WHAT CAN A PARTICLE PHYSICIST LEARN FROM SUPERLIQUID ^{3}He?

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ABSTRACT

The superliquid ^{3}He is shown to be a wide field of application for many recently developed methods in particle physics and field theory:

1) There is a transformation from fundamental to collective fields via path integral techniques just as exists from Thirring to Sine-Gordon fields and as one would like to find from quarks to hadrons.
2) There are many classical field configurations, monopoles, strings, and solitons, which all can be produced and investigated in the laboratory.
3) Topological quantum numbers are helpful in classifying the stable field configurations.

Such applications to realistic situations in other branches of physics, apart from being useful in themselves, may extend our intuition and help us finding new methods and approximation procedures.

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Many arguments used recently in the attempts to deduce the confinement of quarks in Yang-Mills theories [1] rely heavily upon our intuition about flux loops derived from what we know about superconductors and superliquid He-II [2]. I would like to draw your attention to another superliquid, ^{3}He, whose collective description has a non-trivial topological structure and may therefore be a complimentary or even more inspiring source of information. Instead of a pure phase, $e^{i\phi}$, the order parameter describing the condensate of Cooper pairs at each point is given by a complex 3x3 matrix of the specific form

$$A_{a,i} \propto d_a(\phi^{(1)} + i\phi^{(2)})_i \tag{1}$$

Here $\underset{\sim}{\phi}^1, \underset{\sim}{\phi}^2$ are orthogonal unit vectors which can be thought of as the two axes of a dreibein (see Fig.I) with $\underset{\sim}{\ell} \equiv \underset{\sim}{\phi}^{(1)} \times \underset{\sim}{\phi}^{(2)}$ as the third axis while d_a is another

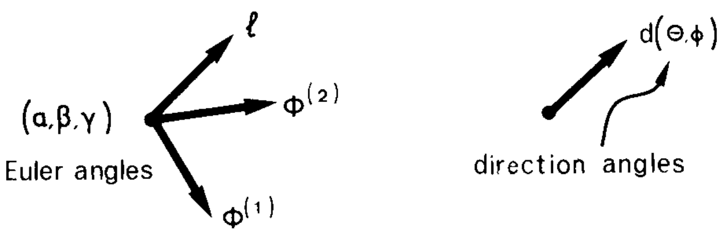

Fig. I

unit vector. Physically, $\underset{\sim}{\ell}$ points in the direction of the angular momentum of the Cooper pairs while $\underset{\sim}{d}$ denotes the axis along which the total spin has a vanishing component. The parameter space of this field is topologically equivalent to $S^2 x SO_3/Z_2$ where Z_2 corresponds to the simultaneous reflection of $\underset{\sim}{\phi}^1, \underset{\sim}{\phi}^2$, and $\underset{\sim}{d}$ (with $\underset{\sim}{\ell}$ staying fixed to preserve the orientation of the dreibein).

Due to this feature there exist many non-trivial topologically inequivalent field configurations very similar to those in gauge theories. Topology is very useful in classifying the different solutions. Contrary to gauge theories, however, many of these fields can be prepared in the laboratory by an appropriate choice of the container walls, external fields, and currents. It is possible to bring the liquid into states which are separated from the ground state by a potential barrier and study the "fate of the false vacuum" [3)].

Finally, the very derivation of the collective field theory from that of the fundamental ^{3}He action is completely analogous to what has recently become popular in 1+1 dimensional theories: The transition from the Thirring model to the Sine-Gordon equation [4)]. Since it is quite plausible that eventually a similar transition will be found from quantum chromodynamics of quarks and gluons to a dual theory of hadrons [5)], it may be an inspiring experience to learn how well a liquid containing many strongly interacting fermions can be described in terms of a few Bose fields [6)]. Also, Landau's way of arguing for an approximation of the original theory by another one formulated in terms of weakly interacting quasiparticles may give related insights into the connection between current and constituent quarks [7].

The fundamental action of ^{3}He is

$$\mathcal{A} = \int d^4x\,\psi^{+\alpha}(x)\left(i\partial_t + \frac{\nabla^2}{2m} + \mu\right)\psi_\alpha(x)$$
$$-\frac{1}{2}\int d^4x\,d^4x'\,\psi^{+\alpha}(x)\,\psi^{+\beta}(x')V(x-x')\psi_\beta(x')\psi_\alpha(x) \tag{2}$$

In addition, there is a weak hyperfine interaction between the nuclear dipole moments which we shall at first neglect. The potential V(r) is displayed on Fig.II.

