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Is 'dark matter' made entirely of the gravitational field?

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Abstract
We argue that part of 'dark matter' is not made of matter, but of the singular world lines and world surfaces in the solutions of Einstein's vacuum field equation $G_{\mu\nu} = 0$. Their Einstein–Hilbert action governs, in a slightly modified form, also their quantum fluctuations in a partition function formed from a sum over all line and surface configurations. For world surfaces, the Einstein–Hilbert action coincides with that of closed bosonic 'strings' in four spacetime dimensions, which appear here in a new physical context.

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The surprisingly large orbital velocities of galaxies in clusters induced F Zwicky in 1933 to postulate the existence of dark matter. A confirmation came from plots of the orbital velocities of stars versus distance inside individual galaxies, whose explanation asked for large amounts of invisible matter in each galaxy. The Friedmann model of the evolution of the Universe indicates that dark matter constitutes a large percentage, roughly 23%, of the mass energy of the Universe. If dark matter is added to the so-called 'dark energy', which accounts for roughly 70% of the energy, the visible matter is practically negligible, which is the reason for ignoring it completely in the most extensive computer simulation reported so far of the evolution of cosmic structures [1], the so-called 'Millenium Simulation'.

There are many speculations as to its composition and we want to propose, in this paper, the simplest possible explanation of at least a part of it.

Let us remember that all static electric fields in nature may be considered as originating from the nontrivial solutions of the Poisson equation for the electric potential $\phi(x)$ as a function of $x = (t, \mathbf{x})$:

$$\Delta\phi(x) \equiv \nabla \cdot \nabla\,\phi(x) = 0. \tag{1}$$

The simplest of them has the form e/r, where $r = |\mathbf{x}|$, and is attributed to point-like electric charges, whose size e can be extracted from the pole strength of the singularity of the electric field $\mathbf{E}$ which points radially outward and has a strength e/r^2. This becomes visible by performing an area integral over the $\mathbf{E}$ field around the singularity which, by the famous Gauss integral theorem,

$$\int_V d^3x\; \nabla \cdot \mathbf{E} = \int_A d^2\mathbf{a} \cdot \mathbf{E}, \tag{2}$$

is equal to the volume integral over $\nabla \cdot \mathbf{E} = -\Delta\phi(x)$. Thus a field that solves the homogeneous Poisson equation can have a nonzero integral $\int_V d^3x\, \Delta\phi(x) = -4\pi e$. This fact is more properly expressed with the help of a Dirac delta function $\delta^{(3)}(\mathbf{x})$ as

$$\Delta\phi(x) = -4\pi e \delta^{(3)}(\mathbf{x}). \tag{3}$$

In the following, it is useful to re-express the Gauss theorem (2) in a one-dimensional (1D) form as

$$\int^R dr\, \partial_r E(r) = E(R). \tag{4}$$

This appears in the radial part of the Gaussian relation

$$\int d^3x\; \nabla{\cdot}\mathbf{E} = -4\pi \int^R dr\; r^2 \nabla{\cdot}\nabla e/r = 4\pi \int^R dr\; \partial_r e, \tag{5}$$

so that we find the electric charge e from the 1D equation

$$\int^R dr\; \partial_r e = e, \tag{6}$$

showing once more that $\nabla \cdot \mathbf{E} = 4\pi e \delta^{(3)}(\mathbf{x})$.

For celestial objects, the situation is quite similar. The Einstein equation in the vacuum, $G_{\mu\nu}=0$, possesses simple nontrivial solutions in the form of the Schwarzschild metric defined by

$$ds^2 = B(r)c^2 dt^2 - A(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (7)$$

with $B(r)=1-r_S/r$, $A(r)=1/B(r)$, where $r_S \equiv 2G_N M/c^2$ is the Schwarzschild radius and G_N is Newton's gravitational constant. Its Einstein tensor has the component

$$G_t^t = \frac{A'}{A^2 r} - \frac{1-A}{Ar^2}. \qquad (8)$$

This vanishes. However, if we calculate the volume integral $\int_V d^3x\,\sqrt{-g}G_t{}^t$, we find that $\int_V d^3x\sqrt{B/A}[A'/Ar-(1-A)/r^2]$, which is equal to $\int^R dr\,\partial_r(r-r/A) = (2G_N/c)\int^R dr\,\partial_r M = (2G_N/c)M$. Using the 1D form (4) of Gauss' integral theorem, we find, also here, a nonzero integral

