A Gauge Invariance in Gribov's Field Theory and the Intercept of the Pomeron.

R. ACHARYA (*) and H. KLEINERT

Institut für Theoretische Physik, Freie Universität Berlin - Berlin

(ricevuto il 9 Settembre 1975)

In Gribov's field theory [1], reggeons on a trajectory $\alpha(t)$ are treated as quasi-particles of « energy » $E = 1 - \alpha(-\boldsymbol{k}^2)$. Diffraction is supposed to arise from the multiple charge of « bare » pomerons which start out as pure poles moving linearly for small t (*i.e.* $\alpha(t) = 1 + \alpha_0' t + \ldots$) and interact via three- and more-point fundamental vertices. The experimental data indicate that, to a very good approximation, the pomeron may be treated as « massless » (mass $\equiv E(k=0) \equiv \Delta_0 = 1 - \alpha(0)$) (**). Diffraction, therefore, becomes an infra-red problem of pomeron field theory [2].

Only one triple-pomeron interaction has been studied so far [2]. It is given by the Lagrangian

$$\mathscr{L}^{\lambda}_{\text{int}} = -i\frac{\lambda}{2}(\psi^{\dagger 2}\psi + \psi^{\dagger}\psi^2)\,. \tag{1}$$

By employing an expansion in $\varepsilon = 4 - D$, this theory was found to be infra-red stable. The scale-invariant propagator has the form

$$G^{(1,1)}_{\text{ren}}(E, \boldsymbol{k}) = E^{-1+\gamma(g_\infty)} F(\boldsymbol{k}^2 E^{-z(g_\infty)})\,, \tag{2}$$

(*) Supported by DFG under grant Kl 256/4; Present address: Physics Department, Arizona State University, Tempe, Arizona 85281, U.S.A.

(**) If $\Delta_0 \neq 0$, such a treatment holds for $\ln s \ll 1/|\Delta_0|$.

(1) V. N. GRIBOV: *Žurn. Ėksp. Teor. Fiz.*, **53**, 654 (1967) (English translation, *Sov. Phys. JETP*, **26**, 414 (1968)); V. N. GRIBOV and A. A. MIGDAL: *Žurn. Ėksp. Teor. Fiz.*, **55**, 1498 (1968) (English translation, *Sov. Phys. JETP*, **28**, 784 (1969)); *Yad. Fiz.*, **8**, 1002, 1213 (1968) (English translation, *Sov. Journ. Nucl. Phys.*, **8**, 583, 703 (1969)); V. N. GRIBOV, E. M. LEVIN and A. A. MIGDAL: *Yad. Fiz.*, **12**, 173 (1970) (English translation, *Sov. Journ. Nucl. Phys.*, **12**, 93 (1971)); *Žurn. Ėksp. Teor. Fiz.*, **59**, 2140 (1970) (English translation, *Sov. Phys. JETP*, **32**, 1158 (1971)); A. A. MIGDAL, A. M. POLYAKOV and K. A. TER-MARTIROSYAN: *Phys. Lett.*, **48** B, 239 (1974).

(2) H. D. I. ABARBANEL and J. B. BRONZAN: *Phys. Lett.*, **48** B, 345 (1974); *Phys. Rev. D*, **9**, 2397, 3304 (1974); R. L. SUGAR and A. R. WHITE: *Phys. Rev. D*, **10**, 4074 (1974); H. D. I. ABARBANEL and R. L. SUGAR: *Phys. Rev. D*, **10**, 721 (1974); J. L. CARDY and A. R. WHITE: *Nucl. Phys.*, **80** B, 12 (1974); C. DETAR: MIT preprint, No. 421 (July 1974); J. BARTELS and R. SAVIT: FNAL-Pub-74/61-THY (1974); A. R. WHITE: FNAL Report, Fermi Lab-Conf-74/77-THY (August 1974); R. C. BROWER and J. ELLIS: *Phys. Lett.*, **51** B, 496 (1974); *Phys. Rev. D*, **10**, 4208 (1974).

