Neutrino Mass Differences and Nonunitarity of Neutrino Mixing Matrix from Interfering Recoil Ions

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Received 17 June 2009, Accepted 15 August 2009, Published 30 October 2009

Abstract: We show that the recent observation of the time modulation of two-body weak decays of heavy ions reveals the mass content of the electron neutrinos via interference patterns in the recoiling ion wave function. From the modulation period we derive the difference of the square masses $\Delta m^2 \approx 22.5 \times 10^{-5}\,\mathrm{eV}^2$, which is about 2.8 times larger than that derived from a combined analysis of KamLAND and solar neutrino oscillation experiments. It is, however, compatible with a data regime to which the KamLAND analysis attributes a smaller probability. The experimental results displayed in Fig. 1 imply that the neutrino mixing matrix violates unitarity by about 10%.

Keywords: Neutrino Mass and Mixing; Interfering Recoil Ions; Weak-interaction
PACS (2008): 14.60.Pq; 12.15.Ff; 23.40.Bw

1. Introduction

At the GSI in Darmstadt, the experimental storage ring ESR permits observing completely ionized heavy atoms I or hydrogen-like heavy ions I_H over a long time [1, 2] and thus to measure the time dependence of their weak two-body decays $I_H \to I + \nu_e$ or $I \to I_H + \bar{\nu}_e$. The first is the well-known electron-capture (EC) process. The virtue of such experiments is that the properties of the neutrino or antineutrino can

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be deduced from measurements of the time dependence of the transition observing only the initial and final ions. The special efficiency of these experiments becomes clear in the Dirac sea interpretation of the second process, where the initial ion simply absorbs a negative-energy antineutrino in the vacuum. Since the vacuum has *all* negative-energy states filled, the vacuum is a source of negative-energy neutrinos of *maximally possible* current density, i.e., the best possible neutrino source in the universe. This is why the ESR experiments yield information on neutrino properties with great precision even if the targets and exposure times are quite small, in particular much smaller than the 2.44×10^{32} proton-yrs (2881 ton-yrs) in the famous KamLAND experiments [3], which are only sensitive to the much less abundant positive-energy neutrinos produced by nuclear reactors.

Apart from the neutrino mass difference, the experiment reveals also another important property of the presently popular neutrino mixing scheme: the matrix which expresses the neutrino flavor states into fixed-mass states must be *nonunitary* to explain the data. The measurement determines the degree of nonunitarity to be roughly 10%.

2. Two-Neutrino Mixing

To illustrate this we consider here at first only the two lightest neutrinos. According to Pontecorvo [4, 5], the Dirac fields of the physical electron and muon-neutrinos $\nu_f = (\nu_e, \nu_\mu)$, the so-called *flavor fields*, are superpositions of neutrino fields $\nu_i = (\nu_1, \nu_2)$ of masses m_1 and m_2:

$$\nu_e(x) = \nu_1(x)\cos\theta + \nu_2(x)\sin\theta, \qquad \nu_\mu(x) = -\nu_1(x)\sin\theta + \nu_2(x)\cos\theta\,, \tag{1}$$

where θ is a mixing angle. This is, of course, the neutrino analog of the famous Cabibbo mixing of up and down quarks. The free Dirac action has the form

$$\mathcal{A} = \sum_f \int d^4x\, \bar\nu_f(x)\,(i\gamma^\mu\partial_\mu - \mathcal{M})\,\nu_f(x),$$

where γ^μ are the Dirac matrices, and $\mathcal{M}$ is a mass matrix, whose diagonal and off-diagonal elements are $m_f = (m_e, m_\mu)$ and $m_{e\mu} = m_{\mu e}$, respectively. The eigenvalues $m_i = (m_1, m_2)$ are related to m_f by [4, 5, 6, 7],

$$m_e = m_1\cos^2\theta + m_2\sin^2\theta, \quad m_\mu = m_1\sin^2\theta + m_2\cos^2\theta,$$
$$m_{e\mu} = m_{\mu e} = (m_2 - m_1)\sin\theta\cos\theta. \tag{2}$$

The weak transition between the electron e and its neutrino ν_e is governed by the interaction

$$\mathcal{A}_{\rm int} = g\int d^4x\, W^-_\mu(x) J^{+\mu}(x) + \text{h.c.} \equiv g\int d^4x\, W^-_\mu(x)\,\bar e(x)\gamma^\mu(1-\gamma_5)\nu_e(x) + \text{h.c.}, \tag{3}$$

where γ_5 is the product of Dirac matrices $i\gamma^0\gamma^1\gamma^2\gamma^3$.

Since the interaction (3) involve only the flavor fields (1), the states of masses m_i will always be produced as coherent superpositions. The weakness of the interaction will allow us to calculate the shape of the mixed wave packet from perturbation theory. Consider the decay $I \to I_H + \bar{\nu}_e$ which is a superposition of the states of masses m_1 and m_2. The formulas will be applicable for electron capture if we exchange M_H by the mass M of the bare ion and deal with outgoing neutrinos.

