Algebra of Regge Residues.

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As higher and higher energies will be reached, the couplings of Regge trajectories to hadrons might become observable to the same degree of accuracy as electromagnetic and weak currents are right now. It is therefore interesting to investigate whether these couplings will display a simple algebraic structure. In such a case, this algebraic structure might be used to anticipate the results of future experiments on hadronic high-energy processes. At the end it might provide us with a key to a deeper understanding of strong-interaction symmetries complementary to current algebra.

It is the purpose of this note to point out that, according to our ideas of duality, such a simple algebra does indeed exist. The structure constants of this algebra are supplied by the vertices of three reggeons at zero mass.

In order to be specific we shall derive here only the subalgebra fulfilled by the trajectories of vector and tensor mesons. For this we consider the completely collinear amplitude

$$\beta(p') + \pi_{b_1}(q_1') + \pi_{b_2}(q_2) \leftarrow \pi_{a_2}(q_2) + \pi_{a_1}(q_1) + \alpha(p) ,$$

where $\beta(p'), \alpha(p)$ are arbitrary resonances (*) while $\pi_a(q)$ stands for the pion triplet. For simplicity we neglect the mass of the pions.

Note that the mass of the two-pion sybsystems is automatically zero, $(q_1 - q_2)^2 = (q_2 - q_1')^2 = 0$, and that $(p + q_1)^2 = (p' + q_1')^2$. In this case, the only variables describing the process are

$$\text{(1)} \qquad \begin{cases} M^2 = (p + q_1 - q_2)^2 , \\ s \;\;\; = (p + q_1)^2 = (p' + q_1')^2 , \end{cases}$$

According to multi-Regge analysis, this amplitude behaves for $M^2 \to \infty$, $s/M^2 \to \infty$ as [1]

$$\text{(2)} \qquad T^{b_1 b_2, a_2 a_1}_{\beta\alpha} \approx -\sum_{j,i,k} \left(\frac{s}{M^2}\right)^{\alpha_j + \alpha_i} \left(\frac{M^2}{m_0^2}\right)^{\alpha_k} \xi_j^* \xi_i \xi_k R^{b_1 a_2}{}_j R_i{}^{b_2 a_1} g^{ji}{}_k R^k_{\beta\alpha} \qquad (\alpha_k > \alpha_j + \alpha_i - 1) ,$$

(*) All external and intermediate resonance will certainly have the same helicity, due to collinearity.

[1] C. E. DE TAR, C. E. JONES, F. E. LOW, J. H. WEIS, J. E. YOUNG and C.-I. TAN: *Phys. Rev. Lett.*, **26**, 675 (1971); M. V. MICHELOFF: *Phys. Rev.*, **184**, 1732 (1969); P. GODDARD and A. R. WHITE: *Nucl. Phys.*, **17** B, 45 (1970); C. E. DE TAR and J. H. WEIS: *Phys. Rev. D*, **4**, 3141 (1971).

where the sum over j, i, k extends over all Regge poles exchanged in the $\pi_{b_1}\bar{\pi}_{a_2}$, $\pi_{b_2}\bar{\pi}_{a_1}$, $\beta\bar{\alpha}$ channels, respectively. The factors

$$\xi_{j,i} = \frac{\tau_{j,i} + \exp[-i\pi\alpha_{j,i}]}{2\sin\pi\alpha_{j,i}}\,, \qquad \xi_k = \frac{\tau_i\tau_j\tau_k + \exp[-i\pi(\alpha_k - \alpha_j - \alpha_i)]}{2\sin\pi(\alpha_k - \alpha_j - \alpha_i)} \tag{3}$$

take care of the signatures τ_j, τ_i, τ_k of the Regge poles and the corresponding residues have been denoted by R. The parameter m_0 is arbitrary, say 1 GeV. All Regge poles are at zero mass (2).

This asymptotic behaviour can be made use of in a finite-energy sume rule (2)

$$\frac{1}{2\pi i}\int\limits_0^N \mathrm{d}M^2 \operatorname{disc} T^{b_1 b_2, a_2 a_1}_{\beta\alpha} \approx \frac{m_0^2}{2\pi}\sum_{j,i,k}\left(\frac{s}{N}\right)^{\alpha_j+\alpha_i}\left(\frac{N}{m_0^2}\right)^{\alpha_k+1}\cdot \\ \cdot\, \xi_j^*\xi_i R_j^{b_1a_2} R_i^{b_2a_1}\frac{g_k^{ji}}{\alpha_k+1-\alpha_j-\alpha_i}R^k_{\beta\alpha}\,. \tag{4}$$

