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Photoproduction in semiconductors by onset of magnetic field

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Abstract – The energy bands of a semiconductor are lowered by an external magnetic field. When a field is switched on, the straight-line trajectories near the top of the occupied valence band are curved into Landau orbits and Bremsstrahlung is emitted until the electrons have settled in their final Fermi distribution. We calculate the radiated energy, which should be experimentally detectable, and suggest that a semiconductor can be cooled by an oscillating magnetic field.

Introduction. – Valence and conduction electrons near the Fermi sphere of a semiconductor have many similarities with the Dirac electrons in the vacuum. In fact, the hole state as a missing state in the valence band is completely analogous to a positron in Dirac's sea of occupied negative-energy electrons. The band width Δ of a superconductor, which is typically of the order of 0.1 eV, corresponds to the energy gap $\Delta = 2m_e c^2 \simeq 1.04\,\mathrm{MeV}$ in Dirac's vacuum, above which electron-positron pairs can be produced. As a consequence, the electromagnetic behavior of a semiconductor at and below room temperature with $k_B T \simeq 0.024\mathrm{eV}$ ($k_B =$ Boltzmann constant) can be studied by the same field-theoretic techniques as a Dirac vacuum for $k_B T \ll 2mc^2$. In particular, one can transfer the results found by Heisenberg and Euler [1,2] for electrons and positron to electrons and holes. A strong electric field larger than $E_c = m_e^2 c^3/e\hbar \simeq 1.3\cdot 10^{18}\,\mathrm{V/m}$ leads to electron-hole pair production.

A magnetic field H lowers the energy of the ground state since the electrons are curved into Landau orbits [3]. This should produce synchrotron radiation. For the magnetic field switched on in the vacuum, this was pointed out in ref. [4] as a consequence of the Euler-Heisenberg calculation [1,2]. However, this effect could become observable only for extremely large magnetic fields which cannot be attained in present-day laboratories. It will play a role mainly in astrophysical events, such as supernova explosions, and during the formation of neutron stars, where magnetic fields reach $H_c = m_e^2 c^3/e\hbar = 4.3\times 10^{13}$ gauss. It may also account for the emission of an anomalous X-ray pulsar [5].

The purpose of this note is to suggest observing this type of synchrotron radiation at presently available magnetic fields of 10^5 gauss by placing a semiconductor in a magnetic field. Moreover, we point out that this may give rise to a novel cooling technique for semiconductors.

Electron and hole states. – The electrons in the highest valence band of a semiconductor occupy Bloch states which look like free-particle states $\psi_{\mathbf{k}}(\mathbf{r},t) = e^{i\mathbf{kx}-i\xi_{\mathbf{k}}t}\psi_{\mathbf{k}}(\mathbf{r})$, where $\mathbf{k}$ is the Bloch momentum and $\xi_{\mathbf{k}}$ the energy measured from the Fermi surface between the bands. Near the top of the band, the energy can be expanded as [6]

$$\xi_{\mathbf{k}} \simeq \Delta_v + \sum_i \frac{k_i^2}{2m_i^*}, \quad (i = x, y, z), \tag{1}$$

where Δ_v is the distance of the top of the valence band from the Fermi level, which is usually close to $\Delta/2$, and m_i are the effective masses in the three space directions. The constant-energy surface $\xi_{\mathbf{k}} = \mathrm{const}$ is in general an ellipsoid. For simplicity, we shall assume axial symmetry with $m_x^* = m_y^* \equiv m_\perp^*$, and shall switch on the magnetic field in the z-direction. Then, the energies (1) are replaced by the Landau energies

$$\xi(n, k_z) = \Delta_v + \left[\frac{k_z^2}{2m_z^*} + (n+\lambda)\omega^*\right], \quad \omega^* = \frac{eH}{m_\perp^*}, \tag{2}$$

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where the constant λ is independent of n, k_z and H [6]. The area of the Landau orbits $A(\xi_{\mathbf{k}}, k_z)$ is quantized,

$$A(n,k_z) = (n+\lambda)\Delta A, \qquad \Delta A \equiv 2\pi eH, \tag{3}$$

where $n=0,1,2,\dots$. Equations (2), (3) are Onsager's famous result.

Given a semiconductor sample of volume $V = L_xL_yL_z$, the electrons of a fixed quantum number n occupy a phase space $L_xL_y\Delta A$. Dividing this by the volume $(2\pi)^2$ per quantum state, we obtains the degeneracy of the states of fixed n:

$$\mathcal{D} = 2\frac{L_xL_y\Delta A}{(2\pi)^2} = 2\frac{L_xL_yeH}{2\pi}, \tag{4}$$

where the factor 2 accounts for the spin degeneracy and $\mathcal{D}$ is independent of n. This is of course the same as for free electrons.

$$\frac{2e}{2\pi\hbar} \simeq 4.84\cdot 10^6 \frac{1}{\text{cm}^2\,\text{gauss}}, \tag{5}$$

the degeneracy for a $1\times 1\,\text{cm}^2$ sample in a field of one kilogauss is about 10^{10}.

