QUARK PAIRS INSIDE HADRONS☆

H. KLEINERT

Institut für Theoretische Physik, Arnimallee 3, 1 Berlin 33, Germany

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Exact PCAC implies the presence of "wee" quark pairs inside any hadronic wave function. If present physics takes place underneath a large energy gap within a type II superconductive world, such quark pairs find a natural quantitative explanation. It can be phrased in terms of a *mixing operator* dressing any constituent quark wave function with the proper amount of pairs. This completes the program of transforming constituent quarks into current quarks (partons).

Experiments suggest the presence of an infinite sea of quark-antiquark pairs of small longitudinal momentum p^+ inside the proton ("wee" quarks). For if hadrons were to consist only of QQQ or $Q\bar{Q}$ wave functions, the structure function $F_2(x)$ of the proton would vanish at $x = -q^2/2pq \approx 0$, resulting in the absence of diffraction scattering [1] and the presence of an exact exchange degeneracy [2] among ρ, $\omega \leftrightarrow A_2$, f and $\pi \leftrightarrow$ B trajectories.

Quantitatively, this effect is rather small: only $\approx 6\%$ of the area of the structure function $F_2(x)$ is arising from antiquarks [3]. Qualitatively, however, it supplies an important information on quark dynamics.

It is the purpose of this note to point out that:

1) Exact PCAC implies a theorem on the presence wee quark pairs.

2) A quantitative description can be obtained by considering the world as a type II superconductor. In particular, there exists a simple *mixing operator* which dresses any constituent quark wave function with the necessary amount of wee pairs in order to respect PCAC.

A quark (or parton) is defined by all observable currents having the form ‡1

$$j^\mu(x) \equiv \bar{\psi}(x)\,\gamma^\mu\,\psi(x), \qquad j_5^\mu(x) = \bar{\psi}(x)\,\gamma^\mu\gamma_5\,\psi(x). \tag{1}$$

Their charges $\int dx^- d^2x^\perp j^+(x^+=0, \boldsymbol{x})$ contain only the "good" components $\psi^{(+)}(x) \equiv (1+\gamma^0\gamma^3)/2\;\psi(x) = \left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)\psi(x)$ of the quark field. These can be expressed, even in the presence of interactions, in the form ‡2:

$$\psi^{(+)}(x^+=0,\boldsymbol{x}) = \sum_{h=-1/2}^{1/2}\int\frac{dp^+d^2p^\perp}{(2\pi)^3}\,\{\exp[-i(p^+x^- - \boldsymbol{p}^\perp\boldsymbol{x}^\perp)]\,w(h)\,Q(p^+,\boldsymbol{p}^\perp;h)$$

$$+\exp[i(p^+x^- - p^\perp x^\perp)]\,w(-h)\,\bar{Q}^\dagger(p^+,\boldsymbol{p}^\perp;h)\}, \tag{2}$$

such that the axial charge

$$\hat{Q}_5(q^+) = \int dx^- d^2x^\perp \exp(iq^+x^-)\,j_5^+(x^+=0,\boldsymbol{x}) \tag{3}$$

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‡1 We omit SU(3) in order to simplify the discussion.

‡2 The spinors $w(1/2) = \binom{1}{0}$, $w(-1/2) = \binom{0}{1}$ are the good components of the infinite helicity spinors: $u(\boldsymbol{p},1/2) = (1, p^\perp/2p^+, M/2p^+, 0)$, $u(\boldsymbol{p},-1/2) = (0, M/2p^+, -p_\perp^*/2p^+, 1)$, $v(\boldsymbol{p},1/2) = (0, -M/2p^+, -p_\perp^*/2p^+, 1)$ $v(\boldsymbol{p},-1/2) = (1, p_\perp/2p^+, -M/2p^+, 0)$ where $p^+ \equiv (p^0+p^3)/2$, $p^\perp = p_\perp \equiv p^1 + ip^2$, $\boldsymbol{p} \equiv (p^+, p^\perp)$.

becomes [‡3] for $q^+ > 0$:

$$\hat{Q}_5(q^+) = \int_{q^+}^{\infty} \frac{dp^+}{2\pi}\int \frac{d^2\boldsymbol{p}^\perp}{(2\pi)^2} \{Q^\dagger(p^+ - q^+, \boldsymbol{p}^\perp)\sigma^3 Q(p^+, \boldsymbol{p}^\perp) + \bar{Q}^\dagger(p^+ - q^+, \boldsymbol{p}^\perp)\sigma^3 \bar{Q}(p^+, \boldsymbol{p}^\perp)\}$$

$$- \int_0^{q^+} \frac{dp^+}{2\pi}\int \frac{d^2\boldsymbol{p}^\perp}{(2\pi)^2} \bar{Q}(-p^+ + q^+, -\boldsymbol{p}^\perp) C Q(p^+, \boldsymbol{p}^\perp) \equiv Q_5(q^+) - \tilde{Q}_5(q^+). \tag{4}$$