In the center-of-mass (CM) frame of the initial bare ion of mass M, the final H-like ion has the same momentum as the antineutrino $\bar{\nu}_i$ $(i = 1, 2)$, whose energy is $\omega_i \equiv \omega_{\mathbf{k}_i,i} = \sqrt{\mathbf{k}_i^2 + m_i^2}$ determined by

$$M \equiv M_H + Q = \omega_i + \sqrt{M_H^2 + \mathbf{k}_i^2} = \omega_i + \sqrt{M_H^2 + \omega_i^2 - m_i^2}, \quad i = 1, 2, \tag{4}$$

so that

$$\omega_i = [(2M_H + Q)Q + m_i^2]/2(M_H + Q). \tag{5}$$

Subtracting ω_2 and ω_1 from each other we find the energy difference

$$\Delta\omega \equiv \omega_2 - \omega_1 = \frac{m_2^2 - m_1^2}{2M} \equiv \frac{\Delta m^2}{2M}. \tag{6}$$

The denominator M is of the order of 100 GeV and much larger than Δm^2, so that $\Delta\omega$ is extremely small. It is the difference of the recoil energies transferred to the outcoming ion by the antineutrinos of masses m_1 and m_2. Without recoil, we would have found the four orders of magnitude larger energy difference at the *same* momentum $\Delta\omega_{\mathbf{k}} = \omega_{\mathbf{k},2} - \omega_{\mathbf{k},1} = (\Delta m^2 + \omega_{\mathbf{k},1}^2)^{1/2} - \omega_{\mathbf{k},1} \approx \Delta m^2/2\omega_{\mathbf{k},1} \approx \Delta m^2/2Q$. This is the frequency with which the incoming negative-energy neutrino current of momentum $\mathbf{k}$ oscillates in the vacuum. Note that although $\Delta\omega$ is small, the momentum difference $\Delta k \equiv k_2 - k_1$ associated with the energies $\omega_{1,2}$ is as large as $\Delta\omega_{\mathbf{k}}$, but has the opposite sign.

3. Experiments

The best experimental results are available for the EC-processes reported in Ref. [1], where an electron is captured from the K-shell and converted into an electron-neutrino which runs off to infinity. On the average, the decay is exponential with a rate expected from a standard-model calculation. In addition, however, the decay rate shows modulations with a frequency $\Delta\omega$. The experimental results are [8]

$$^{140}_{59}\mathrm{Pr}^{58+} \to {}^{140}_{58}\mathrm{Ce}^{58+}: \quad \Delta\omega \approx 0.890(11)\,\mathrm{sec}^{-1} \quad (Q = 3\,386\,\mathrm{keV}), \tag{7}$$

$$^{142}_{61}\mathrm{Pm}^{60+} \to {}^{142}_{60}\mathrm{Nd}^{60+}: \quad \Delta\omega \approx 0.885(31)\,\mathrm{sec}^{-1} \quad (Q = 4\,470\,\mathrm{keV}). \tag{8}$$

In both cases, the period of modulation is roughly 7 sec, and scales with M (see Fig. 1). The decay rate has the form $\lambda(t) = \lambda(0)(1 + a\cos(\Delta\omega t + \Delta\phi)]$ with a modulation amplitude of $a = 0.18(3)$.

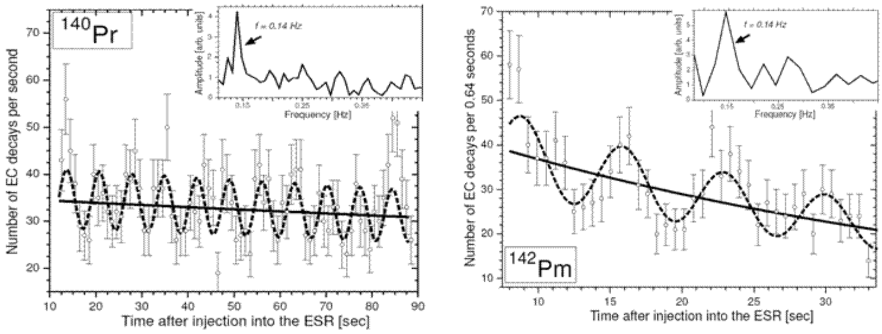

Figure 1 Modulations of decay rate for the processes ${}^{140}_{59}\mathrm{Pr}^{58+} \rightarrow {}^{140}_{58}\mathrm{Ce}^{58+}$ and ${}^{142}_{61}\mathrm{Pm}^{60+} \rightarrow {}^{142}_{60}\mathrm{Nd}^{60+}$. The period is in both cases roughly 7 sec. The inserts show the frequency analyses. Plots are from Ref. [1]. The decay rate is modulated by a factor $1 + a\cos(\Delta\omega t + \Delta\phi)$ with $a = 0.18(3)$.

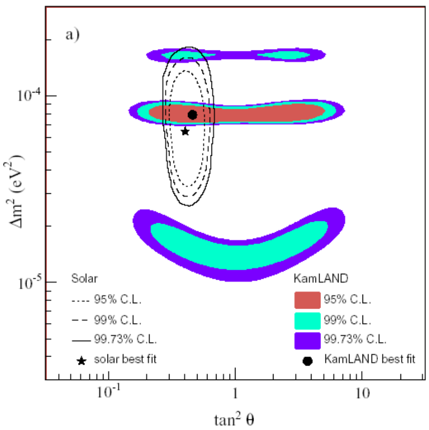

Figure 2 The upper KamLAND regime of 2006 [10] is compatible with our result $\Delta m^2 \approx 22.5 \times 10^{-5}\mathrm{eV}^2$.

We expect theses modulations to be associated with the frequency $\Delta\omega$ of Eq. (6), and thus to give information on Δm^2. Inserting the experimental numbers for $\Delta\omega$ into Eq. (6) and taking into account that the particles in the storage ring run around with 71% of the light velocity with a Lorentz factor $\gamma \approx 1.43$, we find for both processes [9]

$$\Delta m^2 \approx 22.5 \times 10^{-5}\,\mathrm{eV}^2. \qquad (9)$$

This is by a factor ≈ 2.8 larger than the result $\Delta m^2 \approx 7.58^{+0.3}_{-0.3} \times 10^{-5}\mathrm{eV}^2$ favored by the KamLAND experiment [3, 11], but it lies close to their less favored result [10], which the authors excluded by 2.2σ in 2005, and now by 6σ [3] (see Fig. 2).

