Critical fermion density for restoring spontaneously broken symmetry

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We show how the phenomenon of spontaneous symmetry breakdown is affected by the presence of a sea of fermions in the system. When its density exceeds a critical value, the broken symmetry can be restored. We calculate the critical value and discuss the consequences for three different physical systems: First, for the standard model of particle physics, where the spontaneous symmetry breakdown leads nonzero masses of intermediate gauge bosons and fermions. The symmetry restoration will greatly enhance various processes with dramatic consequences for the early universe. Second, for the Gell-Mann–Lèvy σ-model of nuclear physics, where the symmetry breakdown gives rise to the nucleon and meson masses. The symmetry restoration may have important consequences for formation or collapse of stellar cores. Third, for the superconductive phase of condensed-matter, where the BCS condensate at low-temperature may be destroyed by a too large electron density.

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Introduction. Spontaneous symmetry breakdown is one of most important phenomena in many areas of modern physics [1]. In metals, it affects particle number conservation and gives rise to an energy gap that is responsible for superconductivity and the Meissner effect [2]. In the standard model (SM) of particle physics, spontaneous symmetry breakdown produces the masses of elementary fermions and vector bosons, the latter being responsible for the weakness of weak interactions [3]. Recall that electromagnetic cross sections are of the order of α^2/E^2, where E is the energy of the scattering particles, and $\alpha = e^2/\hbar c \approx 1/137$ the fine structure constant. In weak interactions, these are reduced to α^2/M_W^2 or α^2/M_Z^2 where vector boson masses $M_W \approx 80.4\,\text{GeV}$ and $M_Z \approx 91.19\,\text{GeV}$. This causes the slow reaction dynamics of many energy-production processes in astrophysics. In the sun, in which E is of the order of MeV, it is thanks to the largeness of $M_{W,Z}$ that allows a star to live for several billion years. If $M_{W,Z}$ would drop to zero in a phase transition of starts, suns could disappear in short supernova-like explosions.

Recall that in the Gell-Mann-Lèvy σ-model of nuclear physics, a spontaneously broken chiral symmetry is responsible for the nucleon and meson masses, with the pion appearing as a would-be

Nambu-Goldstone boson. The breakdown explains the low pion mass, the observed magnitude of the partial conservation of the axial current (PCAC), and the associated pion decay constant.

Another phenomenon where a spontaneously broken symmetry and generated mass has dramatic consequences is superconductivity. There the mass manifests itself as an energy gap of the conduction electrons, that is responsible for the infinite conductivity below a certain critical temperature T_c.

In a Fermi liquid, a spontaneously broken symmetry is generally due to an attractive potential of a system that causes a dynamical (Cooper) instability, which produces a low-mass pair bound state. On the other hand, the temperature, which represents positive thermal energy and pressure of the system, plays the role of restoring spontaneously broken symmetries. As a result, there exists a critical temperature T_c with a second-order phase transition, below which there exist a symmetric phase. Temperature or pressure can destroy this. Increasing the fermion density causes a higher pressure of the system, which plays the same role as increasing the temperature and restores the spontaneously broken symmetries. If the fermion density is larger than a critical value n_c, the nonzero condensation returns to zero and the symmetric situation is recovered.

In this Letter, we shall demonstrate how this phenomenon occurs and calculate the critical values of fermion density where it happens. We shall discuss the various physical consequences. In addition, we point out the completely opposite directions of influence of higher particle density for fermions and bosons with spontaneously broken symmetries.

Symmetry restoration in U(1)-Higgs Model. To illustrate the basic mechanism, consider first the simplest model with a complex scalar field ϕ of the $U(1)$-symmetric field potential

$$V(\phi^*\phi) = \mu^2\phi^*\phi + \lambda(\phi^*\phi)^2, \qquad \mu^2 < 0, \qquad \lambda > 0\,. \tag{1}$$

We assume that $\mu^2 < 0$, so that the field ϕ has a nonzero expectation value $\langle\phi\rangle \equiv \langle|\phi|\rangle$:

$$\langle\phi\rangle = \sqrt{-\mu^2/2\lambda} \neq 0\,. \tag{2}$$

In the ground state, the U(1)-symmetry is broken. The complex scalar field is usually charged, and coupled to a gauge field, with an Euclidean action

$$\mathcal{A} = \int d^4x \left\{ \frac{1}{2}|D_\mu\phi|^2 + V(\phi^*\phi) + \frac{1}{4}F_{\mu\nu}^2 \right\} ,$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, and $D_\mu \equiv \partial_\mu - ieA_\mu$ denotes the the gauge-covariant derivatives. The A^μ-boson has a Meissner-Higgs mass $m_A^2 = e^2\langle\phi\rangle^2$.