$$\int_V d^3x\sqrt{-g}\,G_t^t = \kappa c M, \qquad (9)$$

where κ is defined, in terms of the Planck length l_P, as

$$\kappa \equiv 8\pi l_P^2/\hbar = 8\pi G_N/c^3. \qquad (10)$$

The other diagonal components of the Einstein tensor have the volume integrals

$$G_r{}^r = \frac{1}{A}\left(-\frac{B'}{Br} - \frac{1-A}{r^2}\right), \qquad (11)$$

$$G_\theta{}^\theta = G_\phi{}^\phi = \frac{1}{A}\left(-\frac{B''}{2B} + \frac{B'^2}{4B^2} - \frac{B'}{2Br} + \frac{A'B'}{4AB} + \frac{A'}{2Ar}\right). \qquad (12)$$

Inserting $AB=1$, these can be rewritten as

$$G_r{}^r = \frac{1}{r^2}\partial_r\left(r - \frac{r}{A}\right) = \frac{2G_N}{c^2}\frac{1}{r^2}\partial_r M, \qquad (13)$$

$$G_\theta{}^\theta = G_\phi{}^\phi = \frac{1}{r^2}\left(\frac{A'r^2}{2A^2}\right) = -\frac{G_N}{c^2}\frac{1}{r^2}\partial_r M. \qquad (14)$$

Performing the spatial integrals over these gives

$$\frac{c}{8\pi G_N}\int_V d^3x\sqrt{-g}G_r{}^r = \int^R dr\,\partial_r M = M \qquad (15)$$

$$\frac{c}{8\pi G_N}\int_V d^3x\sqrt{-g}G_\theta{}^\theta = -\frac{M}{2}, \qquad (16)$$

and the same thing for G_ϕ^ϕ, so that

$$\int_V d^3x\sqrt{-g}\,G_\mu^\mu = \kappa c M. \qquad (17)$$

From this and (9), we identify the mass of the object as M. Another way of obtaining the same result is via the ADM-mass formula [25] of a black hole:

$$M_{\rm ADM} = -\frac{c^2}{16\pi G_N}\int_S\left(\frac{\partial}{\partial x_j}g_{ii} - \frac{\partial}{\partial x_i}g_{ij}\right)dS^j, \qquad (18)$$

where S is an asymptotic surface around the black hole, and dS^j is its normal element. Inserting the long-distance behavior $g_{ij} \approx -\delta_{ij} + 2MG_N\delta_{ij}/c^2 r$, the integral (18) agrees with (17).

If the mass point moves through spacetime along a trajectory parameterized by $x^\mu(\tau)$, it has an energy–momentum tensor

$$T^{\mu\nu}(y) = \frac{M}{\sqrt{-g}}\int_{-\infty}^{\infty} d\tau\,\dot{x}^\mu(\tau)\dot{x}^\nu(\tau)\delta^{(4)}(y - x(\tau)), \qquad (19)$$

where a dot denotes the τ-derivative. We may integrate the associated solution of the homogeneous Einstein equation $G_{\mu\nu}=0$ over spacetime and find, using $\dot{x}^2=1$, that its Einstein–Hilbert action

$$\mathcal{A}_{\rm EH} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R \qquad (20)$$

is equal to the classical action of a point-like particle:

$$\mathcal{A}_{\rm EH}^{\rm worldline} \propto -Mc\int ds. \qquad (21)$$

A slight modification of (21), which is the same classically, but different for fluctuating orbits, also describes the quantum physics of a spin-0 particle [2] in a path integral over all orbits. Thus Einstein's action for a singular worldline in spacetime can be used to define also the quantum physics of a spin-0 point particle.

In addition to point-like singularities, the homogeneous Einstein equation will also possess singularities on surfaces in spacetime. These may be parameterized by $x^\mu(\sigma,\tau)$, and their energy–momentum tensor has the form

$$T^{\mu\nu}(y) = \frac{1}{\sqrt{-g}}\int_{-\infty}^{\infty} d\sigma\,d\tau\,(\dot{x}^\mu\dot{x}^\nu - x'^\mu x'^\nu)\delta^{(4)}(y - x(\sigma,\tau)), \qquad (22)$$

where a prime denotes a σ-derivative. In the associated vanishing Einstein tensor, the δ-function on the surface manifests itself in the nonzero spacetime integral[1]:

$$\int d^4x\sqrt{-g}\,G_\mu{}^\mu \propto \int_A d^2a \equiv \frac{1}{2}\int d\sigma\,d\tau\,(\dot{x}^2 - x'^2). \qquad (23)$$

