prediction of f-meson dominance of the gravitational tensor, and certainly also with (1.8).

Sum rule 4) by itself says

$$g_{f\gamma\gamma}^{(2)}G_{f\mathcal{N}\mathcal{N}}^{(1)}\approx 27\,. \tag{5.2}$$

Combining this with the estimate (1.8) of $|G_{f\mathcal{N}\mathcal{N}}^{(1)}|\approx 25\pm 3$ we obtain the result

$$g_{f\gamma\gamma}^{(2)}\approx 1.1\,, \tag{5.3}$$

which agrees very well with the calculation from finite-energy sum rules in pion Compton scattering (eq. (1.10)).

The third sum rule, on the other hand, gives

$$g_{f\gamma\gamma}^{(1-2)}G_{f\mathcal{N}\mathcal{N}}^{(1)}\approx 12 \tag{5.4}$$

such that

$$g_{f\gamma\gamma}^{(1-2)} \approx \tfrac{1}{2}\,. \tag{5.5}$$

This does not agree completely with the value (1.2) obtained from f dominance. However, $g_{f\gamma\gamma}^{(1-2)}$ has only one third of the size of the estimates (1.10) and (1.2) for $g_{f\gamma\gamma}^{(2)}$, so the qualitative prediction that the f-meson couples predominantly to the s-channel helicity-nonflip amplitude is confirmed.

Our flip-to-nonflip ratio is

$$g_{f\gamma\gamma}^{(1-2)}/g_{f\gamma\gamma}^{(2)} \approx 0.5/1.1\,. \tag{5.6}$$

The sum rule 5) for A_4 gives the same information as that we just obtained since the next sum rule 6) is in moderate agreement (though void of information) (*) and 4) is just a combination of 5) and 6) deserving a higher degree of confidence. The rule 7) for tA_5 yields

$$g_{f\gamma\gamma}^{(2)}\, G_{f\mathcal{N}\mathcal{N}}^{(2)} = -7.55\,, \tag{5.7}$$

which agrees in sign but not in magnitude with sum rule 2). From our Table V we see that the different resonances contribute large terms of alternating signs. This is due to the fact that the factor t increases the weight of higher resonances in such a way that the sum can be made to converge only by changing signs. In the meson channel the weaker of the two coupling constants of the f to nucleons, $G_{f\mathcal{N}\mathcal{N}}^{(2)}$, enters. So probably f will not dominate significantly, the t-channel integral with a weight factor t stressing too much the higher-mass region. For this reason we shall ignore the result (5.4).

Finally the sum rule 9) for $t(A_1 + mA_2)$ can be simplified somewhat if we use the fact

$$G_{f\mathcal{N}\mathcal{N}}^{(1+2)} \approx G_{f\mathcal{N}\mathcal{N}}^{(1)}$$

suggested by (5.1).

Then

$$-2g_{\eta\mathcal{N}\mathcal{N}}\, g_{\eta\gamma\gamma}\, \frac{m}{m_\eta} - 4\,\frac{m}{m_\sigma}\, g_{\sigma\gamma\gamma}\, g_{\sigma\mathcal{N}\mathcal{N}} \approx -1.47 + 0.13\, g_{f\gamma\gamma}^{(1-2)}\, G_{f\mathcal{N}\mathcal{N}}^{(1+2)}\,. \tag{5.8}$$

Let us see what this relation implies for the radiative coupling of σ. If we stick to our own prediction (5.5) and insert $g_{f\gamma\gamma}^{(1-2)} G_{f\mathcal{N}\mathcal{N}}^{(1+2)} \approx 12$; the right-hand side becomes approximately zero. In this case

$$-2g_{\eta\mathcal{N}\mathcal{N}}\, g_{\eta\gamma\gamma}\, \frac{m}{m_\eta} \approx 4\,\frac{m}{m_\sigma}\, g_{\sigma\gamma\gamma}\, g_{\sigma\mathcal{N}\mathcal{N}}\,. \tag{5.9}$$

(*) See, however, our discussion at the end of this Section.

TABLE VI. – *The contributions of the baryon resonances to forward-backward sum rules for*

Amplitude	Asympt. beh.			N		R
	F	B				
$\frac{1}{s-u}A_6^v$	$s^{-2.5}$	$t^{-1.3}$	F		0	0.361
			B		0	0.361
			$F-B$	0	0	0
A_5^v	$s^{-1.5}$	$t^{-1.3}$	F		0	0
			B		0	0
			$F-B$	0	0	0
$\frac{1}{s-u}A_3^v$	$s^{-1.5}$	$t^{-1.3}$	F		0	-0.283
			B		0	-0.283
			$F-B$	0	0	0
$A_4^v+\frac{t}{2(s-u)}A_6^v$	$s^{-1.5}$	$t^{-1.3}$	F		0	-0.527
			B		0	-0.527
			$F-B$	0	0	0
A_4^v	$s^{-1.5}$	$t^{-0.3}$	F		0	-0.527
			B		0	-0.371
			$F-B$	-0.111	0	-0.156
$t\frac{A_6^v}{s-u}$	0	$t^{-0.3}$	F		0	0
			B		0	-0.313
			$F-B$	0.222	0	-0.313
tA_5^v	0	$t^{-0.3}$	F		0	0
			B		0	0
			$F-B$	-7.16	0	0
$\frac{tA_3^v}{s-u}$	0	$t^{-0.3}$	F		0	0
			B		0	0.245
			$F-B$	5.58	0	-0.245
$t(A_1^v+mA_2^v)$	0	$t^{-0.3}$	F		0	0
			B		0	0
			$F-B$	7.58	0	0

sovector amplitudes.