As $q^+ \to 0$, the first operator tends to the conventional axial charge Q_5 which simply counts the difference of up and down quarks (or antiquarks). For a single quark its matrix elements are

$$\langle \boldsymbol{p}', \tfrac{1}{2} | Q_5 | \boldsymbol{p}, \tfrac{1}{2} \rangle = (2\pi)^3\, \delta(p^{+\prime} - p^+)\, \delta(\boldsymbol{p}^{\perp\prime} - \boldsymbol{p}^\perp)\, g_A, \tag{5}$$

with $g_A = 1$ while for a proton one finds from β decay $g_A \approx 1.2$.

The second operator vanishes for single quarks. Between hadrons it can be non-zero only if these do *not* consist of pure QQQ wave functions but contain, in addition, admixtures of $Q\bar{Q}$ pairs with total momentum q^+. If $\tilde{Q}_5(q^+)$ turns out to be non-zero even in the limit $q^+ \to 0$, there must be an infinity in the wee quark distribution at small longitudinal momentum.

The important observation is that in a world in which PCAC is exact, precisely this must happen. In such a world current conservation implies for elastic matrix elements

$$\langle \boldsymbol{p} | \hat{Q}_5(q^+ \to 0) | \boldsymbol{p} \rangle |_{p^\perp = 0} = 0, \tag{6}$$

and therefore for infinitesimal $q^+ \to 0$

$$\langle p' | \tilde{Q}_5(q^+ \to 0) | p \rangle |_{p^\perp = 0} = (2\pi)^3\, \delta(p'^+ - p^+ + q^+)\, \delta^2(\boldsymbol{p}^{\perp\prime} - \boldsymbol{p}^\perp)\, g_A. \tag{7}$$

The simplest example is the axial current between protons

$$\langle \boldsymbol{p}' | j_5^+(0) | \boldsymbol{p} \rangle = \bar{u}(p')\left(\tfrac{1}{2}\gamma^+\gamma_5 g_A - \frac{q^+}{2q^+q^- - q^{\perp 2}} f_\pi g_{\pi NN}\gamma_5\right) u(\boldsymbol{p}), \tag{8}$$

with $M_p g_A = f_\pi g_{\pi NN}$. Inserting $\bar{u}(p', \frac{1}{2})\gamma^+\gamma_5 u(\boldsymbol{p}, \frac{1}{2}) = 1$ and $\bar{u}(\boldsymbol{p}', \frac{1}{2})\gamma_5 u(\boldsymbol{p}, \frac{1}{2})|_{p^{\perp\prime} = p^\perp = 0} = \bar{q}/M_p$ yields directly (6).

Let us now describe such admixtures of wee quark pairs quantitatively. Consider a chirally symmetric world following a Heisenberg-Nambu type of Lagrangian [4] with an ultraviolet cutoff at $p^2 = \Lambda^2$.

$$\mathcal{L}(x) = i\bar{\psi}(x)\partial\!\!\!/\,\psi(x) + \mathcal{L}_{int}(\bar{\psi}(x), \psi(x)), \tag{9}$$

where

$$\mathcal{L}_{int}(\bar{\psi}, \psi) = g[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2] - g'[(\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\psi)^2]. \tag{10}$$

Chiral symmetry is broken spontaneously giving rise to quarks of a large mass M satisfying the "gap" equation

$$1 = 8gi\int_0^{\Lambda^2} \frac{d^4p}{(2\pi)^4}\frac{1}{p^2 - M^2} = \frac{g\Lambda^2}{2\pi^2}\left[1 - \frac{M^2}{\Lambda^2}\log\left(\frac{\Lambda^2}{M^2} + 1\right)\right]. \tag{11}$$

The vacuum of these massive quarks is a coherent superposition of "Cooper" pairs of the original massless quarks.

Consider now a field theoretic analogen to the theory of type II superconductors by allowing for a space-time dependent gap $M(x)$. Let $M(x)$ deviate little from the large constant value M, say by

[‡3] We abbreviate $\left\{\begin{matrix} Q(p^+, p^\perp;\ \ 1/2) \\ Q(p^+, p^\perp; -1/2) \end{matrix}\right\} \equiv Q(p^+, p^\perp)$ and $C = i\sigma^2$.

$$m(x) \equiv M(x) - M \equiv \sigma'(x) + \pi(x)\,\mathrm{i}\gamma_5 + V_\mu(x)\gamma^\mu + A_\mu(x)\gamma^\mu\gamma_5, \tag{12}$$

with $|m(x)| < M$.