In analogy with the line-like case, we obtain for such a singular field an Einstein–Hilbert action (20)

$$\mathcal{A}_{\rm EH}^{\rm world\ surface} \propto -\frac{1}{2\kappa}\int_A d^2a = -\frac{\hbar}{16\pi l_P^2}\int_A d^2a. \qquad (24)$$

Apart from a numerical proportionality factor of the order of one, this is precisely the Nambu–Goto action of a bosonic closed string in four spacetime dimensions:

$$\mathcal{A}_{\rm NG} = -\frac{\hbar}{2\pi l_s^2}\int_A d^2a, \qquad (25)$$

where l_s is the so-called string length l_s, related to the slope parameter $\alpha' = dl/dm^2$ in the string tension $T \equiv 1/2\pi\alpha'\hbar c$ by $l_s = \hbar c\sqrt{\alpha'}$. Note that, in contrast to the worldlines, there is no extra mass parameter M.

[1] For this, one takes the delta function on the surface as defined by Dirac [5] (see his equation (15)) and makes use of the distributional form of Gauss's integral theorem as formulated by Kleinert [5] or on p 253 of the textbook [9].

The original string model was proposed to describe color-electric flux tubes and their Regge trajectories whose slopes α' lie around $1\,\mathrm{GeV}^{-2}$. However, since the tubes are really fat objects, as fat as pions, only very long flux tubes are approximately line-like. Short tubes degenerate into spherical 'MIT-bags' [6]. The flux-tube role of strings was therefore abandoned, and the action (25) was re-interpreted in a completely different fashion, as describing the fundamental particles of nature, assuming l_S to be of the order of l_P. Then the spin-2 particles of (25) would interact like gravitons and define quantum gravity. But also the ensuing 'new string theory' [3] has been criticized by many authors[2]. One of its most embarrassing failures is that it has not produced any experimentally observable results. The particle spectra of its solutions have not matched the existing particle spectra. The proposal of this note cures this problem. If 'strings' describe 'dark matter', there would be no need to reproduce other observed particle spectra. Instead, their celebrated virtue, that their spin-2 quanta interact like gravitons, can be used to fix the proportionality factor between the Einstein action (24) and the string action (25).

It must be kept in mind that just as $-Mc\int ds$ had to be modified for fluctuating paths [2], also the Nambu–Goto action (25) needs a modification if the surfaces fluctuate. That was found by Polyakov when studying the consequences of the conformal symmetry the theory [4]. He replaced the action (25) by a new action that is equal to (25) at the classical level, but contains in $D \neq 26$ dimensions another spin-0 field with a Liouville action.

Since the singularities of Einstein's fields possess only gravitational interactions, their identification with 'dark matter' seems very natural. All visible matter consists of singular solutions to the Maxwell equations and the field equations of the standard model. A grand-canonical ensemble of these and the smooth wave solutions of the standard model explain an important part of the matter in the Friedmann model of cosmological evolution.

But the main contribution to the energy comes from the above singularities of Einstein's equation. Soon after the Universe was created, the temperature was so high that the configurational entropy of the surfaces overwhelmed completely the impeding Boltzmann factors. Spacetime was filled with these surfaces in the same way as superfluid helium is filled with the world surface of vortex lines. In hot helium, these lie so densely packed that the superfluid behaves like a normal fluid [8, 9]. The Einstein–Hilbert action of such a singularity-filled turbulent geometry behaves like the action of a grand-canonical ensemble of world surfaces of a bosonic closed-string model. Note that here these are 2D objects living in four spacetime dimensions, and there is a definite need to understand their spectrum by studying the associated Polyakov action, without circumventing the accompanying Liouville field by escaping into unphysical dimensions.

It should be noted that in the immediate neighborhood of the singularities, the curvature will be so high that Einstein's linear approximation $-(1/2\kappa)R$ to the Lagrangian must break down and will have to be corrected by some nonlinear function of R that starts out like Einstein's, but continues differently. A possible modification was suggested a decade ago [10], and many other options have been investigated since then [11].

After the Big Bang, the Universe expanded and cooled down, so that large singular surfaces shrunk by emitting gravitational radiation. Their density decreased, and some phase transition made the cosmos homogeneous and isotropic on the large scale[3]. But the cosmos remained filled with gravitational radiation and small singular surfaces that had shrunk until their sizes reached the levels stabilized by quantum physics, i.e. when their fluctuating action decreased to order $\hbar$. The statistical mechanics of this cosmos is the analogue of a spacetime filled with superfluid helium whose specific heat is governed by the zero-mass phonons and by rotons. Recall [13] that in this way Landau discovered the fundamental excitations called rotons whose existence he deduced from the temperature behavior of the specific heat. In the Universe, the role of rotons is played by the smallest surface-like singularities of the homogeneous Einstein equation, whose existence we deduce from the cosmological requirement of dark matter.