520	1561	1630	1652	1672	Total ($m_p = 1$)
0.0359	0.0582	—0.0495	0	0.0049	
0.0368	0.0582	—0.0495	0	0.0241	
0.0009	0	0	0	0.0290	0.0281
-0.2194	0	0	0	—0.0541	
-0.1515	0	0	0	0.1115	
-0.0679	0	0	0	—0.1656	—0.233 5
-0.0039	0.0484	—0.0430	0	—0.0012	
-0.0184	0.0484	—0.0430	0	—0.0063	
0.0145	0	0	0	—0.0075	0.0070
0.1602	—0.1029	0.1001	0	0.0357	
0.0910	—0.1029	0.1001	0	0.0539	
0.0693	0	0	0	0.0896	0.1589
0.1602	—0.1029	0.1001	0	0.0357	
0.1096	—0.0700	0.0666	0	—0.0719	
0.0507	—0.0329	0.0335	0	0.1076	—0.1087
0	0	0	0	0	
-0.0371	—0.0658	0.0669	0	0.0360	
-0.0371	0.0658	—0.0669	0	—0.0360	0.5352
0	0	0	0	0	
0.1525	0	0	0	—0.1663	
-0.1525	0	0	0	0.1663	—7.1462
0	0	0	0	0	
0.0185	—0.0548	0.0582	0	—0.0094	
-0.0185	0.0548	—0.0582	0	0.0094	5.3219
0	0	0	0	0	
-0.3691	0	0	0	0.1146	
0.3691	0	0	0	—0.1146	7.8345

The value of $g_{\eta\gamma\gamma}$ is known quite well from the Primakoff production of η-mesons: $|g_{\eta\gamma\gamma}(m/m_\eta)| \approx 0.36$ (see Appendix A). The $\eta\mathcal{N}\mathcal{N}$ coupling, on the other hand, could be close to the SU_3 value ([23])

$$g_{\eta\mathcal{N}\mathcal{N}} = -\frac{1}{\sqrt{3}}\left(1-4\,\frac{F}{F+D}\right) g_{\pi\mathcal{N}\mathcal{N}} = \frac{\sqrt{3}}{5}\, g_{\pi\mathcal{N}\mathcal{N}} \qquad \text{for } \frac{F}{D} = \frac{2}{3}, \tag{5.10}$$

but recent determinations ([24]) find also larger values of

$$g_{\eta\mathcal{N}\mathcal{N}} \approx 10\,. \tag{5.11}$$

If we assume the value for $g_{\sigma\mathcal{N}\mathcal{N}} \approx -13$ as suggested by (1.7) and the σ-model we can estimate

$$|g_{\sigma\gamma\gamma}| \approx \begin{cases} 0.05\,, \\ 0.1\,. \end{cases} \tag{5.12}$$

Hence $g_{\sigma\gamma\gamma}$ is small in both cases, much smaller than the estimate (1.9) from pion Compton scattering and verifying the result (1.1) of Ward identities. If we had over-estimated $g^{(1-2)}_{f\gamma\gamma}$ in our result (5.6), this would not change the picture much. It would increase the first value roughly by 50 % and the second value by 25 %, *i.e.* $g_{\sigma\gamma\gamma}$ remains very small.

For the isovector amplitudes the sum rules we obtain are

1) ★★★ $\dfrac{A_6^v}{s-u}$, $\quad 0 = 0.028\,,$

2) ★★ A_5^v, $\quad 0.37\, g^{(2)}_{A_2\gamma\gamma} G^{(2)}_{A_2\mathcal{N}\mathcal{N}} = -0.234\,,$

3) ★★ $\dfrac{A_3^v}{s-u}$, $\quad 0.046\, g^{(1-2)}_{A_2\gamma\gamma} G^{(1)}_{A_2\mathcal{N}\mathcal{N}} = 0.007\,,$

4) ★★ $A_4^v + \dfrac{t}{2(s-u)}A_6^v$, $\quad 0.184\, g^{(2)}_{A_2\gamma\gamma} G^{(1)}_{A_2\mathcal{N}\mathcal{N}} = 0.16\,,$

5) ★ A_4^v, $\quad 0.184\, g^{(2)}_{A_2\gamma\gamma} G^{(1)}_{A_2\mathcal{N}\mathcal{N}} = -0.11\,,$

6) ★ $\dfrac{tA_6^v}{s-u}$, $\quad 0 = 0.54\,,$

7) ★ tA_5^v, $\quad 0.72\, g^{(2)}_{A_2\gamma\gamma} G^{(2)}_{A_2\mathcal{N}\mathcal{N}} = -7.1\,,$

8) ★ $t\,\dfrac{A_3^v}{s-u}$, $\quad 0.09\, g^{(1-2)}_{A_2\gamma\gamma} G^{(1)}_{A_2\mathcal{N}\mathcal{N}} = 5.3\,,$

9) ★ $t(A_1 + mA_2)^v$, $\quad 0.122\, g^{(1-2)}_{A_2\gamma\gamma} G^{(1+2)}_{A_2\mathcal{N}\mathcal{N}} - 0.242\, g^{(1-2)}_{A_2\gamma\gamma} G^{(1)}_{A_2\gamma\gamma} - 2\, g_{\pi\mathcal{N}\mathcal{N}}\, g_{\pi\gamma\gamma}\dfrac{m}{m_\pi} = 7.84\,.$

Again, 1) is well fulfilled but empty.