By rewriting (9) in the form

$$\mathcal{L}(x) = \mathrm{i}\bar\psi\,\partial\psi - (M + m(x))\bar\psi\psi + [\mathcal{L}_{\mathrm{int}} + (M + m(x)\bar\psi\psi] \equiv \mathcal{L}_m(x) + \mathcal{L}'_{\mathrm{int}}(x), \tag{13}$$

and minimizing $\mathcal{L}'_{\mathrm{int}}$ with respect to the self-energy one finds the relations

$$\sigma(x) \equiv M + \sigma'(x) = 2g\,\mathrm{tr}\,S_m(x,x),\quad \pi(x) = 2\,g\,\mathrm{tr}\,(\mathrm{i}\gamma_5\,S_m(x,x)),$$

$$V^\mu(x) = -(g+4g')\,\mathrm{tr}(\gamma^\mu S_m(x,x)),\qquad A^\mu(x) = (g-4g')\,\mathrm{tr}(\gamma^\mu\gamma_5\,S_m(x,x)), \tag{14}$$

where S_m is the propagator of $\mathcal{L}_m$ satisfying the integral equation

$$S_m = S_M + S_M m S_m;\quad S_M(x,y) \equiv \int \frac{\mathrm{d}^4p}{(2\pi)^4} \exp\{-\mathrm{i}p(x-y)\}\,\frac{\mathrm{i}}{\not p - M}. \tag{15}$$

Neglecting terms of the order $L^{-1} = 16\pi^2/\log(\Lambda^2/M^2)$ one obtains [6] an effective (classical) chiral Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = |(\partial_\mu + 2\mathrm{i}A_\mu(x))\,\phi(x)|^2 + 2M^2|\phi(x)|^2 - |\phi(x)|^4 - \tfrac{1}{3}F^{\mathrm{A}}_{\mu\nu}F^{\mu\nu}_{\mathrm{A}} + \tfrac{2}{3}m^2_{\mathrm{A}}A_\mu A^\mu - \tfrac{1}{3}F^{\mathrm{V}}_{\mu\nu}F^{\mu\nu}_{\mathrm{V}} + \tfrac{2}{3}m^2_{\mathrm{V}}V_\mu V^\mu, \tag{16}$$

where

$$\phi(x) \equiv \sigma(x) + \mathrm{i}\pi(x),\quad F^{\mu\nu}_{\mathrm{V}}(x) = \partial^\mu V^\nu(x) - \partial^\nu V^\mu(x),\quad m^2_{\mathrm{V,A}} = 3/[4L(4g' \pm g)] \tag{17}$$

which describes massless pseudoscalar fields interacting with massive ($m_\sigma = 2M$) scalar and very massive axial vector fields. The vector fields are decoupled as long as no SU(3) symmetry is included. Otherwise the Lagrangian (16) becomes a standard massive Yang-Mills theory.

Of great help in extracting consequences from these Lagrangians is the fact, that only a few percent of hadronic wave functions consists of $\mathrm{Q\bar{Q}}$ pairs. One can therefore approximate physical hadrons extremely well by proceeding in the following fashion:

First one takes quarks of mass M and calculates bound states via the interaction Lagrangian

$$\mathcal{L}_{\mathrm{int}}(\bar\psi_M(x), \psi_M(x)) + M\bar\psi_M(x)\,\psi_M(x). \tag{18}$$

The result is a standard "naive" quark model. Then one turns on the interaction (13). In the vicinity of a constituent quark the degenerate vacuum will be perturbed by a field $m(x) \neq 0$. It's shape is controlled by the Lagrangian $\mathcal{L}_{\mathrm{eff}}$ of eq. (16) with a coupling to the quarks given by

$$\mathcal{L}_{\mathrm{eff,\bar{Q}Q}} = -\frac{1}{L}\,[\sigma'(x)\,\bar\psi(x)\,\psi(x) + \pi(x)\,\bar\psi(x)\,\mathrm{i}\gamma_5\,\psi(x)]$$

$$+\frac{1}{4}\left(\frac{4g'}{g}+1\right)\frac{1}{L}\,V^\mu\bar\psi(x)\gamma_\mu\psi(x) + \frac{1}{4}\left(\frac{4g'}{g}-1\right)\frac{1}{L}\,A^\mu\bar\psi(x)\gamma_\mu\gamma_5\,\psi(x). \tag{19}$$