So far we do not yet understand the origin of this discrepancy. One explanation has been attempted in Ref. [12] where the authors investigate the influence of the strong Coulomb field around the ion upon the process.

4. Entangled Wavefunction

For a theoretical explanation of the modulations, we first simplify the situation and ignore all spins and the finite size of the ions. Then the decay of the initial ion I into the ion I_H plus an electron-antineutrino $\bar{\nu}_e$. can be described by an effective interaction for this process is

$$\begin{aligned}\mathcal{A}_{\rm int} &= g\int d^4x\, I_H^\dagger(x)\nu_e(x)I(x)\\ &= g\int d^4x\left[\cos\theta\, I_H^\dagger(x)\nu_1(x)I(x)+\sin\theta\, I_H^\dagger(x)\nu_2(x)I(x)\right],\end{aligned}\tag{10}$$

where $I(x)$, $\nu_e(x)$, and $I_H(x)$ are the field operators of the involved particles. In the CM frame, the initial ion is at rest, the final moves nonrelativistically. The outgoing wave is spherical. The role of the antineutrino creation operators in ν_1 and ν_2 is simply to create a coherent superposition of two such waves with the two different k- and ω-values calculated above. The combined outgoing wave function will be

$$\begin{aligned}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\big[\cos\theta e^{-iE_{\mathbf{p}}t+i\mathbf{px}}|I_H(\mathbf{p})\rangle e^{-i\omega_{\mathbf{k},1}t+i\mathbf{kx}}|\bar{\nu}_1(\mathbf{k})\rangle\\ +\sin\theta e^{-iE_{\mathbf{p}}t+i\mathbf{px}}|I_H(\mathbf{p})\rangle e^{-i\omega_{\mathbf{k},2}t+i\mathbf{kx}}|\bar{\nu}_2(\mathbf{k})\rangle\big].\end{aligned}\tag{11}$$

The states $|\bar{\nu}_1(\mathbf{k})\rangle, |\bar{\nu}_2(\mathbf{k})\rangle$, in turn, can be reexpressed in terms of the electron- and muon-neutrino states as

$$|\bar{\nu}_1(\mathbf{k})\rangle=\cos\theta|\bar{\nu}_e(\mathbf{k})\rangle-\sin\theta|\bar{\nu}_\mu(\mathbf{k})\rangle,\qquad |\bar{\nu}_2(\mathbf{k})\rangle=\sin\theta|\bar{\nu}_e(\mathbf{k})\rangle+\cos\theta|\bar{\nu}_\mu(\mathbf{k})\rangle.\tag{12}$$

Thus we find for the transition to an electron-neutrino of any momentum the effective action

$$\int\frac{d^3k}{(2\pi)^3}\langle\bar{\nu}_e(-\mathbf{k})|\mathcal{A}_{\rm int}|0\rangle = g\int d^3x\, I_H(x)I(x)v_{\bar{\nu}_e}(x)\tag{13}$$

with a spacetime-dependent potential

$$v_{\bar{\nu}_e}(x)=\int\frac{d^3k}{(2\pi)^3}\left[\cos^2\theta e^{i\omega_{\mathbf{k},1}t+i\mathbf{kx}}+\sin^2\theta e^{i\omega_{\mathbf{k},2}t+i\mathbf{kx}}\right].\tag{14}$$

In Born approximation we find from this the scattering state of the recoiling ion I_H:

$$\langle\mathbf{x}|\psi^{(+)};t\rangle^{\bar{\nu}_e}\equiv-\frac{g}{r}\left[\cos^2\theta\, e^{i(k_1r-\omega_1t)}+\sin^2\theta\, e^{i(k_2r-\omega_2t)}\right].\tag{15}$$

This wave carries a radial current density of ions I_H

$$j_r^{\bar{\nu}_e}=\frac{g^2}{M_Hr^2}\left[\cos^4\theta\, k_1+\sin^4\theta\, k_2+\sin^2\theta\cos^2\theta\,(k_1+k_2)\cos(\Delta k\, r-\Delta\omega\, t)\right].\tag{16}$$

In order to find the decay rate we integrate this over a sphere of radius R surrounding the initial ion, choosing for R any size $\ll 1/\Delta k\approx 10^4\,$m. For this surface we find the outgoing probability current density

$$\dot{P}=4\pi g^2\frac{\bar{k}}{M}\left[1-\tfrac{1}{2}\sin^2(2\theta)+\tfrac{1}{2}\sin^2(2\theta)\cos(\Delta\omega\, t)\right],\tag{17}$$

where we have approximated k_1 and k_2 by their average $\bar{k}$.

This $\dot{P}$ can explain directly the observed modulations of the decay rate of the initial ions. The is only one problem: the amplitude of modulations are predicted to be $a = \frac{1}{2}\sin^2(2\theta)/[1 - \frac{1}{2}\sin^2(2\theta)] \equiv 0.72$. Experimentally, however, a is much smaller. It has the value 0.18(3).