([23]) A. E. S. GREEN and T. SAWADA: *Nucl. Phys.*, **2** B, 267 (1967).
([24]) G. SCHIERHOLZ: *Nucl. Phys.*, **7** B, 483 (1968).

As in the case of the f-meson we can combine sum rules 2) and 4) to obtain the ratio

$$\frac{G^{(1)}_{A_2\mathcal{N}\mathcal{N}}}{G^{(2)}_{A_2\mathcal{N}\mathcal{N}}} \approx -1.3 \,. \tag{5.13}$$

Hence the nonflip-to-flip ratio is

$$G^{(1+2)}_{A_2\mathcal{N}\mathcal{N}}/G^{(2)}_{A_2\mathcal{N}\mathcal{N}} \approx -0.3 \,. \tag{5.14}$$

The numerical value of this ratio is not exactly the same as the one found in photoproduction (10) (≈ 0.1). However, there the saturation is done only with a nucleon and the Δ-resonance and does not deserve as much credibility as the present estimate. Both estimates agree, however, in predicting a dominant s-channel helicity-flip of A_2 exchange at the nucleon vertex.

Sum rule 3) can be used together with the estimate

$$|G^{(1)}_{A_2\mathcal{N}\mathcal{N}}| \approx 6 \tag{5.15}$$

obtained from photoproduction (10) to predict a small s-channel helicity-flip coupling of the A_2 to photons:

$$|g^{(1-2)}_{A_2\gamma\gamma}| \approx 0.025 \,. \tag{5.16}$$

The helicity-nonflip part of $A_2\gamma\gamma$, however, is found from 2) to be larger:

$$|g^{(2)}_{A_2\gamma\gamma}| \approx 0.1 \,. \tag{5.17}$$

Thus A_2 couples only weakly to photons and conserves s-channel helicity (*).

The following one-star sum rules 5), 7) and 8) are all found to give results different from the three- and two-star rules. The reason is, apparently, that the A_2-meson couples too weakly to photon and nucleon vertices (much weaker than f) to allow it to dominate the t-channel contributions in only barely convergent dispersion integrals. The situation is expected to be different only for the last sum rule, where a low-lying pion pole can balance the baryon resonances. Looking at (5.15) and (5.16) we see that the A_2 contribution in sum rule 9) can be neglected and we remain with

$$-2g_{\pi\mathcal{N}\mathcal{N}}\,g_{\pi\gamma\gamma}\frac{m}{m_\pi} = 7.8 \,. \tag{5.18}$$

(*) Using smoothness along the Regge trajectory and vector-meson dominance assumptions one is led to predict s-channel helicity conservation of the p-n photoproduction of ρ^0-mesons. Experimentalists should test this result.

Inserting $g_{\pi\mathcal{N}\mathcal{N}} = 13.5$ we obtain

$$g_{\pi\gamma\gamma} = -0.041 \,, \tag{5.19}$$

which has the correct value and sign (experimental value $= -0.037$) (*). Notice that the nucleon by itself makes up almost all of the s-channel contribution:

$$-2 g_{\pi\mathcal{N}\mathcal{N}} g_{\pi\gamma\gamma} \frac{m}{m_\pi} = 2(2 + K_p) = 7.6 \,.$$

This result is reminiscent of a calculation done by PAGELS [27], also for Compton scattering, but under completely different assumptions. PAGELS finds, for example,

$$-2 g_{\pi\mathcal{N}\mathcal{N}} g_{\pi\gamma\gamma} \frac{m}{m_\pi} \approx (\mu_p^2 - \mu_n^2 - 1) \approx 3.15 \,,$$

which agrees in sign but is off in magnitude by a factor of two.

At this point we should also like to mention a superconvergence type of calculation done by other authors for the Compton amplitude along the line $u = 0$ for certain combinations of helicity amplitudes [28]. There one finds

$$-g_{\pi\gamma\gamma} = 0.039 + 0.007 \,,$$

where the first number comes from the nucleon and the second from continuum corrections.

From the standpoint of determining the meson coupling constants in the t-channel from the values of the electromagnetic multipoles of the baryons in the s-channel we have called our sum rules 1) and 6) empty. They represent, however, quite welcome consistency tests of our input data. Particularly interesting are the isoscalar amplitudes. They receive very little contribution from higher resonances. If we assume a saturation in terms of $\mathcal{N}$ and Δ reso-

---

(*) See ref. [25] for a test of the sign. Notice that the coupling constant $F = g_{\pi\gamma\gamma}/m_\pi$ of ADLER has the same sign as ours. Therefore the Steinberger [26] result of the triangle graph with a single nucleon loop of isospin $\frac{1}{2}$ is $g_{\pi\gamma\gamma} = -(1/4\pi^2)(m_\pi/m_\mathcal{N})(g_{\pi\mathcal{N}\mathcal{N}}/g_A) \approx \approx -0.04$ in rough agreement with experiment (see Appendix A,3)). In reading Gilman's paper on the sign of F one has to be careful since he defines his F opposite to Adler's. From his relation $F = -(1/2\pi^2)(g_{\pi\mathcal{N}\mathcal{N}}/m_\mathcal{N} g_A)$ we see, however, that he also uses $g_{\pi\mathcal{N}\mathcal{N}}$ opposite to Adler's (see Appendix A,3) for this definition).