Relativistic fermions are coupled by an action

$$\mathcal{A}_{\rm f} \;=\; \int d^4x\bar{\psi}[\gamma_\mu(i\partial^\mu - eA^\mu) - m(\phi)]\psi\,, \tag{3}$$

where

$$m(\phi) = m_0 + g\phi\,, \tag{4}$$

m_0 are the bare fermion masses, and g is a Yukawa coupling. In the ground state with $\langle\phi\rangle \neq 0$, the observed fermion masses are given by $m(\langle\phi\rangle) = m_0 + g\langle\phi\rangle$. The Dirac equation for the fermion field ψ leads to the Pauli equation

$$\Big[(i\partial^\mu - eA^\mu)^2 - \frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu} - m^2(\phi)\Big]\psi = 0, \tag{5}$$

where $e\sigma_{\mu\nu}F^{\mu\nu} = 2e(i\alpha\cdot\mathbf{E} + \sigma\cdot\mathbf{B}) = 2e\sigma\cdot\mathbf{B}$ for vanishing electric field $\mathbf{E} = 0$. In the semi-classical description of external fields ϕ and constant $\mathbf{B}$ pointing in the $\hat{z}$-direction, the energy levels are given by

$$\omega^2(\mathbf{p}) = m(\phi)^2 + p_z^2 + p_\perp^2\,, \tag{6}$$

where $p_\perp$ is the squared transverse momentum satisfying $p_\perp^2 = 2eB(n + 1/2 - \hat{s})$, $n = 0, 1, 2, \cdots$, where $\hat{s} = \pm 1/2$ for a spin-1/2 fermion and $\hat{s} = 0$ for a spin-0 boson.

In thermal equilibrium at an inverse temperature β, fermions are described by a Euclidean action

$$A \;=\; \beta(H - \mu N)\,, \tag{7}$$

where μ is the chemical potential. The total particle-number and Hamiltonian are given by the phase space integrals

$$N \;=\; V\int \frac{d^3\mathbf{p}}{(2\pi)^3}\hat{N}_{\mathbf{p}} = V\int \frac{d^3\mathbf{p}}{(2\pi)^3}a^\dagger(\mathbf{p})a(\mathbf{p})\,, \tag{8}$$

$$H \;=\; V\int \frac{d^3\mathbf{p}}{(2\pi)^3}\hat{H}_{\mathbf{p}} = V\int \frac{d^3\mathbf{p}}{(2\pi)^3}\omega(\mathbf{p})a^\dagger(\mathbf{p})a(\mathbf{p})\,, \tag{9}$$

where $a^\dagger(\mathbf{p})$ and $a(\mathbf{p})$ are the creation and annihilation operators of particles in momentum-$\mathbf{p}$. We have suppressed the spin index $\hat{s}$ and the sums over $\hat{s}$, for brevity. In an externalmagnetic field, the momentum integrals are quantized as follows: $\int d^2\mathbf{p}_\perp/(2\pi)^2 = eB/(2\pi)^2\sum_{n,\hat{s}}$. For fermions, there are two possible occupation numbers of each state, so that the particle number operator of each momentum state $\hat{N}_{\mathbf{p}} = a^\dagger(\mathbf{p})a(\mathbf{p})$ has eigenvalues zero or one. For bosons, on the other hand, the operator $N_{\mathbf{p}} = a^\dagger(\mathbf{p})a(\mathbf{p})$ comprises all integer values $0, 1, 2, \ldots$. Their energies $\hat{H}_{\mathbf{p}} = \omega(\mathbf{p})\hat{N}_{\mathbf{p}}$ and partition functions for each momentum state are

$$Z^F_{\mathbf{p}} = \sum_{N_{\mathbf{p}}=0,1} e^{-\beta(H_{\mathbf{p}}-\mu N_{\mathbf{p}})} = 1 + e^{-\beta(\omega(\mathbf{p})-\mu)} , \tag{10}$$

$$Z^B_{\mathbf{p}} = \sum_{N_{\mathbf{p}}} e^{-\beta(H_{\mathbf{p}}-\mu N_{\mathbf{p}})} = \left[1 - e^{-\beta(\omega(\mathbf{p})-\mu)}\right]^{-1} . \tag{11}$$