The discrepancy is explained by a missing contribution to the decay rate. So far we have only included the contribution of the effective action (10). There is, however, also a second effective action which is generated by the matrix elements

$$\int \frac{d^3k}{(2\pi)^3} \langle \bar{\nu}_\mu(-\mathbf{k})|\mathcal{A}_{\rm int}|0\rangle = g \int d^3x\, I_H(x) I(x) v_{\bar{\nu}_\mu}(x) \tag{18}$$

where

$$v_{\bar{\nu}_\mu}(x) = \int \frac{d^3k}{(2\pi)^3} \sin\theta\cos\theta \left[-e^{i\omega_{\mathbf{k},1}t + i\mathbf{kx}} + e^{i\omega_{\mathbf{k},2}t + i\mathbf{kx}}\right]. \tag{19}$$

This can be derived directly from Eqs. (12) and (13). Here the Born approximation yields the scattering state of the recoiling ion I_H:

$$\langle \mathbf{x}|\psi^{(+)}; t\rangle^{\bar{\nu}_\mu} \equiv \frac{g}{r}\sin\theta\cos\theta \left[e^{i(k_1 r - \omega_1 t)} - e^{i(k_2 r - \omega_2 t)}\right]. \tag{20}$$

Its radial current density is now

$$j_r^{\bar{\nu}_\mu} = \frac{g^2}{M_H r^2}\sin^2\theta\cos^2\theta (k_1 + k_2)\left[1 - \cos(\Delta k\, r - \Delta\omega\, t)\right]. \tag{21}$$

Now we have another problem: the modulations of this current cancel the modulations of the current (17). We may suspect that this has to do with the fact that there are three neutrinos which we must take into consideration.

5. Three-Neutrino Mixing

Let us now include all three known neutrinos ν_e, ν_μ, ν_τ. Their fields are denoted by ν_σ with $\sigma = e, \mu, \tau$. These fields are combinations of three fields with definite mass

$$\nu_\sigma = U_{\sigma i}\nu_i, \qquad \sigma = (e, \mu, \tau), \tag{22}$$

The mixing matrix $U_{\sigma i}$ is called Maki-Nakagawa-Sakata matrix, or short MNS-matrix, the neutrino analog of the 3×3 Cabibbo-Kobayashi-Maskawa matrix for the mixing of the quarks d, s, b, It is commonly assumed to be unitary, i.e., to satisfy the relation

$$\sum_\sigma U^*_{\sigma j} U_{\sigma l} = \delta_{jl}. \tag{23}$$

Its standard parametrization is the following product of four simple unitary matrices

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{24}$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$. For quarks, the unitarity relation (23) is presently in the focus of experimental and theoretical studies in many research groups [14]. For leptons, the data have so far been insufficient to test it.

Generalizing (13), (14) and (18), (19), we have from each flavor σ an effective potential

$$\int \frac{d^3k}{(2\pi)^3} \langle \bar{\nu}_\sigma(-\mathbf{k})|\mathcal{A}_{\rm int}|0\rangle = g \int d^3x \, I_H(x) I(x) v_{\bar{\nu}_\sigma}(x) \tag{25}$$

with a potential

$$v_{\bar{\nu}_\sigma}(x) = \int \frac{d^3k}{(2\pi)^3} \sum_{j=0}^{3} U_{ej} U^*_{\sigma j} e^{i\omega_{\mathbf{k},j} t + i\mathbf{kx}}. \tag{26}$$

This produces an outgoing wave function of the ion I_H in the center-of-mass frame due to the potential v_{ν_σ} is

$$\langle \mathbf{x}|\psi^{(+)}; t\rangle^{\bar{\nu}_\sigma} \equiv -\frac{g}{r} \sum_{j=1}^{3} U_{ej} U^*_{\sigma j} e^{i(k_j r - \omega_j t)}, \tag{27}$$

with an ion current density

$$j_r^{\bar{\nu}_\sigma} = \frac{g^2}{M_H r^2} \sum_{j,l=1}^{3} \sum_{\sigma=1}^{3} U_{ej} U^*_{\sigma j} U^*_{el} U_{\sigma l} \, k_j e^{i[(k_j - k_l)r - (\omega_j - \omega_l)t]}. \tag{28}$$

If we sum over all flavors of the antineutrino and use the unitarity relation (23), we obtain the total radial current density

$$j_r \equiv \sum_\sigma j_r^{\bar{\nu}_\sigma} = \frac{g^2}{M_H r^2} \sum_{j=1}^{3} U_{ej} U^*_{ej} \, k_j. \tag{29}$$

As previously for two flavors, the modulations in (28) disappear.

However, the GSI experiments did observe modulations with an amplitude $a \approx 0.18(3)$. Thus we must conclude that the unitarity relation (23) must be violated. Since so far only the lowest possible modulation frequency $\Delta\omega = \omega_2 - \omega_1$ between the two lightest neutrinos has been measured, we may parametrize the right-hand side of the unitarity violation by

$$\sum_\sigma U^*_{\sigma j} U_{\sigma l} = u_0 \delta_{jl} + u_{21}(\delta_{j,2}\delta_{l,1} + \delta_{j,1}\delta_{l,2}) + \ldots, \tag{30}$$

and find

$$j_r = \frac{g^2}{M_H r^2} \left\{ u_0 S_0 \delta_{ij} + 2u_{21} S_{21} \cos[\Delta k r - \Delta\omega t + \Delta\phi] \right\}, \tag{31}$$

where

$$S_0 \equiv \sum_{j=1}^{3} U_{ej} U^*_{ej} k_j, \qquad S_{21} \equiv |U_{e\,2} U^*_{e1}|(k_2 + k_1)/2, \qquad \Delta\phi \equiv \arg U_{e2} U^*_{e1}. \tag{32}$$

Assuming that the violation of unitarity is small, the sums S_0 and S_1 are close to unity. Then we deduce from tha experimental result $a \approx 0.18(3)$ that

$$\frac{u_{21}}{u_0} \approx 10\% \,. \tag{33}$$

A possible origin of this unitarity violation could be that there are more than three families of leptons in nature and that universality of weak interaction is not valid for all of them. If the symmetry between quarks and leptons of the standard model persists to higher energies, we do not expect more than eight lepton families to exist— more than eight quark families would ruin asymptotic freedom and thus confinement. Thus there is room for more than the three quark and lepton families observed so far. Indeed, a fourth set of families is under intense discussion [15] in connection with the new accelerator LHC at CERN. So far, there are only weak bounds on their masses from different sources [3]:

$$m_{t'} \geq 256\,\text{GeV}, \quad m_{b'} \geq 128\,\text{GeV}, \quad m_{\tau'} \geq 100.8\,\text{GeV}, \quad m_{\nu_{\tau'}} \geq 90.3\,\text{GeV}. \tag{34}$$

If any of the heavier leptons is coupled with a coupling constant that does not fit into the CKM scheme, unitarity will certainly be violated. More data will be needed to decide precisely how.