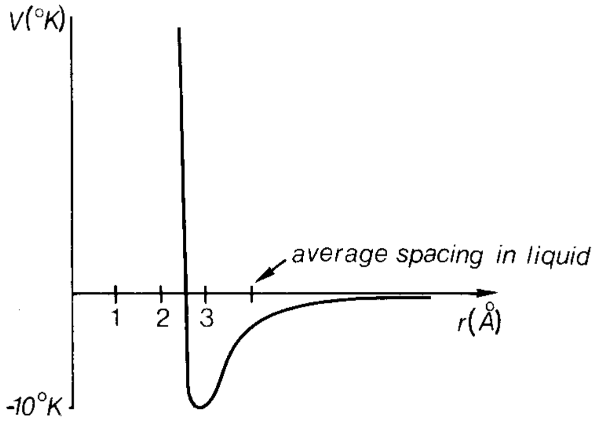

Fig. II

If one calculates the average distance between two atoms one finds $\langle r\rangle \approx 3.5$ Å. Thus the atoms attract each other all the time and one is confronted with a strong-coupling problem. It would be hopeless to attempt a perturbative solution were it not for Landau's important observation:

Instantaneous screening effects dress the particle, moving through the liquid, with a small cloud and generate a quasiparticle. The interaction between the quasiparticles is quite weak. In order to test this idea one may calculate specific heat, susceptibility, and compressibility for a free system of quasiparticles. As the temperature varies, these behave as T, constant, T^{-2}, with coefficients depending on the mass. It turns out that all formulas agree very well with experiment if one uses the quasiparticle mass $m_{Qp} \approx 3m_{^3He}$. Thus Landau's idea of almost free quasiparticles seems to be correct in ^{3}He with the screening cloud consisting, on the average, of two neighbouring atoms. If one now writes an <u>effective</u> action for the quasiparticles again in the form (2), the potential is expected to be strongly screened and of short range. Only the hard core will definitely stick out. Now there is hope of Feynman graphs leading to realistic answers.

Notice that since the fields are now not completely local but describe a time and space average over a quasiparticle size ($\approx$ 3 Å) one cannot use the effective action to answer questions concerning distances of this order. We shall therefore restrict the following discussion to much larger distances, say starting with 100 Å.

In quark physics, something similar and much more drastic seems to happen: Fundamental quarks are strongly interacting objects of small mass ($M_u^o \approx$ 15 MeV) [9]. When moving through the quark matter inside a hadron they seem to become heavy ($M_u \approx$ 310 MeV) and almost free. Thus the current versus constituent quark picture is in complete analogy with Landau's idea of Fermi liquid effects.

The quasiparticle action of the form (2) can most efficiently be treated by approximating the potential with another one of the same characteristic shape but with a squeezed radius

$$V(x) = V_o\delta(x) - \frac{3g}{4p_F^2}\nabla^2\delta(x) \tag{3}$$

Here $V_o > 0$ accounts for the repulsive core and $g > 0$ for the attractive well around the core. If one neglects V_o (which can be taken care of afterwards in form of what is called paramagnon correction) the interaction can be reordered and written as

$$\mathcal{A}_{int} = \frac{1}{2}\frac{3g}{4p_F^2}\int dx\,\psi^{+\alpha} i\overleftrightarrow{\nabla}_i\psi^{+\beta}\psi_\beta i\overleftrightarrow{\nabla}_i\psi_\alpha \tag{4}$$

We have left out a term proportional to $\underset{\sim}{\nabla}(\psi^{+\alpha}\psi_\rho)\underset{\sim}{\nabla}(\psi^{+\beta}\psi_\sigma)$ since for phenomena with wavelength much larger than 3 Å this is neglegible compared to (4): There the derivative $\overleftrightarrow{\nabla}$ stands between the fields which gives at the Fermi surface a momentum $2p_F$. Since $p_F \approx 1/(1.1$ Å$)$ this is indeed large compared with $\underset{\sim}{\nabla}(\psi^{+\alpha}\psi_\beta)\,\underset{\sim}{\nabla}(\psi^{+\beta}\psi_\alpha)$ which goes with the total momentum of the phenomena we want to study ($<$ 1/(100 Å)).

With the interaction (4) we can write the generating functional of the quantum field theory of ^{3}He as the path integral

$$Z = \int\mathcal{D}\psi\mathcal{D}\psi^+\, e^{i\int dx\,\mathcal{L}(x)} \tag{5}$$

with

$$\mathcal{L}(x) = \psi^+(i\partial_t - \xi(-i\underset{\sim}{\nabla}))\psi + \frac{3g}{4p_F^2}\psi^+ i\overleftrightarrow{\nabla}_i \frac{\sigma_a}{2}C^+\psi^+ \psi i\overleftrightarrow{\nabla}_i C\frac{\sigma_a}{2}\psi$$

Notice that we have inserted $C\sigma_a$ matrices ($C \equiv i\sigma^2$) between the fields in the interaction term without changing the expression (4) because of the anticommutativity of the fields.