with an anomalous dimension of the field [3]

$$(3) \qquad -\gamma(g_\infty) = \frac{\varepsilon}{12} + \left(\frac{\varepsilon}{12}\right)^2 \left[\frac{257}{12}\ln\frac{4}{3} + \frac{37}{24}\right] + O(\varepsilon^3)$$

and a critical index

$$(4) \qquad z(g_\infty) = 1 + \frac{\varepsilon}{24} + \left(\frac{\varepsilon}{12}\right)^2 \left[\frac{155}{24}\ln\frac{4}{3} + \frac{79}{48}\right] + O(\varepsilon^3)\,,$$

fixing the behaviour of the interacting Regge trajectory as

$$(5) \qquad \alpha(t) \sim 1 + \text{const}\,(-t)^{1/z(g_\infty)}\,.$$

The scattering amplitude behaves asymptotically conjugate to the propagator (2):

$$(6) \qquad A(s, t) \xrightarrow[s\to\infty]{} s(\ln s)^{-\gamma(g_\infty)} F\big(t(\ln s)^{z(g_\infty)}\big)\,.$$

For $\varepsilon = 2$ this yields

$$(7) \qquad -\gamma(g_\infty) \approx \frac{1}{6} + \frac{7.7}{36} \approx \frac{1}{6} + \frac{1.3}{6}\,,$$

$$(8) \qquad z(g_\infty) \approx 1 + \frac{1}{12} + \frac{3.5}{36} \approx 1 + \frac{1}{12} + \frac{1.16}{12}\,,$$

implying rising total cross-sections and an infinite initial slope of the Regge trajectory.

This model has four unattractive features:

1) The masslessness of the pomeron field is an artifact which has to be put in by hand.

2) The choice of the interaction (1) is not the only possible one for three pomerons. An interaction allowing for simultaneous annihilation

$$(9) \qquad \mathscr{L}^{\lambda'}_{\text{int}} = -i\,\tfrac{1}{6}\,(\lambda'\psi^3 + \lambda'^*\psi^{\dagger 3})$$

can always be added which arbitrarily changes the most interesting prediction (*i.e.* that of $\gamma(g_\infty)$). We see no fundamental reason why three vacuum trajectories should not be able to collide and dissipate into the vacuum.

3) The ε-expansion seems to be an inappropriate tool for the calculation of $\gamma(g_\infty)$, with the ε^2 terms being larger than the ε terms.

4) The model predicts increasing shrinkage for higher energy (due to (8)), while experimentally exactly the opposite happens. At $s \approx 60\ (\text{GeV})^2$ the slope is ~ 0.4, while at $10^3\ (\text{GeV})^2$ it is ~ 0.25.

---

[3] M. BAKER: *Phys. Lett.*, **51** B, 158 (1974); *Nucl. Phys.*, **80** B, 61 (1974); J. B. BRONZAN and J. W. DASH: *Phys. Lett.*, **51** B, 496 (1974); *Phys. Rev. D*, **10**, 4208 (1974).

It is the purpose of this note to show how all these problems can be avoided by postulating a gauge invariance

$$\psi(x,\tau)\to\psi(x,\tau)+i\varrho \tag{10}$$

with arbitrary real ϱ, for the most general pomeron Lagrangian. If we accept the concept of diffraction formulated above, this general Lagrangian reads

$$\mathscr{L}=\frac{i}{2}\,\psi^\dagger\overset{\leftrightarrow}{\partial}_\tau\psi-\alpha_0'\nabla\psi^\dagger\nabla\psi+\mathscr{L}^\lambda_{\text{int}}+\tilde{\mathscr{L}}^{\lambda'}_{\text{int}}\,. \tag{11}$$