6. Comments

It is noteworthy that this analysis, in which we extract the properties of the unobserved antineutrino from the behavior of the ion, corresponds precisely to the usual entanglement analysis of decay processes such as $\pi^0 \to \gamma + \gamma$. There the measurement of the polarization of *one* photon tells us immediately the polarization properties of the other, unobserved photon.

A few comments are in place on several recent publications [16, 17, 18, 19, 20, 21] which deny the relation between neutrino oscillations and the nonexponential decay seen in the GSI experiment for various reasons. In Ref. [16], the basic argument is that the antineutrino oscillations set in *after* their emission, so that they cannot be observed in the GSI experiment. The present discussion shows that although the forst part of this argument is true, the conclusion depends on the unitary assumption of the mixing matrix.

Finally we should point out that similar oscillation phenomena in the associate production of particles together with an oscillating partner have been proposed and controversially discussed before by many authors in the production of muons together with antineutrinos in the decay $\pi^- \to \mu^- + \bar{\nu}_\mu$ [23], and in the production of Λ hyperons together with neutral Kaons [24], In the latter case the oscillation would come from a nonunitarity of the quark mixing matrix, which seems to be much smaller than that of the neutrino mixing matrix reported here.

Acknowledgment

We are grateful to Günter Kaindl for organizing regular scientific evening talks in the Magnus-Haus Berlin of the German Physical Society. This brought us together on March 11, 2008, in a lecture by P.K. on the GSI experiments, leading to the interpretation presented here. We also appreciate financial support by the WE-Heraeus-Stiftung for the 'Nachsitzung' in the Remise of the Magnus-Haus.

Appendix: Properties of Outgoing Wave

The interaction is time-dependent and we must adapt the scattering theory to this situation. Recall briefly the theory for a time-independent interaction, where the scattering amplitude is obtained from the standard limiting formula

$$\begin{aligned}\langle \mathbf{p}'|\hat{S}|\mathbf{p}\rangle &= \lim_{t\to\infty}\langle \mathbf{p}'|\mathbf{p}^{(+)}(t)\rangle = \lim_{t\to\infty}\langle \mathbf{p}'|\hat{U}_I(t,0)|\mathbf{p}^{(+)}\rangle = \lim_{t\to\infty}\langle \mathbf{p}'|e^{i\hat{H}_0 t}e^{-i(\hat{H}_0+\hat{V})t}|\mathbf{p}^{(+)}\rangle \\ &= \lim_{t\to\infty} e^{i(E_{\mathbf{p}'}-E_{\mathbf{p}})t}\langle \mathbf{p}'|\mathbf{p}^{(+)}\rangle. \end{aligned} \tag{35}$$

Here $|\mathbf{p}'\rangle$ denotes an eigenstate of the free Hamiltonian $\hat{H}_0$ with momentum $\mathbf{p}'$ and energy $E_{\mathbf{p}'} = \mathbf{p}'^2/2M$, and $|\mathbf{p}^{(+)}\rangle$ is an eigenstate of the interacting Hamiltonian $\hat{H}_0 + \hat{V}$ with momentum $\mathbf{p}$ and energy $E_{\mathbf{p}}$. It solves the Lippmann-Schwinger equation:

$$|\mathbf{p}^{(+)}\rangle = |\mathbf{p}\rangle + \frac{1}{E_{\mathbf{p}} - \hat{H}_0 + i\eta}\hat{V}|\mathbf{p}^{(+)}\rangle, \tag{36}$$

which is verified by multiplying both sides by $E - \hat{H}_0$ from the left. Inserting this into (35) leads to

$$\langle \mathbf{p}'|\hat{S}|\mathbf{p}\rangle = \langle \mathbf{p}'|\mathbf{p}\rangle + \lim_{t\to\infty}\frac{e^{i(E_{\mathbf{p}'}-E_{\mathbf{p}})t}}{E_{\mathbf{p}} - E_{\mathbf{p}'} + i\eta}\langle \mathbf{p}'|\hat{V}|\mathbf{p}^{(+)}\rangle, \tag{37}$$

where $\eta > 0$ is an infinitesimally number. The second term contains the T-matrix $T_{\mathbf{p}',\mathbf{p}} \equiv \langle \mathbf{p}'|\hat{V}|\mathbf{p}^{(+)}\rangle$ which describes true scattering. In the absence of neutrino oscillations, $|\mathbf{p}\rangle$ is simply the initial ion at rest, and $\langle \mathbf{p}'|$ the state with the ion I_H with momentum $\mathbf{p}+\mathbf{k}$ and the antineutrino $\bar{\nu}_e$ with momentum $-\mathbf{k}$. The limit $t \to \infty$ in the prefactor can simply be taken after rewriting it as $-i\int_{-\infty}^{t} dt e^{i(E_{\mathbf{p}'}-E_{\mathbf{p}}-i\eta)t}$, which obviously tends to $-2\pi i\delta(E_{\mathbf{p}'} - E_{\mathbf{p}})$ in the limit $t \to \infty$. The δ-function ensures the conservation of energy in the process. This is, of course, the standard derivation of *Fermi's Golden Rule* which we repeated here to clarify that it is applicable only to processes in which the final state is an eigenstate of the free-particle Hamiltonian operator $\hat{H}_0$.