([25]) F. GILMAN: SLAC PUB-591 (1969) and to be published (1970) (see also S. OKUBO: *Phys. Rev.*, **179**, 1829 (1969)).
([26]) J. STEINBERGER: *Phys. Rev.*, **76**, 1180 (1969).
([27]) H. PAGELS: *Phys. Rev.*, **158**, 1566 (1967).
([28]) S. R. CHOUDHURY and R. RAJARAMAN: *Phys. Rev.*, **169**, 1218 (1968).

nances only, we are able to express the coupling completely in terms of the nucleon magnetic moments. In this way we find from $tA_6^s/(s-u)$ that the E_{1+}/M_{1+} ratio of the electromagnetic coupling of the Δ-resonance is very small:

$$E_{1+}/M_{1+} \approx -6\,\% \, .$$

The smallness of E_{1+}/M_{1+} stems from an extreme sensitivity of the Δ contribution to E_{1+}/M_{1+}, which is shown in detail in Appendix B. From $A_6^s/(s-u)$ one finds that the magnetic moment of the Δ-resonance has to be [29]

$$G_M \approx 3.5$$

in order to balance the nucleon contribution, which is in excellent agreement with experiment [30].

In this context it is worth-while to recall that A_6 also allows for superconvergence relation in the forward direction which provides for a similar consistency check of the electromagnetic couplings. Historically, this relation was first derived by DRELL and HEARN [31] as a sum rule expressing the anomalous magnetic moment of the nucleon in terms of a sum over resonance multipoles. In this case this sum can also be written as an integral over the photoabsorption cross-section (*):

$$\frac{2\pi^2\alpha}{m^2}K^2 = \int_0^\infty \frac{d\nu}{\nu}\,[\sigma(\nu, 1) - \sigma(\nu, -1)] \, .$$

Here $\sigma(\nu, 1)$, $\sigma(\nu, -1)$ are the cross-sections for absorbing a photon of helicity $+$ or -1 on a nucleon of helicity $\frac{1}{2}$.

In the narrow-resonance approximation our input data yield the anomalous magnetic moments

$$K_n \approx 1.9 \, , \qquad K_p \approx 2.2 \, .$$

It is interesting that the result for the neutron is better than that for the proton, which is wrong by 20 %. Since the Roper resonance contributes consid-

---

([29]) W. W. ASH, K. BERKELMAN, C. A. LICHTENSTEIN, A. RAMANAUSKAS and R. H. SIEMANN: *Phys. Lett.*, **24** B, 165 (1967).

([30]) *Compilation of coupling constants and low energy parameters*, in *Springer Tracts in Modern Physics*, Vol. **55** (1970).

([31]) S. D. DRELL and A. HEARN: *Phys. Rev. Lett.*, **16**, 908 (1966).

(*) Since $\frac{1}{2}(s-m^2)^2A_6$ is equal to the difference of the helicity amplitudes $f_{\frac{1}{2},1;\frac{1}{2},1} - f_{\frac{1}{2},-1,\frac{1}{2},-1}$.

erably in the neutron case we consider the good result for K_n as an indication for the correctness of our large input value of

$$M_{1-}^{\gamma n\to\pi^- p} \approx -1.1\sqrt{\mu b}\,.$$

6. – Conclusion.

By a systematic application of forward-backward dispersion relation we have been able to obtain many estimates on the coupling constants of the important mesons exchanged in the t-channel of Compton scattering. These estimates agree, to a large extent, with theoretical expectations.

The predictions for the couplings of $f\gamma\gamma$ and $f\mathcal{N}\mathcal{N}$ show roughly the properties predicted from f dominance of gravity, in particular s-channel helicity conservation. The vertex $A_2\mathcal{N}\mathcal{N}$ flies predominantly the nucleon helicity in accordance with exchange degeneracy of A_2 and meson. The $A_2\gamma\gamma$ vertex, on the other hand, is predicted to conserve s-channel helicity. (Also it is found to be very small.) The radiative decay of $\sigma\gamma\gamma$ comes out extremely small in agreement with σ dominance of the trace of the energy-momentum tensor.

Our calculation has been done with a sharp-resonance approximation. From this we expect errors of at least 10 %. The errors coming from the uncertainty of the multipole amplitudes are probably larger than that. In addition, the assumption of meson dominance of the t-channel contributions will bring along errors of $(10\div20)$ %, even for the best sum rules. As a consequence, the exact numerical values should not be taken too seriously. What we believe in is roughly the size of our results with, say, 50 % errors possible in the worst cases. If the meson contributions are large and the higher resonances add little, the error bars are expected to be somewhat smaller. An example is the $\pi\gamma\gamma$ coupling which agrees extremely well with experiment.