These yield the Fermi $(+)$ and Bose-Einstein $(-)$ distributions

$$\langle N_{\mathbf{p}}\rangle = \frac{1}{Z^{F,B}_{\mathbf{p}}} \sum_{N_{\mathbf{p}}} N_{\mathbf{p}} e^{-\beta(H_{\mathbf{p}}-\mu N_{\mathbf{p}})} = \frac{1}{e^{\beta(\omega(\mathbf{p})-\mu)} \pm 1} . \tag{12}$$

Each partition function $Z^{F,B}_{\mathbf{p}}$ defines a grand-canonical free energy $\beta\Omega^{F,B}_{\mathbf{p}} \equiv -\log Z^{F,B}_{\mathbf{p}}$. The total grand-canonical free energy Ω is the sum of all possible microscopic states, determining the pressure of the system $p = -\Omega/V$

$$p^F = \frac{2}{\beta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \ln\left[1 + e^{-\beta(\omega(\mathbf{p})-\mu)}\right] , \tag{13}$$

$$p^B = -\frac{1}{\beta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \ln\left[1 - e^{-\beta(\omega(\mathbf{p})-\mu)}\right] , \tag{14}$$

where the factor 2 of Eq. (13) accounts for the two spin directions of fermions.

We consider the case of fermions, the pressure terms depend via $m(\phi)$ of Eq. (4) on the scalar field ϕ and thus modify the field potential (1), which in the presence of fermions is changed to

$$\begin{aligned} V_{\text{eff}} &= V(\phi^*\phi) - p \\ &= V(\phi^*\phi) - \frac{2}{\beta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \ln\left[1 + e^{-\beta(\omega(\mathbf{p})-\mu)}\right] . \end{aligned} \tag{15}$$

Because the second term vanishes for large fields ϕ, this effective potential is positive definite ($V_{\text{eff}} > 0$). However, the expectation value $\langle\phi\rangle$, i.e., the location of the ground state in the field configurations, can be changed depending on the chemical potential, temperature and magnetic field. In particular, the spontaneously broken $U(1)$-symmetry can be restored, if $\langle\phi\rangle$ becomes zero for some critical values μ_c, β_c and B_c. In the following, we shall focus our attention upon this restoration process and its observable consequences for our Universe.

To start with we assume the temperature to be zero and that there is no magnetic field. Then the effective potential (15) becomes

$$\begin{aligned} V_{\text{eff}} &= V(\phi^*\phi) + V_2(m(\phi)) \\ &= V(\phi^*\phi) + 2 \int_{|\mathbf{p}| \le P_F} \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\omega(|\mathbf{p}|) - \mu\right] , \end{aligned} \tag{16}$$

where P_F is the Fermi momentum and $\mu = \mathcal{E}_F = \sqrt{P_F^2 + m^2(\phi)}$ the associated Fermi energy. The fermion density is

$$n = 2\int_{|\mathbf{p}|\leq P_F} \frac{d^3\mathbf{p}}{(2\pi)^3} = \frac{1}{3\pi^2} P_F^3 \,. \tag{17}$$

The momentum integral of Eq. (16) yields with the abbreviations $\hat{m} \equiv m(\phi)/P_F$ and $V_2(m(\phi)) \equiv (P_F^4/24\pi^2)v_2(\hat{m})$ the result:

$$\begin{aligned} v_2(\hat{m}) = & \Big\{ -8 + 3\sqrt{1+\hat{m}^2}(2+\hat{m}^2) \\ & + 3\hat{m}^4 \Big\{ \log\hat{m} - \log(1+\sqrt{1+\hat{m}^2}) \Big] \Big\} \,. \end{aligned} \tag{18}$$

The reduced potential v_2 has the Taylor expansion

$$v_2(\hat{m}) = -2 + 6\hat{m}^2 + \frac{3}{4}\hat{m}^4 \left(1 + 2\log\frac{\hat{m}^2}{4}\right) + \dots \,. \tag{19}$$