Another way of deriving this result is based on the spatial wave function associated with the state $|\mathbf{p}^{(+)}\rangle$. One multiplies Eq. (36) by the state $\langle \mathbf{x}|$ from the left and obtains the wave funcion

$$\langle \mathbf{x}|\mathbf{p}^{(+)}\rangle = \langle \mathbf{x}|\mathbf{p}\rangle + \int d^3x' G(E_{\mathbf{p}}; \mathbf{x}, \mathbf{x}')\hat{V}(\mathbf{x}')\langle \mathbf{x}'|\mathbf{p}^{(+)}\rangle, \tag{38}$$

where

$$G(E;\mathbf{x},\mathbf{x}') \equiv \langle \mathbf{x}|\frac{1}{E-\hat{H}_0+i\eta}|\mathbf{x}'\rangle = \int \frac{d^3p'}{(2\pi)^3}\frac{e^{i\mathbf{p}'(\mathbf{x}-\mathbf{x}')}}{E-\mathbf{p}'^2/2M+i\eta} \approx -2M\,\frac{e^{ip'_E r}}{4\pi r}e^{-ip'_E\hat{\mathbf{x}}\mathbf{x}'}, \tag{39}$$

with $r \equiv |\mathbf{x}-\mathbf{x}'|$, $\hat{\mathbf{x}} \equiv \mathbf{x}/r$ and $p'_E = \sqrt{2ME}$.

In Born approximation, one inserts on the right-hand side of Eq. (38) a plane wave $\langle \mathbf{x}'|\mathbf{p}^{(+)}\rangle \approx \langle \mathbf{x}'|\mathbf{p}\rangle = e^{i\mathbf{x}'\mathbf{p}}$, and Eq. (38) becomes

$$\langle \mathbf{x}|\mathbf{p}^{(+)}\rangle = \langle \mathbf{x}|\mathbf{p}\rangle - 2M\,\frac{e^{ip'_{E_\mathbf{p}}r}}{4\pi r}\int d^3x' e^{-i(\mathbf{p}'-\mathbf{p})\mathbf{x}'}V(\mathbf{x}'), \tag{40}$$

where $\mathbf{p}'$ is short for the momentum of the outgoing particle of energy $E_\mathbf{p}$ in the direction of $\mathbf{x}$: $\mathbf{p}' \equiv p'_{E_\mathbf{p}}\hat{\mathbf{x}}$. Thus, in Born approximation, the amplitude for the final particle to emerge with momentum $\mathbf{p}'$ is proportional the Fourier transform of the potential at the momentum transfer $\Delta\mathbf{p} \equiv \mathbf{p}'-\mathbf{p}$. If the potential is a plane wave of momentum $-\mathbf{k}$, i.e., if

$$V(\mathbf{x}) = \frac{g}{(2\pi)^3}e^{i\mathbf{k}\mathbf{x}} \tag{41}$$

then the final state has the wave function

$$\langle \mathbf{x}|\mathbf{p}^{(+)}\rangle = \langle \mathbf{x}|\mathbf{p}\rangle - 2Mg\,\frac{e^{ip'r}}{4\pi r}\delta^{(3)}(\mathbf{p}'-\mathbf{p}+\mathbf{k}). \tag{42}$$

Let us adapt this formalism to the oscillating situation. According to Eqs. (13), (14), the emission of an antineutrino of mass m_1 and momentum $-\mathbf{k}$ is described by the time-dependent interaction potential

$$v_{\bar{n}u_e}(\mathbf{x},t) = \frac{e^{i\mathbf{k}\mathbf{x}}}{(2\pi)^3}e^{i\omega_{\mathbf{k},1}t}, \qquad \omega_{\mathbf{k},1} = \sqrt{\mathbf{k}^2+m_1^2}, \tag{43}$$

where we have dropped the factor $\cos^2\theta$ accompanying the coupling g, for brevity. As before, the incoming ion I has the momentum $\mathbf{p}$. Its energy is $\mathbf{p}^2/2M$. The outgoing ion I_H has the momentum $\mathbf{p}'$ and an energy $E_{\mathbf{p}'} = M_H - M + \mathbf{p}'^2/2M_H$.

Let us first adapt the Lippmann-Schwinger approach. We introduce the time-dependent interacting state $|\mathbf{p}^{(+)}(t)\rangle$, which is an eigenstate of the full Hamiltonian, and satifies the time-dependent Schrödinger equation

$$i\partial_t|\mathbf{p}^{(+)}(t)\rangle = [\hat{H}_0 + \hat{v}(\mathbf{x},t)]|\mathbf{p}^{(+)}(t)\rangle, \qquad \hat{H}_0 = \hat{\mathbf{p}}^2/2M. \tag{44}$$

The formal solution of this is

$$|\mathbf{p}^{(+)}(t)\rangle \equiv \hat{U}(t)|\mathbf{p}\rangle, \qquad \hat{U}(t) \equiv \hat{T}e^{-i\int_0^t dt'[\hat{H}_0+\hat{V}(\mathbf{x},t)]}, \tag{45}$$

An implicit exression for this state can be written, by analogy with (36), as

$$|\mathbf{p}^{(+)}(t)\rangle = |\mathbf{p}\rangle e^{-iE^0_\mathbf{p}t} + \frac{1}{i\partial_t - \hat{H}_0 + i\eta}\hat{V}(\mathbf{x},t)|\mathbf{p}^{(+)}(t)\rangle, \qquad E^0_\mathbf{p} = \mathbf{p}^2/2M. \tag{46}$$