Due to the smallness of $m(x)$, all non-linear terms in σ', π, V_μ, A_μ of $\mathcal{L}_{\mathrm{eff}}$ can be neglected. Thus π and σ' fields satisfy

$$\Box\,\pi(x) \approx -\frac{1}{2L}\,\bar\psi_M(x)\,\mathrm{i}\gamma_5\,\psi_M(x);\qquad (\Box + 4M^2)\,\sigma'(x) \approx -\frac{1}{2L}\,\bar\psi_M(x)\,\psi_M(x). \tag{20}$$

The perturbed *states* can be calculated from the original ones by applying the Möller operator:

$$\Omega^{(+)} = \exp\left[-\mathrm{i}T\int_{-\infty}^{0}\mathrm{d}^3x\;\bar\psi_M(x)\,m(x)\,\psi_M(x)\right]. \tag{21}$$

It can be considered as a *mixing operator* adding an infinity of quark-antiquark pairs to any constituent quark wave function.

Let us now go to the infinite momentum frame. As far as the operators $Q(p^+, \boldsymbol{p}^\perp; h)$ at $x^+ = 0$ are concerned, massive and massless quark operators become indistinguishable‡4. Thus any charge at $x^+ = 0$ can directly be evaluated on the final mixed wave function. The massive constituent quarks have turned into current quarks.

For a single quark $Q(\boldsymbol{P})$, the $Q\bar{Q}Q$ admixture of pseudoscalar pairs given by (21) is‡5 (with $q^- \equiv (\boldsymbol{P}_\perp^2 + M^2)/2P^+ - [(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2 + M^2]/2(P^+ - q^+)$):

$$-\frac{1}{2L}\int \frac{d^3q}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\left[\frac{\boldsymbol{p}_\perp^2 + M^2}{2p^+} + \frac{(\boldsymbol{q}_\perp - \boldsymbol{p}_\perp)^2 + M^2}{2(q^+ - p^+)} - q^-\right]^{-1} (2q^+q^- - \boldsymbol{q}^{\perp 2})^{-1}$$

$$\times\, \bar{u}(\boldsymbol{P} - \boldsymbol{q})\, \mathrm{i}\gamma_5\, u(\boldsymbol{P})\, \bar{u}(\boldsymbol{q} - \boldsymbol{p})\, \mathrm{i}\gamma_5\, v(\boldsymbol{p})\, Q^+(\boldsymbol{q} - \boldsymbol{p})\bar{Q}^+(\boldsymbol{p})\, Q^+(\boldsymbol{P} - \boldsymbol{q})|0\rangle. \tag{22}$$

If we evaluate the operator $\tilde{Q}_5(q^+)$ we find that eq. (7) is indeed fullfilled and hence PCAC is respected.

The admixture of pairs has the probability $\approx 1/L = 16\pi^2/\log(\Lambda^2/M^2)$.

This remains small for a large cutoff such that our approximation of small $m(x)$ is self-consistent. The value of $1/L$ can be adjusted to the experimentally observed number ($\approx 6\%$).

Notice that the description presented here bears a strong analogy to presently fashionable studies of extended hadronic objects and of "solitons" in non-linear field theories [7]. Also solitons can be considered as "gap waves" in a type II superconductor.

A remark is in order concerning the program of transforming constituent quarks into current quarks [8]. This program is certainly completed by the operator (21). When calculating axial charges one has to keep in mind that any quark wave function is commonly constructed in terms of canonical quark spinors $u(\boldsymbol{p}, s_3)$. The operator Q_5 of eq. (4), on the other hand, deals with infinite momentum helicities h. A Wigner rotation

$$Q^+(\boldsymbol{p}; s_3) = \sum_{h=-1/2}^{1/2} Q^+(\boldsymbol{p}; h)\, W_{h,s_3}(\boldsymbol{p}), \tag{23}$$

with

$$W_{h,s_3}(p) = \bar{u}(\boldsymbol{p}, h)\, u(\boldsymbol{p}, s_3), \tag{24}$$

has to be performed before applying Q_5. In this connection, W_{h,s_3} is called the Melosh transformation.

It will be interesting to investigate the modification of chiral wave functions, the detailed distribution of pairs, the properties of diffraction scattering and the amount of breakdown of exchange degeneracy by means of such considerations.

‡4 In the infinite momentum frame, vacua corresponding to fermions of different mass, which are originally connected by a Bogolubov type of transformation, become equivalent. Thus also the sea of Cooper pairs disappears. The reason is the clear frequency separation of creation and annihilation operators in $\psi(x)$ of eq. (2).

‡5 Here $\boldsymbol{p} \equiv (p^+, \boldsymbol{p}^\perp)$; $d^3p \equiv dp^+ d^2p^\perp$.

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