We shall ignore the bare masses m_0 of fermions in Eq. (4), which makes $\hat{m}^2$ proportional to ϕ^2. From the quadratic term $6\,\hat{m}^2$ in Eq. (19), we can immediately see that for sufficiently high P_F, the location of the minimum of the combined field potential (16) will moves from the $\phi \neq 0$-value of the broken U(1)-symmetry to the origin $\phi = 0$, thereby restoring the spontaneously broken U(1)-symmetry. This happens at

$$-\mu^2 = \frac{P_F^2}{4\pi^2} g^2 \,, \tag{20}$$

which can be estimated by the quasicritical point where the curvature term changes its sign.

If it were not for the $\hat{m}^4 \log \hat{m}^2$-term in Eq. (19), the Taylor expansion (19) would be of the Landau-type and the phase transition would be of the second order. The logarithmic term destroys the Landau expansion. Indeed, suppose P_F is chosen so large that the ϕ^2 in Eq. (16) vanishes, at which the coefficient of ϕ^2 in Eq. (16) changes from negative values to positive values. Then the fourth-order term

$$\lambda\phi^4 + \frac{1}{32\pi^2} g^4\phi^4 \left(1 + 2\log\frac{g^2\phi^2}{4}\right) \tag{21}$$

has a minimum at

$$\phi_s^2 = \frac{4}{g^2} e^{-1/2} e^{-4/g^2} e^{-18\pi^2\lambda/g^4} \,, \tag{22}$$

and the minimal value of the field potential is

$$-\frac{1}{8}\phi_s^4 \left(\lambda + \frac{2g^2}{\pi^2}\right) \,. \tag{23}$$

For the fourth-order term coupling $\lambda \sim \mathcal{O}(1)$ and the Yukawa coupling $g \sim \mathcal{O}(1)$, $\phi_s \sim 0$ in unit of P_F, this implies that for slightly positive curvature, the field *jumps* from its nonzero expectation value to zero in a first-order phase transition. Note that the argument can only be used qualitatively, as been emphasized by Colemen and Weinberg [4, 5]. The reason is that the new minimum lies in a field regime where the perturbation theory is no longer reliable [6].

The condition (20) yields the critical fermion-number density

$$n_c = \frac{8\pi}{3}\left(\frac{\sqrt{-\mu^2}}{g}\right)^3 = \frac{8\pi}{3}\left(\frac{\sqrt{-\mu^2}}{m}\right)^3 \langle\phi\rangle^3 \,, \tag{24}$$

where we express the Yukawa coupling g in terms of the fermion mass $m(\phi)$ and the expectation value $\langle\phi\rangle$ as $g = m\sqrt{-2\lambda/\mu^2}$. When the fermion number-density is smaller than this critical density n_c, we are in the broken phase of spontaneous symmetry breaking by the Nambu-Higgs-Englert mechanism. When the fermion number-density is larger than this critical density n_c, symmetries are restored and we are in the symmetric phase. In our calculations, we do not consider the temperature and external magnetic field, however their effects are clear that the critical density decreases as the temperature increases, instead, as the magnetic field strength decreases.

We shall now apply these considerations to the possible realizations in our Universe: (i) the standard model of particle physics in the early Universe; (ii) the Gell-Mann-Lèvy σ-model of nuclear physics and its application in the stellar dynamics; (iii) the superconductivity of condensate matter physics.

The standard model for particle physics. The standard model of electroweak interactions has the local gauge symmetry $SU_L(2)\otimes U_Y(1)$ with respect to the fermions of each generation, the lightest being (e, μ, u, d). The gauge fields A^i_μ $(i = 1, 2, 3)$ of the left-handed gauge group $SU_L(2)$ with the coupling g, and the gauge field B_μ of the hyper-charge $U_Y(1)$ with the coupling g' form the combinations to yield physical $W^\pm$-, Z-bosons and photon fields.