We are looking forward to a thorough phase-shift analysis of Compton scattering which can be inserted in our dispersion relation in order to obtain better estimates on the contribution of the baryon resonances. On the t-channel side, however, an improvement will be very hard since the background is difficult to determine. We know from the same kind of analysis of $\pi\mathcal{N}$ scattering that the intrinsic uncertainty coming from the meson side of the sum rule is the major problem in the application of this method.

* * *

One of the authors (T.H.C.) wishes to thank the Deutsche Forschungsgemeinschaft for generous support and the Institut für Theoretische Physik der Freien Universität Berlin for its warm hospitality.

APPENDIX A

Couplings and widths.

1) σ: $$\mathscr{L} = g_{\sigma\mathscr{N}\mathscr{N}}\sigma\bar{N}N + g_{\sigma\gamma\gamma}\frac{e^2}{2m_\sigma}F_{\mu\nu}F^{\mu\nu}.$$

The σ-model predicts $g_{\sigma\mathscr{N}\mathscr{N}} = -g_{\pi\mathscr{N}\mathscr{N}} = -m/f_\pi \approx -10$.

On the other hand, σ dominance of θ gives $g_{\sigma\mathscr{N}\mathscr{N}} \approx (m/m_\sigma)g_{\sigma\pi\pi}$ (which becomes in the σ-model again $-m/f_\pi$, since there $g_{\sigma\pi\pi} = -(m/f_\pi)(1-\mu^2/m_\sigma^2) \approx \approx -m_\sigma/f_\pi$). Using the experimental width $\Gamma_{\sigma\pi\pi} \approx 400$ MeV (with mass ≈ 700) and accepting the sign of the σ-model we have $g_{\sigma\pi\pi} \approx -5.5$. Then the determination (1.7) from $\pi\mathscr{N}$ backward scattering amounts to $g_{\sigma\mathscr{N}\mathscr{N}} \approx -13$, in rough agreement with the first part of the σ-model prediction.

The radiative coupling corresponds to a width of

$$\Gamma_{\sigma\gamma\gamma} = \frac{m_\sigma}{4}e^4\frac{g_{\sigma\gamma\gamma}}{4\pi} \approx 0.11 g_{\sigma\gamma\gamma}^2 \text{ keV}.$$

While σ dominance of θ predicts $\Gamma_{\sigma\gamma\gamma} = 0$, the finite-energy sum rule estimate gives $g_{\sigma\gamma\gamma} \approx 0.45$ corresponding to $\Gamma_{\sigma\gamma\gamma} \approx 22$ keV.

2) η: $$\mathscr{L} = g_{\eta\mathscr{N}\mathscr{N}}\eta\bar{N}N + g_{\eta\gamma\gamma}\frac{e^2}{2m_\eta}\varepsilon_{\alpha\beta\gamma\delta}\,\partial^\alpha A^\beta\,\partial^\gamma A^\delta, \qquad \varepsilon_{0123} = 1.$$

The $\eta\mathscr{N}\mathscr{N}$ coupling is unknown. SU_3 gives the relation

$$g_{\eta\mathscr{N}\mathscr{N}} = -\frac{1}{\sqrt{3}}\left(1 - 4\frac{F}{F+D}\right)g_{\pi\mathscr{N}\mathscr{N}}.$$

Assuming the F/D ratio $\frac{2}{3}$ one finds the rough estimate $(\sqrt{3}/5)\,g_{\pi\mathscr{N}\mathscr{N}} \approx 5$.

The $\eta\gamma\gamma$ coupling leads to a width of

$$\Gamma_{\eta\gamma\gamma} = \frac{m_\eta}{16}e^4\frac{g_{\eta\gamma\gamma}^2}{4\pi} \approx 23 g_{\eta\gamma\gamma}^2 \text{ GeV}.$$

Experimentally, $\Gamma_{\eta\gamma\gamma} = (2.63 \pm 59) \times 37\%$ keV. Hence $|g_{\eta\gamma\gamma}| \approx 0.21$. Some people prefer to use the dimensional coupling $F_\eta = g_{\eta\gamma\gamma}/m_\eta \approx 0.33$ (GeV)$^{-1}$.

3) π: The coupling is written the same way as above. Here $g_{\pi\mathscr{N}\mathscr{N}} = 13.4$ is well known. For $\pi\gamma\gamma$ one finds from the experimental width of 7.8 eV ($\hat{=}\,\tau = 0.84 \cdot 10^{-16}$ s), $|g_{\pi\gamma\gamma}| \simeq 0.037$ or $|F_{\pi\gamma\gamma}| \approx 0.274$ (GeV)$^{-1}$. From constructive interference with the photoproduction amplitude one concludes that $-g_{\pi\gamma\gamma}$ has the same sign as $g_{\pi\mathscr{N}\mathscr{N}}$ which we choose positive (*).

(*) Defined by $\mathscr{L} = g_{\pi\mathscr{N}\mathscr{N}}\bar{N}i\gamma_5\boldsymbol{\tau}N\boldsymbol{\pi}$.

