Challenge to find Quasicrystals with Seven-Fold Symmetry

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We propose that exerimentalists should look for materials exhibiting sevenfold quasicrytalline symmetry, and show a picture of the expected density distribution.

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Recently, Dan Shechtman was awarded the Nobel-Prize for his 1984 discovery of a quasicrystalline phase with five-fold symmetry [1] in a sputtered Al-Mn alloy. While the solid-state community considered this discovery initially with great skepticism [2], as is now vividly described in many newspaper articles and in Shechtman's Wikipedia page [3], the existence of material with such a symmetry had been proposed three years earlier by Kleinert and Maki in 1981 [4]. In that paper, an extremely simple order parameter was composed of a sum of exponentials

$$\phi(\mathbf{x}) = \sum_{k=0}^{N-1} e^{i\mathbf{x}\mathbf{p}_k} \tag{1}$$

with momenta $\mathbf{p}_k$ which are all equally long and point into the directions observable in Bragg scattering. These were allowed to point along the edges of all Platonic Solids, in particular of the icosahedron as shown in Fig. 1.

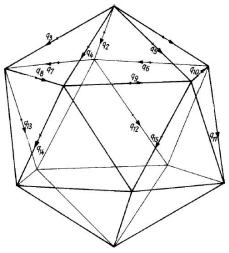

Figure 1: Momenta along the edges of the icosahedral Platonic Solid (from [4]).

In two dimensions, the simplest set of momenta with fivefold symmety is

$$\mathbf{p}_k = (\cos\alpha_k, \sin\alpha_k), \qquad \alpha_k = 2\pi k/N, \tag{2}$$

which for $N = 3$ produces the well-known triangular density distribution shown in Fig. 2a. For $N = 5$ it yields the five-fold symmetry of [4] shown in Fig. 2b.

It is now suggestive to look also for materials exhibiting the sevenfold symmetry. For them there is no three-dimensional regular structure. Most simply, it could consist of equally spaced layers of two-dimensional material. The associated density distribution is illustrated in Fig. 3. It will be interesting to see which elements or compounds qualify for this symmetry.

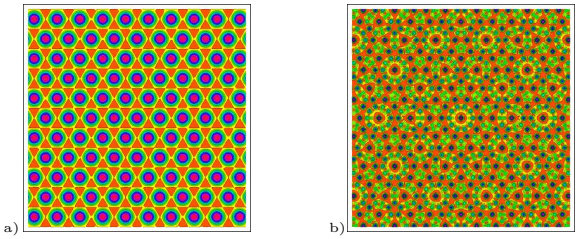

Figure 2: Density $\rho(\mathbf{x}) = |\phi(\mathbf{x})|^2$ obtained from order parameter (1) for $N = 3$ and $N = 5$, respectively.

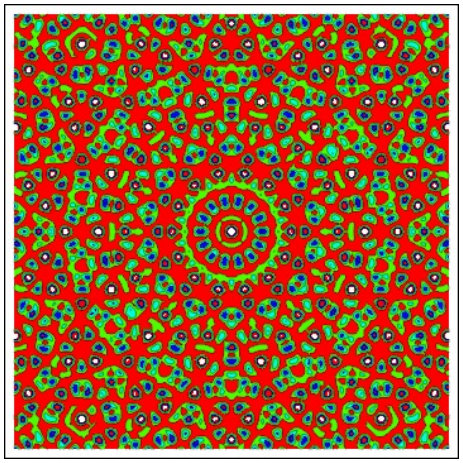

Figure 3: Density $\rho(\mathbf{x}) = |\phi(\mathbf{x})|^2$ obtained from order parameter (1) for $N = 7$. Note the similarity with sevenfold Mandalas [5].

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[1] D. Shechtman, I. Blech, D. Gratias, J. W. Cahn, Phys. Rev. Lett. **53**, 1951 (1984).

[2] Here a short collection of comments by his colleagues:

- The head of the NBS group of researchers told him: "Dr. Shechtman, you are embarrassing the group and

I have to ask you to transfer to another research group."

- The editors of the Journal of Applied Physics wrote: "The paper will not interest physicists. We recommend that you send it to a metallurgical journal."
- "Shechtman is talking nonsense", at a scientific conference where Shechtman was sitting in the audience. "There are no such things as quasicrystals there are only quasi-scientists."

See http://www1.technion.ac.il/en/technion/sci-tech/111006-nobel.

[3] http://en.wikipedia.org/wiki/Dan_Shechtman. On this page he recounts his experience of "several years of hostility".

[4] H. Kleinert and K. Maki, Fortschr. Phys. 29, 1 (1981) (see p. 242 and 243). See also the internet page http://klnrt.de/ic.

[5] For a collection of such religious symbols see http://www.shutterstock.com/pic-82325917/stock-vector-seven-fold-mandalas-with-variations.html.