The letter ξ stands short for the energy measured from the chemical potential:

$$\xi(-i\underset{\sim}{\nabla}) \equiv -\frac{\underset{\sim}{\nabla}^2}{2m} - \mu. \tag{6}$$

In order to allow for temperature ensembles the fields are periodic in Euclidean time such that the energy integrals of the Feynman rules are sums over Matsubara frequencies

$$i\int \frac{dk^4}{2\pi} \longrightarrow T \sum_{k^o = i\left\{\begin{matrix}2n+1\\2n\end{matrix}\right\}\pi T} \quad \text{for} \left\{\begin{matrix}\text{fermions}\\\text{bosons}\end{matrix}\right\} \tag{7}$$

We now transform (5) into Bose form by using the standard trick of multiplying Z with a constant gaussian functional integral [10,5,6]

$$1 \equiv \int \mathcal{D}A\,\mathcal{D}A^+ \, e^{-\frac{i}{3g^2}\int dx \left| A_{a,i} - \frac{3g}{2p_F}\psi i\overleftrightarrow{\nabla}_i C\frac{\sigma_a}{2}\psi\right|^2} \tag{8}$$

Now the exponent of (5) times (8) can be written as

$$\mathcal{L}(x) = \frac{1}{2}\, f^{+}(x) \begin{pmatrix} i\partial_t - \xi & i\overleftrightarrow{\nabla}_i \sigma_a A_{ai} \\ i\overleftrightarrow{\nabla}\sigma_a A_{ai}^{+} & i\partial_t + \xi \end{pmatrix} f(x) - \frac{1}{3g} A_{ai}^{+}(x) A_{ai}(x) \tag{9}$$

where $f(x) \equiv \begin{pmatrix} \psi(x) \\ c^{+}\psi^{+}(x) \end{pmatrix}$. The fields f can be integrated out (for details see Ref. 6)) and the functional becomes a pure integral over A fields

$$Z = \int \mathcal{D}A\, \mathcal{D}A^{+}\, e^{\, i\mathcal{A}_{\text{coll}}[A]} \tag{10}$$

with the collective action

$$\mathcal{A}_{\text{coll}}[A] = -\frac{i}{2}\, \text{tr}\log \begin{pmatrix} i\partial_t - \xi & i\overleftrightarrow{\nabla}\sigma_a A_{ai} \\ i\overleftrightarrow{\nabla}\sigma_a A_{ai}^{+} & i\partial_t - \xi \end{pmatrix} - \frac{1}{3g} \int dx A_{ai}^{+}(x) A_{ai}(x) \tag{11}$$

This depends only on the complex 3x3 Bose field A_{ai} and describes completely the ^{3}He liquid for phenomena varying over distances > 100 Å, say. In regions of small A one can expand (11) in a power series. The lowest terms are for time independent fields

$$\begin{aligned} \mathcal{A} = \int dx\, \Big\{ & \frac{1}{3}\Big(1 - \frac{T}{T_c}\Big) A_{ai}^{+} A_{ai} \\ & - \xi_o^2 (\partial_i A_{ai}^{+} \partial_j A_{aj} + \partial_i A_{aj}^{+} \partial_j A_{ai} + \partial_i A_{ai}^{+} \partial_j A_{aj}) \\ & - \frac{2}{5}\xi_o^2 \Big[\beta_1 A_{ai}^{+} A_{bj} A_{ai}^{+} A_{bj} + \beta_2 (A_{ai}^{+} A_{ai})^2 \\ & + \beta_3 A_{ai}^{+} A_{aj} A_{bi}^{+} A_{bj} + \beta_4 (A_{ai}^{+} A_{bi} A_{bj}^{+} A_{aj}) \\ & + \beta_5 A_{ai}^{+} A_{bi} A^{+}_{aj} A_{bj} \Big] \Big\} \end{aligned} \tag{12}$$