4) f: $$\mathscr{L} = -e^2 \frac{g^{(2)}_{f\gamma\gamma}}{4m_f} f^{\mu\nu} F_{\mu\varrho} F^{\varrho}{}_{\nu} + e^2 \frac{g^{(1-2)}_{f\gamma\gamma}}{4m_f^3} \partial_\mu F_{\varrho\varkappa} \partial_\nu F^{\varrho\varkappa} f^{\mu\nu} + $$
$$+ \bar{N} [G^{(1)}_{f\mathcal{N}\mathcal{N}} i(\overleftrightarrow{\partial}_\mu \gamma_\nu + \overleftrightarrow{\partial}_\nu \gamma_\mu)/4m - G^{(2)}_{f\mathcal{N}\mathcal{N}} \overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}_\nu / 4m^2] N f^{\mu\nu} .$$

The prediction from f dominance of gravity gives $G^{(1)}_{f\mathcal{N}\mathcal{N}} \approx 9$, $G^{(2)}_{f\mathcal{N}\mathcal{N}} = 0$. The determinations from backward dispersion relations (1.8) lead to $G^{(1)}_{f\mathcal{N}\mathcal{N}} \approx 25 \pm 4$, $G^{(2)}_{f\mathcal{N}\mathcal{N}} \approx -6 \pm 8$.

The $f\gamma\gamma$ couplings enter the $f\gamma\gamma$ width as

$$\Gamma_{f\gamma\gamma} = \frac{m_f}{80 \cdot 16\pi} e^4 \left(g^{(2)2}_{f\gamma\gamma} + \frac{1}{6} g^{(1-2)^2}_{f\gamma\gamma} \right) \approx 2.64 \left(g^{(2)^2}_{f\gamma\gamma} + \frac{1}{6} g^{(1-2)^2}_{f\gamma\gamma} \right) .$$

Therefore the f dominance prediction of $g^{(1)}_{f\gamma\gamma} \approx 1.6$, $g^{(1-2)}_{f\gamma\gamma} = 0$ gives $\Gamma_{f\gamma\gamma} \approx 7$ keV.

5) A_2: For A_2 we can write a similar coupling as for f. Here the combined use of forward and backward dispersion relations in photoproduction gives the estimates [10]

$$G^{(2)}_{A_2\mathcal{N}\mathcal{N}} \approx 6 , \qquad G^{(1+2)}_{A_2\mathcal{N}\mathcal{N}} \approx 0 .$$

Appendix B

The expressions (2.28) and (2.29) of the forward and backward amplitudes A_F and A_B close to the poles can be simplified for the special cases:

1) $\frac{1}{2}^+$ *resonances of mass* M.

There $G^+ = 0$, $G^-_{\substack{p\\n}} = ((M-m)/\sqrt{2Mm}) K_{\substack{p\\n}}$ and

$$A_{\substack{F\\B}} = \frac{1}{M^2 - s} \frac{4Mm}{(M+m)^2} \begin{Bmatrix} -1/2m \\ 1/2m^2 \\ 1/2m \\ 1/2mM \\ 0 \\ -1/mM \end{Bmatrix} K^2 .$$

If the isospin of the resonance is $\frac{1}{2}$, K^2 has to be replaced by $(K_p^2 + K_n^2)/2 = K_s^2 + K_v^2$ in the amplitudes A^s and by $(K_p^2 - K_n^2)/2 = 2K_s K_v$ in A^v. If it is $\frac{3}{2}$ only one A^s receives a contribution; $A^v = 0$.

With the aid of the factors $P^\pm|_{F,\text{th}}$ and $P^\pm|_{B,\text{th}}$ derived in Sect. **3** the contribution to the unsubtracted dispersion relation (3.3) is given (see (3.17)) by

$$\begin{Bmatrix} A_1 \\ A_2 \\ \frac{1}{s-u} A_3 \\ A_4 \\ A_5 \\ \frac{1}{s-u} A_6 \end{Bmatrix} = \frac{4m}{M(M+m)^2} \begin{Bmatrix} -1/2m \\ 1/2m^2 \\ 0 \\ 1/2Mm \\ 0 \\ 0 \end{Bmatrix} K^2 .$$

If we multiply the amplitudes by t we find their contribution to be

$$\begin{Bmatrix} t\ A_1 \\ t\ A_2 \\ \frac{t}{s-u} A_3 \\ t\ A_4 \\ t\ A_5 \\ \frac{t}{s-u} A_6 \end{Bmatrix} = \frac{4m}{M(M+m)^2} \begin{Bmatrix} -\frac{1}{2M^2 m}(M^4-m^4) \\ \frac{1}{2M^2m^2}(M^4-m^4) \\ \frac{1}{2m} \\ \frac{1}{2M^3 m}(M^4-m^4) \\ 0 \\ -\frac{1}{Mm} \end{Bmatrix} K^2 .$$

As discussed in Sect. **3**, however, only some of these amplitudes behave sufficiently well to justify inserting them in (3.3).

2) $\frac{3}{2}^+$ *resonance of mass* M.