Energy difference. – Summing the energy spectrum (2) over all states in phase space, we obtain the energy of the semiconductor sample,

$$E^H_{\text{tot}} = 2\frac{V\Delta A}{(2\pi)^2}\sum_n \int \frac{\mathrm{d}k_z}{2\pi}\xi(n,k_z). \tag{6}$$

From this we have to subtract the energy at $H=0$:

$$E_{\text{tot}} = 2V\int \frac{\mathrm{d}k_x\mathrm{d}k_y\mathrm{d}k_z}{(2\pi)^3}\left[\Delta_v + \left(\frac{k^2_\perp}{2m^*_\perp} + \frac{k^2_z}{2m^*_z}\right)\right], \tag{7}$$

where $k^2_\perp = k^2_x + k^2_y$. To subtract this from (6), we express $k^2_\perp$ with the help of a continuous number $n = k^2_\perp/2m^*_\perp\omega^*$ as $\int \mathrm{d}k_x\mathrm{d}k_y = \Delta A\int_0^\infty \mathrm{d}n$, and rewrite (7) as

$$E_{\text{tot}} = 2\frac{V\Delta A}{(2\pi)^2}\int_0^\infty \mathrm{d}n\int\frac{\mathrm{d}k_z}{(2\pi)}\xi(n,k_z). \tag{8}$$

The energies have ultraviolet divergencies at large k_z, which we regularize with a smooth cutoff function $f(k_z)$ equal to unity for small $|k_z| \ll \Lambda_z$ and vanishing for large $|k_z| \gg \Lambda_z$. The cutoff Λ_z is roughly equal to π/a where a is the lattice spacing. In this way, we obtain convergent k_z-integrals

$$F(n) \equiv \int_0^\infty \frac{\mathrm{d}k_z}{2\pi} f(k_z)\xi(n,k_z), \tag{9}$$

and a finite difference between the energies (6) and (8)

$$\Delta E \equiv E^H_{\text{tot}} - E_{\text{tot}} = 4\frac{V\Delta A}{(2\pi)^2}\left[\sum_n F(n) - \int_0^\infty \mathrm{d}nF(n)\right].$$

We can now use the Euler-MacLaurin formula and the Bernoulli numbers $B_2 = 1/6$, $B_4 = -1/30, \dots$ to obtain

$$\Delta E = 4\frac{V\Delta A}{(2\pi)^2}\left[-\frac{1}{2!}B_2F'(0) - \frac{1}{4!}B_4F'''(0) + \cdots\right], \tag{10}$$

where

$$F'(0) = \omega^*\int_0^\infty \frac{\mathrm{d}k_z}{2\pi}f(k_z), \quad F'''(0) = 0, \quad \dots, \tag{11}$$

so that

$$\Delta E = -V\alpha H^2\frac{2}{3m^*_\perp}\int_0^\infty\frac{\mathrm{d}k_z}{2\pi}f(k_z). \tag{12}$$

Choosing $f(z) = \exp(-k^2_z/\Lambda^2_z)$, we find

$$\Delta E = -V\alpha H^2\frac{\Lambda_z}{m^*_\perp}\frac{1}{6\pi^{1/2}}. \tag{13}$$

In principle, we must sum over all different energy bands in eq. (12), but only the bands above and below the Fermi surface will contribute.

The energy difference (12) is negative. This has several consequences:

i) The semiconductor acts as a paramagnetic medium with permeability

$$\mu = 1 - \frac{\alpha}{6\pi}\frac{\Lambda_z}{m^*_\perp} \approx 1 - \frac{\alpha}{6\pi}\frac{\pi}{am^*_\perp}. \tag{14}$$

This should be checked by experiment.

ii) When turning on the magnetic field, the energy difference should be released by the semiconductor. This can proceed by phonon and photon production. The first will heat the system, the second could in principle be detected if there is enough surface. In the following section, we compute rate and spectrum of spontaneous photon emission.