If we saturate the left-hand side with resonances only, we obtain

$$\sum_{m_\gamma^2}^{N}(T^\dagger)^{b_1a_2}_{\beta\gamma}T^{b_2a_1}_{\gamma\alpha}\,, \tag{5}$$

where $T^{ba}_{\gamma\alpha}$ denote the amplitudes for the processes

$$\pi_b(q_2) + \gamma(p+q_1) \leftarrow \pi_a(q_1) + \alpha(p)\,. \tag{6}$$

At large s these amplitudes are themselves expected to show Regge behaviour and the sum can be rewritten as

$$\sum_{\substack{m_\gamma^2\\ j,i}}^{N}\left(\frac{s}{m_0^2}\right)^{\alpha_j}\xi_j^* R_j{}^{b_1a_2}R^j_{\beta\gamma}R^i_{\gamma\alpha}R_i{}^{b_2a_1}\xi_i\left(\frac{s}{m_0^2}\right)^{\alpha_i}. \tag{7}$$

In the limit $s\to\infty$ we may presumably restrict the sum over i, j to the leading trajectories only. In our particular process these are vector and tensor trajectories (*). If we, moreover, go to amplitudes of definite signature in all channels, only one Regge pole remains for every index i, j, k and comparison of (4) and (7) yields

$$\sum_\gamma [R^j_{\beta\gamma}R^i_{\gamma\alpha}]_\mp = \frac{m_0^2}{\pi}\frac{(N/m_0^2)^{\alpha_k+1-\alpha_j-\alpha_k}}{\alpha_k+1-\alpha_j-\alpha_i}g_k^{ji}R^k_{\beta\alpha}\,, \tag{8}$$

(2) M. B. Einhorn, J. Ellis and J. Finkelstein: SLAC-PUB-1006 (1972); A. I. Sanda: NAL preprint THY-19 and 25 (1971).

(*) Within the spirit of dual resonance models we shall discard the diffractive part of the amplitudes everywhere (see ref. (3)). Obviously, the structure we are discussing has an explicit representation in the N-point Veneziano model (4).

(3) H. Harari: *Phys. Rev. Lett.*, **20**, 1395 (1968); H. Harari and Y. Zarmi: *Phys. Rev.*, **187**, 2230 (1969).

(4) Chan Hong-Mo: CERN preprint; P. Olesen: CERN preprint TH 1376.

where commutator and anticommutator correspond to $\tau_k = \mp \tau_j \tau_i$. Thus we find the important result that the Regge couplings form an algebra with the structure constants

$$c_k^{ji} = \frac{m_0^2}{\pi} \left(\frac{N}{m_0^2}\right)^{\alpha_k+1-\alpha_j-\alpha_i} \frac{1}{\alpha_k + 1 - \alpha_j - \alpha_k} g_k^{ji} . \tag{9}$$

Now all leading Regge trajectories involved here have intercept $\alpha_j \approx \alpha_i \approx \alpha_k \approx \frac{1}{2}$, so the structure constants are, up to a factor $\varkappa = (2/\pi)(N/m_0^2)^{\frac{1}{2}} m_0^2$, identical with the three-reggeon coupling g_k^{ji}.

If we make the very mild assumption that these couplings are SU_2 symmetric, the residues of vector and tensor trajectories will form the algebra of $U_2 \times U_2$ ($V^i \equiv (\omega, \varrho^a)$, $T^i \equiv (f, A_2^a)$):

$$\left\{ \begin{aligned} [V^i, V^j] &= \varkappa g^{\varrho\varrho}{}_{\varrho}\, i\varepsilon_{ijk} V^k , \\ [V^i, T^j] &= \varkappa g^{\varrho A_2}{}_{A_2}\, i\varepsilon_{ijk} T^k , \\ [T^i, T^j] &= \varkappa g^{A_2 A_2}{}_{\varrho}\, i\varepsilon_{ijk} V^k . \end{aligned} \right. \tag{10}$$

Notice that the Jacobi identity enforces $g^{\varrho\varrho}{}_{\varrho} = g^{\varrho A_2}{}_{A_2}$.

This looks very close to what was conjectured a long time ago by CABIBBO, HORWITZ and NE'EMAN (« HCN ») ([5]). The correspondence becomes almost exact if we assume, in addition, that also the third $g^{A_2 A_2}{}_{\varrho}$ equals the first two

$$g^{A_2 A_2}{}_{\varrho} = g^{\varrho\varrho}{}_{\varrho} = g^{\varrho A_2}{}_{A_2} = g . \tag{11}$$

Then V, T differ from the couplings of « HCN » only by an overall constant $\varkappa g$. This normalization is an important result of our derivation. If larger and larger saturation schemes are employed, the products VV, VT and TT grow like $N^{\frac{1}{2}}$. Clearly, if the scheme is infinitely large, the algebra does not exist any longer. Truncation is necessary to make the matrix products finite. This is similar ([6]) to standard finite-energy sum rules ([7]), which become meaningless at $N = \infty$. Physically this divergence means that the excitation of a particle via V, T exchange to higher resonances decreases very slowly with increasing resonance mass ($\sim [m_Y]^{-\frac{1}{2}}$).