One introduces a doublet of complex scalar Higgs fields: $\phi = (\phi^+, \phi_0)$, of hypercharge $Y = +1$ which is assumed to have the field potential (1). For $\mu^2 < 0$, ϕ acquires nonvanishing vacuum expectation value, which may be assumed to be real and to point in the direction of ϕ_0:

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} + \mathcal{O}(\hbar),\ \ v = \sqrt{2}\langle\phi\rangle = \sqrt{-\mu^2/\lambda}\ . \tag{25}$$

The nonabelian symmetry is broken, but the symmetry under the abelian subgroup $U_{\rm em}(1)$ is preserved. The $W^\pm$-, Z^0-vector bosons and fermions acquire their masses. The size of $v \approx 246.2\,{\rm GeV}$ is fixed by the measured masses of the vector bosons $v = 2M_W \sin\theta_W/e$, where $\cos\theta_W = M_W/M_Z$, The size of the Higgs mass $m_{_H}$ determines $-\mu^2$ to be

$$-\mu^2 = m_{_H}^2/2, \quad m_{_H}^2/2 = \lambda v^2 . \tag{26}$$

The phase transition takes roughly place if the condition (24) is fulfilled:

$$n = n_c = \frac{8\pi}{3}\left(\frac{\sqrt{-\mu^2}}{2m\lambda}\right)^3 = \frac{8\pi}{3}\left(\frac{m_{_H}}{2m}\right)^3 v^3 . \tag{27}$$

Suppose that only top-quark mass m_t is originated from the spontaneous symmetry breaking, while other fermions masses from the explicit symmetry breaking [7], then the critical density (27) yields,

$$n_c = \frac{8\pi}{3}\left(\frac{m_{_H}}{2m_t}\right)^3 v^3 \approx 5.89 \times 10^6 \mathrm{GeV}^3 . \tag{28}$$

where $m_t \approx 173.1$GeV and $m_{_H} \approx 126\,$GeV.

The number-density that is larger than the critical density (28) can only be realized in the early Universe, which starts from the Planck density $M^3_{\rm planck} \sim 10^{57}\mathrm{GeV}^3$, and the critical density (28) is in the era of electroweak interactions of the temperature $10^2 - 10^7$GeV. All gauge symmetries are preserved, all fields of gauge bosons, fermions and Higgs are massless, they are radiation fields. There is no any mass-threshold for pairs of particle-antiparticle creation, and the pair-production rate is greatly enhanced. All energy is deposited in radiation fields of massless particles. Beside, because of massless $W^\pm$- and Z^0-bosons, the cross-sections of weak-interaction become ($g = e/\sin\theta_W$)

$$\sigma_{\rm weak} \approx \frac{4\alpha^2}{\sin^4\theta_W\, E^2} , \tag{29}$$

which is larger than the cross-sections $\sigma \approx \alpha^2/E^2$ of electromagnetic interaction in high energies E, where $\sin^2\theta_W$ is no longer 0.231 but smaller than one. This could bring an immense enhancement of the reaction speeds of the elementary processes. As one of consequences, all neutrinos are thus trapped inside the system. This implies that in this symmetric phase of high density, all fields are radiation fields giving very high pressure on the system for then expansion. As the fermion number-density decreases, the system undergoes the phase transition to the broken phase where fermions and $W^\pm$- and Z^0-bosons become massive. This phase transition should have dramatic effects on the early Universe evolution. These discussions can be generalized to the spontaneous breaking scales of the theory of the $SU(5)$ (or $SO(10)$) grand unification or supersymmetry.

The Gell-Mann-Lévy-model for nuclear physics. The forces between nucleons can very well be described by an effective Lagrangian involving the nucleon fields proton p and neutron n coupled to various meson fields. The main effects of this model become visible by concentrating upon the nuclear and meson fields alone, which historically was done by Gell-Mann and Lèvy [8] in their famous σ-model in 1960. It contains a fermion doublet $\psi = (p, n)$ of proton and neutron, whose bare masses are $m^0_{_N}$, a triplet of pseudo-scalar pions $\vec{\pi}$, and a scalar field σ. The corresponding Lagrangian is written as

$$\mathcal{L} = \bar{\psi}[i\gamma_\mu\partial^\mu - m^0_{_N} - g(\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5)]\psi + \frac{1}{2}[(\partial\vec{\pi})^2 + (\partial\sigma)^2] - V(\sigma, \vec{\pi}), \qquad (30)$$

with $V(\sigma, \vec{\pi}) = \frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$. This exhibits an $SU(2)$-symmetry to accommodate baryons (p, n) in its fundamental representations and mesons $(\sigma, \vec{\pi})$ in its adjoint representation O(3). Analogous calculations for this Lagrangian lead to the effective potential $V_{\rm eff}(\sigma, \vec{\pi})$ in the form of Eq. (15) with the replacement $m^2(\phi) \to (m^0_{_N} + g\sigma)^2 + g^2\vec{\pi}^2$.