Spontaneous photon emission. – Let us turn the magnetic field $H(t)$ adiabatically on over a long time interval $\Delta t = t^+ - t^-$:

$$H(t) = \begin{cases} 0, & t = t^- \to -\infty, \\ H, & t = t^+ \to +\infty, \end{cases} \tag{15}$$

so that the time variation of $H(t)$ is much slower than the time scale of Bloch electron states in the semiconductor, which is characterized by the period $\tau_\perp = 2\pi/\omega^*$ of the motion in the xy-plane, and by the time scale $\tau_z = \hbar/(m^*_zc^2)$ in the $\hat{\mathbf{z}}$-direction. The normalized initial and final Bloch states are

$$\begin{aligned}\psi_i &= \frac{1}{V^{1/2}}\exp\{i[k_xx + k_yy\bar{k}_zz - \xi_{\mathbf{k}}t]\},\\ \psi_f &= \frac{\chi(y)}{(L_xL_z)^{1/2}}\exp\{i[k'_xx + \bar{k}'_zz - \xi_n(k'_z)t]\},\end{aligned} \tag{16}$$

where the normalized function $\chi(y)$ are [3],

$$\begin{aligned}\chi(y) &= N^{1/2}_\chi e^{-\xi^2/2}H_n(\xi), \quad N_\chi = \frac{(eH)^{1/2}}{2^nn!\pi^{1/2}},\\ \xi &= (eH)^{1/2}\left[y - \frac{k'_x}{eH}\right],\end{aligned} \tag{17}$$

and $H_n(\xi)$ is the Hermite polynomial. The energies of these states are (1) and (2), respectively. Due to axial symmetry w.r.t. z-direction, we start from an initial state with $k_y = 0$, which will remain zero, as seen most easily form the semiclassical equation of motion: $\mathrm{d}\mathbf{k}/\mathrm{d}t = (e/m)\mathbf{k}\times\mathbf{H}$.

The probability amplitude for spontaneous photon emission is [7]

$$J_{fi}(q) = -ie\int \mathrm{d}t\mathrm{d}^3x\psi_f^*\psi_i\left(\frac{1}{2\omega_q V}\right)^{1/2}\exp\{i(\omega_q t - \mathbf{q}\cdot\mathbf{x})\}, \tag{18}$$

where $(\omega_q = |\mathbf{q}|, \mathbf{q})$ is the photon energy-momentum, and the photon field is normalized to one photon energy ω_q crossing a unit area per unit time. The integral over t, x, z gives rise to δ-functions for energy and x, z momentum conservations. The integral over y is done using the formula (eq. (7.376) in [8])

$$\int \mathrm{d}y e^{-iq_y y}e^{-\xi^2/2}H_n(\xi) = (-i)^n\left(\frac{2\pi}{eH}\right)^{1/2}\mathcal{A} \times H_n(\beta q_y), \tag{19}$$

where $\beta^2 \equiv 1/eH$ and $\mathcal{A} \equiv \exp\{-iq_y(k'_x/eH) - \beta^2 q_y^2/2\}$. As a result, eq. (18) becomes

$$J_{fi}(q) = (-i)^{n+1}e\left(\frac{N_\chi}{2\omega_q V L_y L_x^2}\right)^{1/2}\left(\frac{2\pi}{eH}\right)^{1/2} \times\mathcal{A}H_n(\beta q_y) \times(2\pi)^2\delta[\xi_n(k'_z) - \xi_{\mathbf{k}} + \omega_q]\delta(k_x - k'_x - q_x), \tag{20}$$

where $q_z = 0$ and $\omega_q = (q_x^2 + q_y^2)^{1/2}$ for $k_z = k'_z$. The photons are emitted perpendicular to the $\hat{\mathbf{z}}$-direction, as in synchrotron radiation. The squared transition amplitude is

$$|J_{fi}(q)|^2 = e^2\left(\frac{N_\chi}{2\omega_q V L_y L_x}\right)\frac{2\pi}{eH} \times H_n^2(\beta q_y)\Delta t\exp(-\beta^2 q_y^2) \times(2\pi)^2\delta[\xi_n(k'_z) - \xi_{\mathbf{k}} + \omega_q]\delta(k_x - k'_x - q_x). \tag{21}$$

This has to be summed over all final states to yield summing n with degeneracy $\mathcal{D}$ (4), and obtain,

$$\sum_f |J_{fi}(q)|^2 = e^2\left(\frac{1}{\omega_q V}\right)(|e|H)^{1/2}\Delta t\exp(-\beta^2 q_y^2) \times\sum_{n=0}^{\infty}\frac{1}{2^n\pi^{1/2}n!}H_n^2(\beta q_y)(2\pi)\delta[\xi_n(k_z) - \xi_{\mathbf{k}} + \omega_q]. \tag{22}$$

The δ-function for the total energy conservation can be rewritten as

$$(2\pi)\delta[\xi_n(k_z) - \xi_{\mathbf{k}} + \omega_q] = (2\pi)\delta\left[(n + \sigma^*)\omega^* - \frac{k_x^2}{2m_\perp^*} - \omega_q\right] = (\omega^*)^{-1}\delta_{n,n_{k_x}}, \tag{23}$$

where $n_{k_x} \geqslant 1$ is the integer closest to $(\omega^*)^{-1}(k_x^2/2m_\perp^* + \omega_q) - \sigma^*$. From (22) we obtain the probability of spontaneous photon emission from the semiconductor per unit time

$$\frac{\mathrm{d}N_\gamma}{\mathrm{d}t} = \frac{e^2}{2^{n_{k_x}}\pi^{1/2}n_{k_x}!}\left(\frac{eH}{\omega_q^2}\right)^{1/2}H_{n_{k_x}}^2(\beta q_y)\exp(-\beta^2 q_y^2), \tag{24}$$

where we can replace q_y by ω_q since $q_x = 0$. Multiplying (24) by ω_q yields the emitted energy flux.