This can again be verified by multiplication from the left with $i\partial_t - \hat{H}_0$. Multiplying (46) by $e^{iE^0_{\mathbf{p}}t}$, we obtain

$$e^{iE^0_{\mathbf{p}}t}|\mathbf{p}^{(+)}(t)\rangle = |\mathbf{p}\rangle + \frac{1}{i\partial_t + E^0_{\mathbf{p}} - \hat{H}_0 + i\eta} e^{iE^0_{\mathbf{p}}t}\hat{V}(\mathbf{x}, t)e^{-iE_{\mathbf{p}}t}|\mathbf{p}^{(+)}(0)\rangle, \qquad (47)$$

To lowest approximation, we replace $|\mathbf{p}^{(+)}(0)\rangle$ by $|\mathbf{p}\rangle$ and insert (43) to find

$$e^{iE^0_{\mathbf{p}}t}|\mathbf{p}^{(+)}(t)\rangle = |\mathbf{p}\rangle + g\int_{-\infty}^{\infty} dt'\, \hat{G}(t,t')e^{i\mathbf{k}\hat{\mathbf{x}}}e^{i\omega_{\mathbf{k},1}(t'-t_0)}|\mathbf{p}\rangle e^{-i(E_{\mathbf{p}}-E^0_{\mathbf{p}})t'}, \qquad (48)$$

where $\hat{G}(t,t')$ is the Fourier representation of the operator $(i\partial_t + E^0_{\mathbf{p}} - \hat{H}_0 + i\eta)^{-1}$:

$$\hat{G}(t,t') \equiv \int_{-\infty}^{\infty} \frac{dE}{2\pi} \frac{e^{-iE(t-t')}}{E + E^0_{\mathbf{p}} - \hat{H}_0 + i\eta} \qquad (49)$$

Performing the integral over t' in (48) yields

$$e^{iE^0_{\mathbf{p}}t}|\mathbf{p}^{(+)}(t)\rangle = |\mathbf{p}\rangle + g\,\frac{e^{-i(E_{\mathbf{p}}-E^0_{\mathbf{p}}-\omega_{\mathbf{k},1})t}}{E_{\mathbf{p}} - \hat{H}_0 - \omega_{\mathbf{k},1} + i\eta} e^{i\mathbf{k}\hat{\mathbf{x}}}|\mathbf{p}\rangle e^{-i\omega_{\mathbf{k},1}t_0}. \qquad (50)$$

We multiply this equation from the left and insert in front of the right-hand state $|\mathbf{p}\rangle$ a completeness relation $\int d^3x|\mathbf{x}'\rangle\langle\mathbf{x}'| = 1$. Approximating the matrix elements $\langle\mathbf{x}|(E_{\mathbf{p}} - \hat{H}_0 + i\eta)^{-1}|\mathbf{x}'\rangle$ as usual in the large-$\mathbf{x}$ regime by

$$\langle\mathbf{x}|\frac{1}{E_{\mathbf{p}} - \hat{H}_0 - \omega_{\mathbf{k},1} + i\eta}|\mathbf{x}'\rangle = \int \frac{d^3p'}{(2\pi)^3}\frac{e^{i\mathbf{p}'(\mathbf{x}-\mathbf{x}')}}{E_{\mathbf{p}} - \mathbf{p}'^2/2M - \omega_{\mathbf{k},1} + i\eta} \approx -2M\frac{e^{ip'_k r}}{4\pi r}e^{-ip'_k\hat{\mathbf{x}}\mathbf{x}'}, \qquad (51)$$

where p'_k is the momentum of the ion I_H which conserves the energy, i.e., ${p'_k}^2/2M = E_{\mathbf{p}} - \omega_{\mathbf{k},1}$. With this we obtain

$$\langle\mathbf{x}|e^{iE^0_{\mathbf{p}}t}|\mathbf{p}^{(+)}(t)\rangle = \langle\mathbf{x}|\mathbf{p}\rangle - 2M\frac{g}{(2\pi)^3}\frac{e^{ip'_k r}}{4\pi r}\int d^3x' e^{-ip'_k\hat{\mathbf{x}}\mathbf{x}'}e^{i(\mathbf{p}+\mathbf{k})\mathbf{x}'}e^{-i\omega_{\mathbf{k},1}t_0}. \qquad (52)$$

We now perform the integral over $\mathbf{x}'$. For this we assume the inital state to have zero momentum, $\mathbf{p}+\mathbf{k} = 0$. The integral over $\mathbf{x}'$ forces the momentum of the outcoming ion I_H to be equal to $\mathbf{k}$. The integral over $\mathbf{x}'$ creates a δ-function $(2\pi)^3\delta^{(3)}(p'_k\hat{\mathbf{x}} - \mathbf{p} - \mathbf{k})$, so that we obtain

$$\langle\mathbf{x}|e^{iE^0_{\mathbf{p}}t}|\mathbf{p}^{(+)}(t)\rangle = \langle\mathbf{x}|\mathbf{p}\rangle - 2M\frac{g}{4\pi r}e^{-ip'_k\hat{\mathbf{x}}\mathbf{x}'}\delta^{(3)}(\mathbf{p}' - \mathbf{p} + \mathbf{k})e^{-i\omega_1 t_0}. \qquad (53)$$

Note that since energy and momentum are balanced, then $\omega_{\mathbf{k},1} = \omega_1$ of Eq. (5).

Consider now the case of two oscillating mass states and let us study the temporal behavior of the emerging energy distribution. The experiment does not explore the limit of very large times but measures the t-dependence starting from small t after the ion enters the storage ring. Instead of the limiting energy conservation δ-function $-2\pi i\delta(E_{\mathbf{p}'} - E_{\mathbf{p}})$ in (37), it observes an approximation to it valied for short times.