In order to illustrate what kind of structure we shall have to expect as a solution of eq. (10) we consider a simple model of infinitely rising baryon resonances of octets and decuplets with $m_n^2 = m_0^2 n$, $n = 1, 2, 3, \ldots$. If v^i, a^i denote the standard $(1, \frac{1}{2})$ representation of $U_2 \times U_2$, then

$$V^i = G v^i , \qquad T^i = G a^i , \tag{12}$$

with

$$G_{mn} = \frac{g}{\pi} m_0^2 [m \cdot n]^{-\frac{1}{4}} , \tag{13}$$

([5]) N. CABIBBO, L. HORWITZ and Y. NE'EMAN: *Phys. Lett.*, **22**, 336 (1966).

([6]) H. KLEINERT: *Phys. Lett.*, **39** B, 511 (1972); H. KLEINERT and L. R. RAM MOHAN: *Nucl. Phys.*, **52** B, 253 (1973).

([7]) R. DOLEN, D. HORN and C. SCHMID: *Phys. Rev.*, **166**, 1768 (1968).

represents a solution of the algebra (10) upon truncation of all matrices at a large value of N, since

$$\sum_{l=1}^{N/m_0^2} G_{kl} G_{lm} \approx \frac{2}{\pi} \left(\frac{N}{m_0^2}\right)^{\frac{1}{2}} m_0^2 g G_{km} \,. \tag{14}$$

It is obvious that the same method can be applied to other multiparticle processes as well in order to obtain information on commutators among more Regge couplings. For example the trajectories of ρ_a, f, π_a, A_{1_a} (here a denotes only the isospin index) can be shown to transform like the generators L_{bc}, L_{45}, L_{a4}, L_{a5} of the group O_5. Since the π and A_1 trajectories have zero intercept, the commutators $[\pi_b, \pi_a]$, $[\pi_b, A_{1_a}]$, and $[A_{1_b}, A_{1_a}]$ grow like $N^{\frac{3}{2}}$ for increasing saturation schemes. All the others grow, as before, like $N^{\frac{1}{2}}$

Finally let us remark that the study of single-particle production amplitudes fixes also the commutators between particle couplings and Regge residues. In particular, one can in this way extend the algebra of Regge couplings presented here by the algebra of vector and axial vector charges of current algebra.

The result of such a discussion is the same as was obtained by the first of ref. ([6]). There it was found that f and π_a are proportional to m_4^2 and m_a^2, respectively, which are members of a $(\frac{1}{2}, \frac{1}{2})$ representation of chiral $SU_2 \times SU_2$ (*).

Similarly the coupling of the ρ trajectory was found to be, up to a factor, $[m_b^2, m_a^2]$, which transforms like $(1, 0) + (0, 1)$ under chiral $SU_2 \times SU_2$. Its chiral partner $[m_4^2, m_a^2]$ is, certainly, the coupling of the A_1 trajectory (**).

From these remarks it is obvious that our findings concerning the commutation rules among the Regge coupling can be read as the condition that m_4^2, m_a^2, $[m_b^2, m_a^2]$ and $[m_4^2, m_a^2]$ form, up to a normalization depending on the size of the saturation scheme N, the algebra of O_5.

Notice that all these operators can be written as appropriate commutators of X_a and m^2.

The resulting « supergroup » formed by vector and axial charges T_a, X_a and Regge couplings of ρ_a, A_{1_a}, π_a, f is $SU_2 \times SU_2 \times O_5$.

A study of the complete structure will be given elsewhere ([8]).

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(*) We recall that the absence of exotic exchanges forces the mass operator to be the sum of an $SU_2 \times SU_2$ singlet m_0^2 and the 4-th component of a $(\frac{1}{2}, \frac{1}{2})$ representation m_4^2. The isovector partner of m_4^2 is $m_a^2 = -i[X_a, m^2]$ which is by PCAC identical with the pion coupling.

(**) This particular point has also been noticed by R. CARLITZ: CERN preprint TH 1495. This author does not take proper account, however, of the significance of the power of N appearing in the commutator $[X_a, \varrho_b] \propto N^{-\frac{1}{2}} i\varepsilon_{abc} A_{1_c}$. Even though the right-hand side becomes suppressed for large N, it is needed in any *finite saturation scheme* to counter-balance the increasing commutator $[X_a, A_{1_b}] \propto N^{\frac{1}{2}} i\varepsilon_{abc}\varrho_c$ in a Jacobi identity for $[X_a, [\varrho_b, A_{1_c}]]$.

([8]) H. KLEINERT: *Lectures Presented at the Frascati Meeting, March 26, 1973*. This paper also shows the connection of our algebra with the algebra of bilocal form factors of Fritzsch and Gell-Mann.