Here all terms have been omitted which would be «irrelevant» in the infra-red (4,5). Examples are terms like $\alpha_0''\nabla^2\psi^\dagger\nabla^2\psi$ and others of higher power in ∇ arising in an expansion of $\alpha(\nabla^2)$, or terms like $\lambda''(\psi^\dagger\psi)^2$, $\lambda'''(\psi^\dagger\psi)^3$ etc. Their dimensions are larger than those of the slope and three-point coupling terms. Therefore, in the solution of the Callan-Symanzik equation, the «effective values» of their coupling constants $\bar{\alpha}_0''$, $\bar{\lambda}''$, $\bar{\lambda}'''$, ... would tend to zero in the infra-red (*). A possible term $\beta_0'(\nabla\psi\nabla\psi+\nabla\psi^\dagger\nabla\psi^\dagger)$ looks «relevant» at first sight. However, it can be removed by a transformation $\psi=\cosh\theta\chi+\sin\theta\chi^\dagger$ (**).

The invariance (10) fixes the interaction uniquely to be the combination

$$\mathscr{L}^\lambda_{\text{int}}+\tilde{\mathscr{L}}^\lambda_{\text{int}}=-\frac{i}{6}\,\lambda(\psi+\psi^\dagger)^3\,, \tag{12}$$

and forbids a mass term. Notice there *is* a quadratic term allowed by the symmetry: $-\Delta_0(\psi+\psi^\dagger)^2$. However, this does *not* introduce a nonzero intercept, but changes the trajectory to the square-root form $E\sim(\alpha_0'/\Delta_0)\sqrt{-t}$ for small t, which is ruled out by assumption.

In this model we find that to lowest order $\beta(g)$ has an infra-red stable zero at (***)

$$\frac{g_\infty^2}{(8\pi)^2}=\frac{\varepsilon}{22}+O(\varepsilon^2)\approx\frac{1}{11}\,. \tag{13}$$

But contrary to the first model (1), the anomalous dimension vanishes:

$$\gamma(g_\infty)=0+O(\varepsilon^2)\,. \tag{14}$$

Thus, the cross-sections *do* remain constant in spite of the infra-red pile-up of cuts in the angular-momentum plane. Notice that, unlike in relativistic theories, a canonical dimension does *not* imply the theory to be free.

---

(4) K. Wilson: *Phys. Rev. B*, **4**, 3174, 3184 (1971); K. Wilson and J. Kogut: *Phys. Rep.*, **12** C, 75 (1974); S. Ma: *Rev. Mod. Phys.*, **45**, 589 (1973); G. Parisi: *Lectures at the Cargese Summer Institute* (1973); B. Schroer: *V Simposio Brasilero de Fisica Teorica* (Rio de Janeiro, 1974).

(5) R. Jengo: *Phys. Lett.*, **51** B, 143 (1974); G. Calucci and R. Jengo: *Nucl. Phys.*, **84** B, 413 (1975).

(*) If one wants to construct pomeron Lagrangians giving different results in the infra-red, one necessarily has to depart from the bare pomeron being a pole moving linearly for small t. These arguments are certainly true only for sufficiently small ε. For large ε it might, in principle, happen that the anomalous dimensions of the different terms cross over and a neglected term becomes relevant. We ignore such possibilities.

(**) Terms of such a form are necessary, on the other hand, as counter-terms in order to renormalize the theory.

(***) This is an improvement with respect to the previous number (2), which was $\varepsilon/6\approx\frac{1}{3}$.