The situation can also be illustrated by a further analogy with a many-body system. The defects in a crystal whose 'atoms' have a lattice spacing l_P simulate precisely the mathematics of a Riemann–Cartan spacetime, in which disclinations and dislocations define curvature and torsion [9, 14, 15]. Thus we may imagine a model of the universe as a 'floppy world crystal' [16], a liquid-crystal-like phase [17] in which a first melting transition has led to correct gravitational $1/r$-interactions between disclinations. The initial hot universe was filled with defects—it was a 'world liquid'. After cooling down to the present liquid-crystal state, there remained plenty of residual defects around, which form our 'dark matter'.

We know that the cosmos is filled with a cosmic microwave background (CMB) of photons of roughly 2.725 K, the remnants of the big bang. These contribute to the Friedmann equation of motion a constant $\Omega_{\mathrm{rad}}h^2 = (2.47 \pm 0.01) \times 10^{-5}$, where $h = 0.72 \pm 0.03$ is the Hubble parameter, defined in terms of the Hubble constant H by $h \equiv H/(100\,\mathrm{km\,Mpc^{-1}\,s^{-1}})$. The symbol Ω denotes the energy density divided by the so-called critical density $\rho_c \equiv 3H^2/8\pi G_N = 1.88 \times 10^{-26} h^2\,\mathrm{kg\,m^{-3}}$ [18]. The baryon density contributes $\Omega_{\mathrm{rad}}h^2 = 0.0227 \pm 0.0006$ or 720 times as much, whereas the dark matter contributes $\Omega_{\mathrm{dark}}h^2 = 0.104 \pm 0.006$ or 4210 times as much. If we assume for a moment

[2] See the list of critics in the Wikipedia article on string theory; also see Schroer [7].

[3] In this aspect, there are parallels with the work of Huang *et al* [12]. However, their turbulent baby universe is filled with tangles of vortex lines of some scalar field theory, whereas mine contains only singularities of Einstein's equation. A bridge may be found by recalling that the textbook [8] explains how tangles of line-like defects can be described by a complex *disorder field theory*, whose Feynman diagrams are direct pictures of the world lines. Thus, if Huang *et al* were to interpret their scalar field as a disorder field of the purely geometric objects of my theory, the parallels would be closer. Note that in two papers with Halperin [12], Huang manages to make his scalar field theory asymptotically free in the ultraviolet (although at the unpleasant cost of a sharp cutoff introducing forces of infinite range). This property allows him to deduce an effective *dark energy* in the baby universe. With our purely geometric tangles, such an effect may be reached using a lattice gauge formulation of Einstein's theory [9, 15], sketched at the end of the text.

that all massive strings are frozen out and that only the subsequently emitted gravitons form a thermal background[4], then, since the energy of massless states is proportional to T^4, the temperature of this background would be $T_{\rm DMB} \approx 4210^{1/4} \approx 8T_{\rm CMB} \approx 22\,{\rm K}$. In general, we expect the presence of also other singular solutions of Einstein's equation to change this result.

There is an alternative way of deriving the above-described properties of the fluctuating singular surfaces of Einstein's theory. One may rewrite Einstein's theory as a gauge theory [9, 15] and put it on a spacetime lattice [20]. Then the singular surfaces are built explicitly from plaquettes, as in lattice gauge theories of asymptotically free non-Abelian gauge theories [21]. In the Abelian case, the surfaces are composed as shown in [22]; for the non-Abelian case, see [23]. An equivalent derivation could also be given in the framework of *loop gravity* [24]. But that would, require a separate study, which is beyond the scope of this paper.

Summarizing, we have seen that the Einstein–Hilbert action governs not only the classical physics of gravitational fields[5] but also, via the fluctuations of its line- and surface-like singularities, the quantum physics of dark matter. A string-like action, derived from it for the fluctuating surface-like singularities, contains interacting spin-2 quanta that define a finite quantum gravity.

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[4] We do this, although the weakness of gravitational interactions may be an obstacle to thermal equilibration. See [19].

[5] Other studies of purely gravitational field configurations as dark matter see [26].