Here we have

1) divided out an overall factor

$$N(0) \equiv \frac{mp_F}{2\pi^2} = \frac{3}{4}\frac{\varrho}{m}\frac{1}{T_F} \qquad (13)$$

which is the density of states at the surface of the Fermi sphere,

2) introduced the critical temperature T_c as a solution of the gap equation

$$T_c \equiv \omega_{cutoff}\, 2\frac{e^\gamma}{\pi} e^{-1/g N(0)} \qquad (14)$$

with the parameter ω_{cutoff} which limits the energy integration. This cutoff is determined by the frequency at which the quasiparticle approximation breaks down (say $\frac{1}{10}$ MHz),

3) used the length parameter $^{+)}$

$$\xi_o = \sqrt{\frac{7\zeta(3)}{48\pi^2}}\, \frac{v_F}{T_c} \approx .134\, \frac{v_F}{T_c} \approx 150\ \mathring{A} \qquad (15)$$

called coherence length for reasons to be seen shortly

4) set $v_F \approx 3x10^3$cm $\equiv 1$ $^{++)}$ thus converting freely energy into length^{-1} units ($\hbar$ = $K_{Boltzmann} \equiv 1$ from the beginning). If mK is used as energy unit, the frequency and length units become 131.6 MHz and 305 Å, respectively.

+) The constants are $\zeta(3) \equiv \sum_1^\infty \frac{1}{n^3} \approx 1.202$, $v_F \equiv \frac{p_F}{m}$

++) This is at melting pressure. At zero pressure $v_F \approx 5.5x10^3$cm/sec.

The coefficients of the quartic term are actually determined by the calculation as

$$-2\beta_1=\beta_2=\beta_3=\beta_4=-\beta_5=1 \tag{16}$$

It turns out, though, that the effect of the repulsive core and, to a smaller extent, also of the remaining part of the neglected quasiparticle interaction do cause some changes in β (up to 30%). Therefore we keep β as parameters. The same corrections also modify the coefficients of the derivative terms but only by a few percent such that they can be neglected.

Notice that higher derivatives as well as higher powers in A_{ai} carry, for dimensional reasons, one more power of ξ_0 each. Thus for wavelength $> \xi_0$ higher derivatives can be neglected. How about higher powers in A? Looking at the potential we see that there is a phase transition as $T < T_c$ since then the mass term picks up the wrong sign. The fields A will fluctuate around a new minimum determined by the quartic interaction. Its size is of the order of

$$|A| \sim \frac{1}{\xi} \equiv \frac{1}{\xi_0}\sqrt{1-\frac{T}{T_c}} \tag{17}$$

for dimensional reasons. Thus, as one is close enough to T_c, also higher powers in A can be neglected. The action (12) is complete as far as the long wavelength limiting behaviour of the system is concerned. In the modern jargon of critical phenomena, it contains all infrared relevant terms [11].

What information can be extracted from the collective field theory (12) of ^{3}He? Obviously , one is confronted with an $SU_2 x SU_2 x U(1)$ σ type of model whose critical phenomena have been studied in the literature [11]. For brevity, we shall focus here only on the classical aspects:

As $T < T_c$ the vacuum settles at a new minimum determined by β_i. There is no complete analysis of the potential in the 18 parameter space of A_{ai}.[12] However, there are two minima which describe very well all properties of the two main phases of ^{3}He, A and B (see Fig.III). These minima

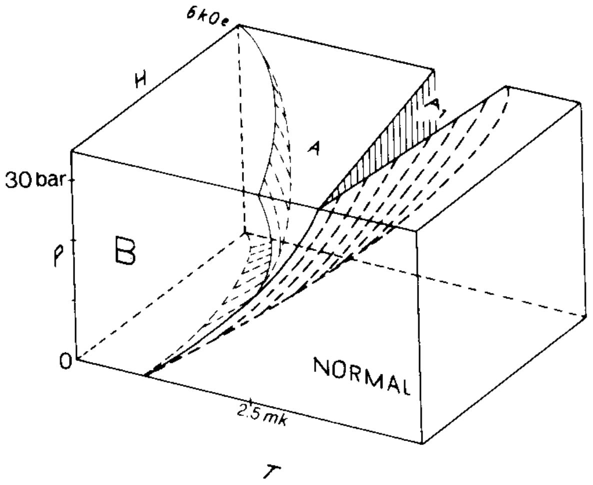

Fig. III

can be parametrized by

$$A^o_{a,i} = \frac{1}{2}\sqrt{\frac{5}{6}}\ \frac{1}{\xi}\ d_a(\phi^{(1)}+i\phi^{(2)})_i \tag{18}$$

and

$$A^{o}_{a,i} = \frac{1}{2}\frac{1}{\sqrt{3}}\frac{1}{\xi} R_{a,i}(\mathbf{n},\theta)e^{i\phi} \tag{19}$$

The first minimum has a degeneracy corresponding to a space $SO_3 x S^2/Z_2$ as discussed in the beginning. Similarly, the second space is described by $SO_3 x U(1)$.