Inserting $n=-1$ and $J=\frac{3}{2}$ in (2.28) and (2.29) one obtains

$$A_r = \frac{m}{M(M^2-s)p^2} \begin{cases} 2M\left\{(G^-)^2 + \frac{M}{m} G^+ G^- \sqrt{3}\right\}, \\ -2\frac{M}{m}\left\{(G^-)^2 + \frac{M}{m}\frac{E}{p} G^+ G^- \sqrt{3} + \frac{M}{p}(G^+)^2\right\}, \\ -2M(G^-)^2, \\ (G^-)^2 + 2\frac{m}{p} G^+ G^- \sqrt{3} + \frac{M^2+3m^2}{2Mp}(G^+)^2, \\ \frac{4}{p}\left\{-G^+ G^- \sqrt{3} - \frac{m}{M}(G^+)^2\right\}, \\ -2\{(G^-)^2 - (G^+)^2\}, \end{cases}$$

and

$$A_B = -\frac{m}{M(M^2-s)p^2}\left|\begin{array}{l} M\left\{(G^-)^2 - 2\dfrac{p}{m}G^+G^-\sqrt{3} + (G^+)^2\right\}, \\[2ex] \dfrac{M}{m}\left\{-(G^-)^2 + 2\dfrac{E^2}{mp}G^+G^-\sqrt{3} + \dfrac{3M^2+m^2}{2pM}(G^+)^2\right\}, \\[2ex] -M\left\{(G^-)^2 + 2\dfrac{E}{m}G^+G^-\sqrt{3} + (G^+)^2\right\}, \\[2ex] 2\left\{(G^-)^2 - \dfrac{m}{M}\dfrac{E}{p}G^+G^-\sqrt{3} - \dfrac{E^2}{Mp}(G^+)^2\right\}, \\[2ex] \dfrac{4}{p}\left\{G^+G^-\sqrt{3} + \dfrac{m}{M}(G^+)^2\right\}, \\[2ex] -4\left\{(G^-)^2 + \dfrac{m}{M}G^+G^-\sqrt{3} + \dfrac{E}{M}(G^+)^2\right\}. \end{array}\right.$$

The momentum and energy factors are at the resonance position

$$p = \frac{M^2 - m^2}{2M}, \qquad E = \frac{M^2 + m^2}{2M},$$

respectively. It turns out that these mass factors render A extremely sensitive to the electric-quadrupole moment of the Δ-resonance. The magnetic and electric couplings of A are usually defined by [29]

$$G^+ = -\frac{1}{2\sqrt{2}}\frac{p}{m}(\sqrt{3}\,G_M + G_E),$$

$$G^- = \frac{1}{2\sqrt{2}}\frac{p}{m}(G_M - \sqrt{3}\,G_E).$$

Experimentally one finds G_E small compared with G_M. The standard statement given in analyses of photo- and electroproduction is $E_{1+}/M_{1+} \approx -4.6\,\%$ [30]. Using the relations among the multipole amplitudes M_{1+}, E_{1+} and G^+, G^- as derived in Sect. **4** we see that

$$-\frac{1}{\sqrt{3}}\frac{G_E}{G_M} = \frac{E_{1+}}{M_{1+}}.$$

If we call this ratio ε, we can rewrite A_F in the form

$$A_F = \frac{G_M^2}{8m^2(M^2-m^2)} \cdot \begin{cases} 2M\left\{\dfrac{M+m}{m}\,9\varepsilon^2 - \dfrac{M-m}{m}\,6\varepsilon - \dfrac{3M-m}{m}\right\}, \\[2mm] 2\dfrac{M}{m}\left\{-\dfrac{M^3+M^2m+Mm^2-m^3/3}{m(M^2-m^2)}\,9\varepsilon^2 + \right. \\ \qquad\qquad \left. + \dfrac{M^3+M^2m^2+Mm^2+m^3}{m(M^2-m^2)}\,6\varepsilon + \dfrac{3M-4Mm-m^2}{m(M+m)}\right\}, \\[2mm] -2M(3\varepsilon+1)^2, \\[2mm] 4\left\{\dfrac{M(M+3m)}{M^2-m^2}\,3\varepsilon^2 - \dfrac{m}{M-m}\,6\varepsilon + \dfrac{M-2m}{M+m}\right\}, \\[2mm] 8\left\{-\dfrac{3M+m}{M^2-m^2}\,3\varepsilon^2 + \dfrac{1}{M-m}\,6\varepsilon + \dfrac{3}{M+m}\right\}, \\[2mm] 4\{-3\varepsilon^2 - 6\varepsilon + 1\}. \end{cases}$$

For A_B the mass factors become quite lengthy. In order to keep them brief we shall abbreviate the expression

$$\alpha_n M^n + \alpha_{n-1} M^{n-1} m + \ldots + \alpha_1 M m^{n-1} + \alpha_0 m^n$$

by an $(n+1)$-tuple $[\alpha_n, \alpha_{n-1}, \alpha_{n-2}, \ldots, \alpha_1, \alpha_0]$. Then

$$A_B = -\frac{G_M^2}{8m^2(M^2-m^2)} \cdot \begin{cases} \dfrac{1}{m}\{[-3, 4, 3]\,3\varepsilon^2 + [1, 0, -1]\,6\varepsilon + [3, 4, -2]\}, \\[2mm] \dfrac{1}{m^2}\left\{\dfrac{[3, 0, 6, 4, 0]}{[1, 0, -1]}\,3\varepsilon^2 - \dfrac{[1, 4, 2, 2, 1]}{[1, 0, -1]}\,6\varepsilon - \dfrac{[3, -5, 1, -3]}{[1, 1]}\right\}, \\[2mm] -\dfrac{1}{m}\{[3, 4, 3]\,3\varepsilon^2 - [1, 0, 1]\,6\varepsilon - [3, -4, 3]\}, \\[2mm] \dfrac{1}{M^2}\left\{\dfrac{[5, -6, -8, -6, -1]}{[1, 0, -1]}\,3\varepsilon^2 + \dfrac{[3, 2, 0, 2, 1]}{[1, 0, -1]}\,6\varepsilon + \dfrac{[-1, 5, -3, 3]}{[1, 1]}\right\}, \\[2mm] 8\left\{\dfrac{3M+m}{M^2-m^2}\,3\varepsilon^2 - \dfrac{1}{M-m}\,6\varepsilon - \dfrac{3}{M+m}\right\}, \\[2mm] -\dfrac{2}{M^2}\{[7, 6, 1]\,3\varepsilon^2 - [-1, 2, 1]\,6\varepsilon + [5, -6, 3]\}. \end{cases}$$