Note that the emitted photon energies ω_q are mostly smaller than the magnetic energy scale $(eH)^{1/2}$. For large n_{k_x}-values, the rate (24) has the power-like suppression $[\omega_q/(eH)^{1/2}]^{2n_{k_x}}/n^{k_x}! \ll 1$. The leading contribution to energy production comes from the electrons with $n_{k_x} = 1$ which yield (recalling that $H_1(x) = 2x$),

$$\frac{\mathrm{d}E_\gamma}{\mathrm{d}t} \simeq \frac{2e^2}{\pi^{1/2}}\left(\frac{eH}{\omega_q^2}\right)^{\frac{1}{2}}\exp\left(-\frac{\omega_q^2}{eH}\right)\frac{\omega_q^3}{eH}, \tag{25}$$

showing a power behavior $\propto \omega_q^2$ in the low energy and an exponential falloff $\exp[-\omega_q^2/(eH)]$ at high photon energies. The spectrum is maximum at $\omega_q = \omega_q^{\max} = (eH)^{1/2}$. Today's laboratories reach $H = 10^5$ gauss for which $\sqrt{eH} \simeq 0.244\,\mathrm{eV}$ [1]. Thus, in present magnetic fields, most photons are infrared.

Integrating (25) over all photon energies, $\int \mathrm{d}\omega_q/(2\pi c)$, we obtain the total energy flux

$$\frac{\mathrm{d}E_\gamma}{\mathrm{d}t} \simeq \alpha\, eH. \tag{26}$$

There are two degenerate valence band maxima, both located at $\mathbf{k} = 0$, in silicon as well as germanium. In the quadratic approximation (1), these are spherically symmetric, *i.e.* $m_\perp^*$ is assumed to be close to m_z^*. The effective masses $m_\perp^*$ are $0.49m_e$ and $0.1m_e$ in silicon, and $0.28m_e$ and $0.044m_e$ in germanium [6]. For a rough estimate, we take $m_\perp^* = 0.1m_e c^2$ and $\sqrt{eH\hbar c} \simeq 0.244\,\mathrm{eV}$ for $H = 10^5$ gauss. The characteristic time scales are $t^* = 2\pi/\omega^* \simeq 3.3\cdot 10^{-7}\,\mathrm{s}$ and $\hbar/(m_\perp^* c^2) = 1.3\cdot 10^{-20}\,\mathrm{s}$, so that rate (26) is of the order of

$$\frac{\mathrm{d}E_\gamma}{\mathrm{d}t} \simeq 6.6\cdot 10^{11}\,\mathrm{eV\,s^{-1}}. \tag{27}$$

This should be observable by infrared light detectors placed around the sample orthogonal to the direction of the magnetic field $H \approx 10^5$ gauss.

Remarks. – In general, the rate and spectrum of spontaneous photon emission will depend on the time dependence of the magnetic field $H(t)$. Our calculation yields only a first estimate based on the adiabatic condition $\Delta t \gg t^* = 2\pi/\omega^* = 3.3\cdot 10^{-7}\,\mathrm{s}$, which is fulfilled in most experiments. On these time scales, one need not

[1] See the webpage of the National High Magnetic Field Laboratory at `http://www.magnet.fsu.edu`.

worry about photons generated by induction. These have a typical energy $\hbar/\Delta\tau$, which is much smaller than the typical energy $\sqrt{eH\hbar c}$ of the photons in (27). Of course, the experiments should be performed at low temperature to reduce the background of thermal photons, which is about $10^8/\text{cm}^3$ at room temperature.

For a periodically oscillating magnetic field $H(t) = H_0\cos(\omega_H t)$ with a period $t_H = 2\pi/\omega_H \gg t^*$, we can still apply the adiabatic approximation. In the phase that the magnetic field increases from 0 to maximum H_0, the spontaneous photon emission occurs as discussed. If these photons are not kept inside the sample by either a large opacity or reflection on the walls of a cavity, they will stream away and carry off energy. In the phase where the magnetic field decreases from the maximum H_0 to 0, the semiconductor may absorb heat from the environment. As a consequence, a periodically oscillating magnetic field $H(t)$ should be able to cool a semiconductor. If the sample is sufficiently thin the electron phonon coupling should not produce enough phonons to destroy the effect by re-heating. Hence there is a good chance that this process may have technical applications.

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