When the nucleon density is small and its effect on the field potential is negligible, the σ-model undergoes the spontaneous symmetry breaking for $\mu < 0$, developing expectational values $\langle\sigma\rangle = v/\sqrt{2}$, $v = \sqrt{-\mu^2/\lambda}$ and $\langle\vec{\pi}\rangle = 0$. The particle spectra are the massive σ-field $m^2_\sigma = -2\,\mu^2$, three Goldstone pions $m_\pi = 0$, and nucleons of physical masses $m_{_N} = m^0_{_N} + g\langle\sigma\rangle$. Recall that applying this model to the up and down quarks, the bare quark masses m_0 are of the order of 10 MeV, while the physical quark masses m is of the order of 1/3 of the nucleon mass. The model was successful in explaining the smallness of the pion mass, PCAC, the pion decay, as well as the low-energy theorems for pion-pion or the pion-nucleon scattering.

As the density of nucleons increases, the field potential $V(\sigma, \vec{\pi})$ is changed to $V_{\rm eff}(\sigma, \vec{\pi})$ of Eq. (15), the phenomenon of phase transition discussed above must occur at the critical density near to the nuclear density

$$n_c = \frac{\pi}{3}\left(\frac{m_\sigma}{m_{_N} - m^0_{_N}}\right)^3 v^3 \approx \frac{\pi}{3}\left(\frac{m_\sigma}{m_{_N}}v\right)^3 \approx n_{\rm nucl}, \qquad (31)$$

where we consider $m_{_N} \gg m^0_{_N}$, $m_\sigma \approx 400$MeV and $v \approx 283\,$MeV for $\lambda \sim \mathcal{O}(1)$. In the symmetric phase, the expectational value $\langle\sigma\rangle = 0$ or the small value of Eq. (22), from the quadratic terms of nucleon and meson fields in the Lagrangian with effective potential $V_{\rm eff}(\sigma, \vec{\pi})$, we obtain that nucleons have their bare masses $m^0_{_N}$, and the σ- and $\vec{\pi}$-fields have their effective masses $m^2_{\sigma,\pi} = \mu^2 + (gP_F/2\pi)^2 > 0$. The $SU(2)$-symmetry is completely restored, however, the symmetry is realized by different massive spectra of baryons and mesons. This second-order phase transition forms a sharp surface of a nucleus with the width $\sim m^{-1}_{\sigma,\pi}$, that distinguishes the broken-symmetry phase, where nucleons and mesons interact as a dilute gas, from the symmetric phase, where nucleons and mesons couple together as a liquid at the nuclear density. In the formation or collapse of stellar cores near to the nuclear density, as the nucleon density increases due to an attractive gravitational force and becomes overcritical, the second-order phase transition may take place, the massive spectra of particles are changed and baryons may become relativistic particles if $m^0_{_N} \ll P_F$, which may result in important physical consequences. In addition, since electrons do not associate to this transition, strong electric fields over the critical value $E_c = m_e^2/e^2$ of Sauter, Euler and Heisenberg could possibly be developed [9].

The superconductivity of condensed matter. To study the effects of fermion density on the spontaneous symmetry breaking and Cooper-pair formation in the BCS model for superconductivity, the Euclidean action (7) should be

$$A = \int_0^\beta d\tau \int d\mathbf{x} \Big[\mathcal{K} - g\bar{\psi}_\uparrow(x)\bar{\psi}_\downarrow(x)\psi_\downarrow(x)\psi_\uparrow(x)\Big], \tag{32}$$

where $x = (\mathbf{x}, \tau)$, the kinetic term $\mathcal{K} = \bar{\psi}_\sigma(x)[\partial_\tau - \nabla^2/2m - \mu]\psi_\sigma(x)$ and the attractive interaction g. The standard approach of introducing $\Delta(x) = \psi_\downarrow(x)\psi_\uparrow(x)$, and expressing Eq. (32) in terms of quadratic fermion fields, integrating out fermion fields leads the effective action (see for example Refs. [10–13])