Inserting the physical masses we are led to the following expression for the Δ contribution to the forward-backward sum rules ($m=1$):

$$\begin{Bmatrix} A_1 \\ A_2 \\ \dfrac{A_3}{s-u} \\ A_4 \\ A_5 \\ \dfrac{A_6}{s-u} \end{Bmatrix}^{\Delta}_{F\text{-}B} = \frac{G_M^2}{8M} \begin{Bmatrix} 168.8\,\varepsilon^2 - 4.15\,\varepsilon - 5.10 \\ -197.0\,\varepsilon^2 + 80.14\,\varepsilon - 0.053 \\ -118.2\,\varepsilon^2 + 1.11\,\varepsilon + 0.56 \\ 144.2\,\varepsilon^2 - 32\,\varepsilon - 0.31 \\ -93.2\,\varepsilon^2 + 87.2\,\varepsilon + 6 \\ -156.4\,\varepsilon^2 - 20\,\varepsilon - 0.6 \end{Bmatrix}$$

and

$$\begin{Bmatrix} tA_1 \\ tA_2 \\ tA_3/(s-u) \\ tA_4 \\ tA_5 \\ tA_6/(s-u) \end{Bmatrix}_{F\text{-}B} = \frac{G_M^2}{8M} \begin{Bmatrix} -9.2\,\varepsilon^2 - 4.4\,\varepsilon - 7.5 \\ -112.9\,\varepsilon^2 + 135.6\,\varepsilon - 1.5 \\ 40.4\,\varepsilon^2 - 16.4\,\varepsilon - 3 \\ 50.53\,\varepsilon^2 - 81.2\,\varepsilon - 1.4 \\ -161.9\,\varepsilon^2 + 151.4\,\varepsilon + 10.4 \\ 72.8\,\varepsilon^2 - 13.1\,\varepsilon + 4.3 \end{Bmatrix}.$$

The large factors in front of ε^2 teach us that Nature has very carefully avoided bringing our sum rules out of balance by keeping ε very small. This can be seen best if we take sum rules 1) and 6) in which no well-known low-lying mesons appear. Thus in $A_6^s/(s-u)$ the nucleon does not contribute at all and the higher resonances contribute very little. The term $\propto \varepsilon^2$, however, enters with a factor 156. The value $\varepsilon \approx -5\%$ which we have inserted is small enough to satisfy the sum rule. It is amusing to note that if one were to assume saturation with $\mathcal{N}$ and Δ only the Δ contribution would have to vanish and one would predict an E/M ratio of $\varepsilon = -6.4\%$. Consider the other amplitude $tA_6^s/(s-u)$. Also here higher resonances can be neglected as we can see in Table V. The nucleon, however, shows a large contribution. If we ask the Δ-resonance to balance completely the nucleon contribution, we find for the E/M ratio of $\varepsilon = -4.6\%$ a magnetic moment of $G_M = 3.5$ which agrees extremely well with experiment [30]. In the case of our theoretical prediction $\varepsilon_{th} = -6.4\%$ we would find $G_M = 3.4$.

● RIASSUNTO (*)

Si usa la tecnica di uguagliare le relazioni di dispersione non sottratte nelle direzioni in avanti ed all'indietro alla soglia allo scopo di ottenere regole di somma per le ampiezze

(*) *Traduzione a cura della Redazione.*

invarianti dello scattering di Compton. Supponendo che f, σ, η e π, A_2 dominino la parte assorbitiva del canale t, e che le risonanze barioniche saturino gli integrali sui tagli dei canali s ed u, si riesce ad esprimere le costanti di accoppiamento di questi mesoni ai fotoni ed ai nucleoni in termini delle ampiezze multipolari elettromagnetiche delle risonanze barioniche. Si confrontano i risultati con le stime ottenute con altri metodi.

Комптоновское рассеяние и константы связи f, σ, η, A_2 и π с фотонами и нуклонами.

Резюме (*). — Мы используем технику приравнивания безвычитательных дисперсионных соотношений для направлений вперед и назад на пороге, чтобы получить правила сумм для инвариантных амплитуд комптоновского рассеяния. Предполагая, что f, σ, η и π, A_2 доминируют в абсорбционной части t-канака и барионные резонансы насыщают интегралы сверх s- и u-канальных разрезов, мы можем выразить константы связи этих мезонов с фотонами и нуклонами через электромагнитные мультипольные амплитуды барионных резонансов. Полученные здесь результаты сравниваются с оценками, полученными другими методами.

(*) *Переведено редакцией.*