What will happen to this result to higher order in ε? The vanishing of $\gamma(g_\infty)$ in this model can be traced to the gauge invariance (10), according to which there exists a conserved gauge current

$$(\varrho(\boldsymbol{x}, \tau), j^i(\boldsymbol{x}, \tau)) = \left(\psi + \psi^\dagger, \frac{\alpha'_0}{i} \nabla(\psi - \psi^\dagger)\right) \tag{15}$$

with

$$\partial_\tau \varrho + \nabla_i j^i = 0 . \tag{16}$$

As a consequence, there is a Ward identity for the renormalized vertices

$$-iE\{G^{(1,1)}_{\text{ren}}(E, \boldsymbol{k}) + G^{(0,2)}_{\text{ren}}(E, \boldsymbol{k})\}|_{\boldsymbol{k}=0} = Z^{-1} , \tag{17}$$

which in combination with (2) and a corresponding scaling law for $G^{(0,2)}_{\text{ren}}$ directly leads to $\gamma(g_\infty) = 0$, in the limit $E \to 0$ (*). Obviously *this* argument holds to any order in the ε-expansion since Z is finite for $\varepsilon \neq 0$ (in our case even for $\varepsilon = 0$).

The critical exponent of the trajectory is in this model

$$z(g_\infty) = 1 - \frac{\varepsilon}{44} + O(\varepsilon^2) \approx \frac{21}{22} , \tag{18}$$

implying a zero slope for $t = 0$ in agreement with the decreasing shrinkage at high energy. The triple-pomeron vertex can be found to vanish as

$$\Gamma^{(2,1)}(\xi \boldsymbol{k}_1, \xi \boldsymbol{k}_2, \xi \boldsymbol{k}_3)|_{\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{k}_3} \sim [\xi^2]^{(1-(D/4)z-(3/2)\gamma)/z} \sim \xi^{2/21} \tag{19}$$

as all three E_i-variables move *along* the trajectory (5).

Once the correct zero-mass Lagrangian has been found, a more realistic modification thereof can be constructed by adding a small « mass » term $-\Delta_0 \psi^\dagger \psi$, with $\Delta_0 \approx -0.06$. This term will now govern the infra-red behaviour and bring the total high-energy cross-section (**) to the experimentally observed asymptotic form $\sigma_T \approx 27 s^{0.06}$ mb. The gauge invariance becomes softly broken such that the current is only partially conserved (« PCGC »):

$$\partial_\tau \varrho + \nabla_i j^i = \frac{\Delta_0}{i} (\psi - \psi^\dagger) . \tag{20}$$

Ward identities and theorems on soft-pomeron emission follow in the familiar fashion.

It should be pointed out that the new interaction $\tilde{\mathscr{L}}_{\text{int}}$ leads, in perturbation expansion, to left-hand cuts in E in the propagators for $k^2 > 0$. The final propa-

(*) $G^{(n,m)}$ is the usual Green's function of n fields ψ and m fields $\psi^\dagger$. Notice that $G^{(0,2)}$ does not vanish in our case due to $\tilde{\mathscr{L}}_{\text{int}}$.

(**) Only at much higher energies ($s \gg e^{1/0.06}$ (GeV)^2) will the Froissart bound $s \leqslant 60 (\ln s/s_0)^2$ mb be enforced by unitarity.

gator (2) does not necessarily inherit this undesirable property since F is an unknown function. In particular, other processes which are ignored in any pure pomeron theory, such as multiparticle s-channel exchanges, can be expected to generate « hiding cuts » guaranteeing a proper analytic structure (*). At any rate the prediction (6) at $t=0$ is completely consistent with Froissart's bound (**).

(*) For a discussion of the implications of the trajectories of the form (5) for the partial-wave amplitudes, see ref. (6).

(6) R. OEHME: University of Chigaco preprint EFI 74/49 (1974); see also, R. OEHME: *Phys. Rev. D*, **4**, 1485 (1971); *Springer Tracts in Mod. Phys.*, **61**, 109 (1972) and references quoted therein; D. HORN and F. ZACHARIASEN: *Hadron Physics at Very High Energy* (Reading, Mass., 1973).

(**) For dynamical attempts at explaining the intercept $\alpha(0)\approx 1$ see ref. (7).

(7) G. VENEZIANO: *Phys. Lett.*, **43** B, 413 (1973); H. LEE: *Phys. Lett.*, **30**, 719 (1973).