$$A_{\rm eff} = \int_0^\beta d\tau \int d\mathbf{x} \left\{ \frac{|\Delta(x)|^2}{g} - \frac{1}{\beta}{\rm Tr}\ln\left(\beta\mathbf{G}^{-1}\right) \right\}, \tag{33}$$

where $\mathbf{G}^{-1}[\Delta(x)]$ is the inverse operator of quadratic fermion fields. Adopt the approximation of a uniform static saddle point $\Delta(x) \approx \Delta_0$, that satisfies the saddle-point condition $\delta A_{\rm eff}(\Delta_0)/\delta\Delta_0 = 0$, which can be written as the renormalized gap-equation

$$\frac{m}{4\pi a_s} = \sum_{\mathbf{k}} \left[\frac{1}{2\epsilon_{\mathbf{k}}} - \frac{\tanh(\beta E_{\mathbf{k}}/2)}{2E_{\mathbf{k}}}\right], \quad \frac{m}{4\pi a_s} = \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}} - \frac{1}{g}, \tag{34}$$

where $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta_0^2}$, $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ and $\epsilon_{\mathbf{k}} = |\mathbf{k}|^2/2m$. The second equation in Eq. (34) regulates the ultraviolet behaviors in the gap-equation by going from g to the renormalized coupling g_R in terms of the observable s-wave scattering length a_s. The fermion number $N = -\partial\Omega/\partial\mu \approx -\partial\Omega(\Delta_0)/\partial\mu$ leading to the number equation

$$n = \frac{1}{V}\sum_{\mathbf{k}} \left[1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\tanh\left(\frac{\beta E_{\mathbf{k}}}{2}\right)\right]. \tag{35}$$

For the sake of simplicity to illustrate the critical density, we consider the zero-temperature case $T = 0$, $\mu = \mathcal{E}_F = P_F^2/2m = (3\pi^2 n)^{2/3}/2m$. In the weak coupling BCS limit $1/(k_F a_s) \to -\infty$, solving Eqs. (34) and (35) one obtains [11] that $\Delta_0(0)/\mathcal{E}_F = 1.1\exp[-\pi/(2P_F|a_s|)]$. This implies that $\Delta_0(0)/\mathcal{E}_F \approx 0$ for $P_F \approx (2a_s)^{-1}$, i.e., the critical density for restoring the symmetry

$$n_c \approx \frac{1}{3\pi^2}\left(\frac{1}{2a_s}\right)^3. \tag{36}$$

This physically means that a restoration of the spontaneously broken symmetry may be caused by the degenerate pressure of fermions [see Eq. (13)], with the dramatic consequence of destroying the supercurrent. Although calculations are made in the zero-temperature case, we can expect that the presence of temperature will enhance restoration, since above T_c, superconductivity is completely destroyed.

Conclusion and remarks. In this Letter we point out the fact that the number density or degenerate pressure of many-fermion systems plays the same role of temperature acting on a spontaneous breakdown of symmetries, and the increase of fermion density implies the decrease of the value of critical temperature T_c for a phase transition. We obtain the critical fermion density in the zero-temperature case and discuss possible observable consequences in our Universe.

To end this Letter, we draw attention to the fact that the phenomena act in precisely the opposite directions of what we see in many-boson systems upon a spontaneous breakdown of a symmetry. As far as the ground state is concerned, this has been well known since the beginning of quantum field theory. A boson field at zero temperature in the vacuum has zero-point energy $\omega\hbar/2$, where $\omega = \sqrt{\mathbf{p}^2 + m^2}$, whereas fermions have the same energy with the opposite sign. But that the effects are opposite also in interacting systems deserves to be emphasized. What has been known since its discovery, the famous phenomenon of Bose-Einstein Condensation (BEC) is caused by squeezing at low temperature too many bosons into a small volume. Thus the exceeding of a critical density leads to a nonzero expectation value of the boson field, i.e., to a spontaneous breakdown of the symmetry of particle number conservation. This is precisely the opposite direction of the influence of a fermionic particle density, which might be seen from the opposite sign of the fermionic pressure (13) and the bosonic pressure (14). It is interesting to see these phenomena in the quark-gluon plasma due to opposite effects of quark density and gluon density.

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