To find it we insert, instead of (43), the mixed potential (14) into Eq. (47), so that the time-dependent factor in the resulting equation of type (48) has the form

$$-i \int_0^t dt \left[\cos^2\theta\, e^{i(E_{1,\mathbf{k}'}-E_{\mathbf{k}})t} + \sin^2\theta\, e^{i(E_{2,\mathbf{k}'}-E_{\mathbf{k}})t}\right], \tag{54}$$

where $E_{i,\mathbf{k}'} \equiv \sqrt{\mathbf{k}'^2 + M_H^2} + \omega_{\mathbf{k}',i}$ and $E_{\mathbf{k}} = M_H + Q$. Since $m_i^2 \ll Q \ll M_H$, we can approximate $E_{i,\mathbf{k}'} - E_{\mathbf{k}} \approx \omega' - \omega_i$ where $\omega_i \approx Q + m_i^2/2M_H$. Let us write $\omega_{1,2} \equiv \bar{\omega} \mp \frac{1}{2}\bar{\Delta} m^2/2M_H = \bar{\omega} \mp \frac{1}{2}\bar{\Delta}\omega$. Then (54) becomes, with the abbreviations $C \equiv \cos^2\theta$ and $S \equiv \sin^2\theta$,

$$-\left(C\frac{e^{i(\omega'-\omega_1)t}-1}{\omega'-\omega_1} + S\frac{e^{i(\omega'-\omega_2)t}-1}{\omega'-\omega_2}\right)$$
$$= -i\left\{Ce^{i(\omega'-\omega_1)t/2}\frac{\sin[(\omega'-\omega_1)t/2]}{(\omega'-\omega_1)/2} + Se^{i(\omega'-\omega_2)t/2}\frac{[\sin(\omega'-\omega_2)t/2]}{(\omega'-\omega_2)/2}\right\}. \tag{55}$$

The absolute square of (55) mutiplied by some factor $|T|^2$ determines probability $P(t)$ to find the initial ions in the ring at the time t. The integral over the final momenta is dominated by the immediate neighborhood of the poles at $\omega_{1,2}$ where $|\mathbf{k}| \equiv Q$. There we may ignore the $\mathbf{k}$-dependence of $|T|^2$, approximating it by a constant, and obtain the probability for the ion I_H to emerge with an energy $M_H + Q - \omega'$ in the center-of-mass frame

$$P^{\omega'}(t) \approx \left\{C^2 s_1^2(\omega') + S^2 s_2^2(\omega') + 2CS\cos(\bar{\Delta}\omega t/2) s_1(\omega') s_2(\omega')\right\}|T|^2, \tag{56}$$

where $s_i(\omega') \equiv \sin[(\omega'-\omega_i)t/2]/[(\omega'-\omega_i)t/2]$. For large t, the limiting relation $\sin^2 at/a^2 \to t\pi\delta(a)$ allows us approximate $s_i^2(\omega') \approx 2\pi t\delta(\omega'-\omega_i)$, and thus the first two terms in (56) by

$$P_{12}^{\omega'}(t) \approx 2\pi t[C^2\delta(\omega'-\omega_1) + S^2\delta(\omega'-\omega_2)]|T|^2. \tag{57}$$

If this is integrated over $\int d^3k'/(2\pi)^3 \approx Q^2\int d\omega'/2\pi^2$, the probabilities of ν_1- and ν_2-decays simply add, thereby yielding the ordinary β-decay rate $I \to I_H + \bar{\nu}_e$ without mixing. Consider now the third term in (56). Here the integral over all ω' yields

$$\int d\omega' P_3^{\omega'}(t) \approx 2CS\cos(\bar{\Delta}\omega t/2)\, 2\pi\frac{\sin(\bar{\Delta}\omega t/2)}{\bar{\Delta}\omega/2}|T|^2. \tag{58}$$

Thus we obtain for the total decay rate as a function of time

$$\dot{P}(t) = \int d\omega' \left[\dot{P}_{12}^{\omega'}(t) + \dot{P}_3^{\omega'}(t)\right] \approx 2\pi\left[1 + 2CS\cos(\bar{\Delta}\omega t)\right]|T|^2. \tag{59}$$

It would be interesting to observe experimentally the predicted distribution (56) of antineutrino energies ω' by measuring the recoil momenta $\mathbf{k}$ of the final ions I_H. The distribution consists of two peaks associate with the emission of the antineutrinos $\bar{\nu}_1$ and $\bar{\nu}_2$. Centered between them lies the oscillating distribution proportional to $s_1(\omega')s_2(\omega')$ shown in Fig. 1.

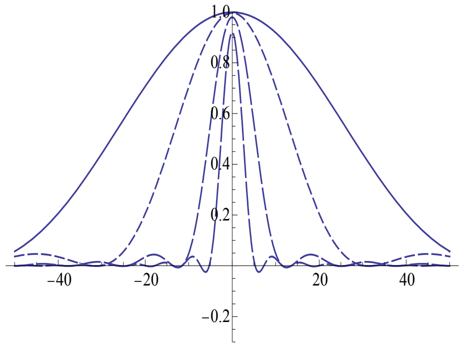

Figure 3 Distribution of the temporally oscillating part of the antineutrino center-of-mass energies ω' in the decay $I \to I_H + \bar{\nu}_e$ for different times t [the function $s_1(\omega')s_2(\omega')$ in Eq. (56)].

Note that the usual Feynman diagrams in momentum space cannot be used to describe the observed oscillations as done in Ref. [17], since they imply taking the limit $t \to \infty$ in which the oscillations disappear. Only diagrams in *spacetime* involving a *propagator matrix* in ν_1, ν_2 field space with off-diagonal matrix elements are applicable, and these reproduce the above-calculated oscillations.

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