Great simplicity is gained by restricting oneself to phenomena with wavelength $\gg \xi_o$. Then the energy density stays much smaller than $1/\xi_o$ and the field is pinned down tightly at the minimum of the potential valley. The size of A is fixed and the energy depends only on the changes in the directions of the field vectors. It is therefore often referred to as bending energy. In the standard field theory of the σ model this corresponds to the transition from the linear to the non-linear σ model by letting $m_\sigma \to \infty$ while keeping $\langle\sigma\rangle$ fixed. Now the only degrees of freedom left are the directions of the different vacua with possible smooth spatial variations.

By concentrating on such low-energy phenomena it becomes important to include a weak force which was left out in the beginning but which now has relatively large effects: The hyperfine interaction due to the magnetic dipole-dipole forces in the nuclei. If we calculate its collective form (by taking it through the path integral transformation [6]) we find that it wants to align $\underset{\sim}{d}$ and $\underset{\sim}{\ell}$ vector in the A phase via

$$\mathcal{A}_d = \frac{1}{6}\frac{\xi_o^2}{\xi^2}\frac{1}{\xi_d^2}\int dx(\underset{\sim}{d}\cdot\underset{\sim}{\ell})^2 \tag{20}$$

where the length parameter ξ_d characterizes the strength of the interaction. The microscopic calculation of ξ_d [6]

agrees with the experimental determination

$$\xi_d \approx 10^{-3} \text{ cm} \approx 300\, \xi_o \tag{21}$$

Thus the dipole interaction drives $\underset{\sim}{d}$ parallel to $\underset{\sim}{\ell}$ and becomes important at length scales 10μ. This alignment force is easily understood. In the Cooper pair the configuration $S \parallel L$ is clearly of higher energy than $S \perp L$ since equal magnetic poles are always adjacent to each other while in $S \perp L$ they are in line for half the orbit (see Fig.IV). But $S \perp L$ means $d \parallel \ell$ since $\underset{\sim}{d}$ is the axis

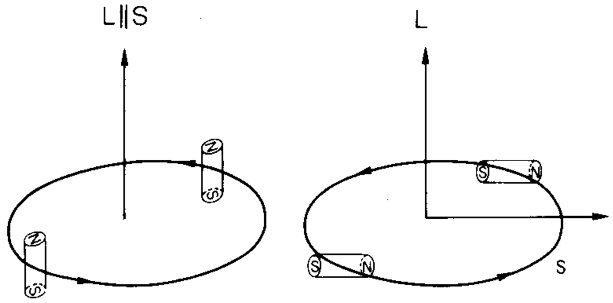

Fig. IV

along which $S_3 = 0$.

The collective free energy can now be written (dropping an overall factor $\frac{1}{6}\frac{\xi_o^2}{\xi^2}$ and surface terms) as

$$f = \frac{1}{2}\left(|\partial\Phi|^2 + \frac{1}{2}|\partial_i \Phi_j|^2 + |\underset{\sim}{\Phi}\underset{\sim}{\partial} d_a|^2 + (\partial_i d_a)^2 \right) - \frac{1}{\xi_d^2}(\underset{\sim}{\ell}\cdot\underset{\sim}{d})^2 \tag{22}$$

Similarly one can write f for the B phase in terms of the fields $\underset{\sim}{n}$, θ, and ϕ and a dipole energy which drives the angle θ to $\theta_d \approx \cos\left(-\frac{1}{4}\right)$ with a length scale of the same order of magnitude.

This non-linear σ model (22) (and a corresponding version for the B phase) can now be investigated with popular methods of field theory: It contains the Sine-Gordon equation for particular field configurations thereby giving rise to solitons. The topology of the parameter space allows for the existence of non-trivial field configurations for the ground state.

Before we present a few of the phenomena we have to realize that the superliquid is always in a finite container, usually with a size of the order of cm. This imposes boundary conditions upon the field lines. It can easily be derived that $\underset{\sim}{\ell}$ has to stand orthogonal to the walls. Physically, this is obvious from the fact that the size of the Cooper pairs is given by the coherence length and is therefore a few hundred times larger than the atomic distance. Thus if the liquid is to stay "super" up to the walls, the orbital planes of the pairs have to be parallel to the walls in order to avoid break-up (see Fig.V).

Another important external effect is given by a magnetic field. Since this introduces a quantization axis for the spin the vector d along which $S_3 = 0$ is forced orthogonal to H (see Fig.VI).

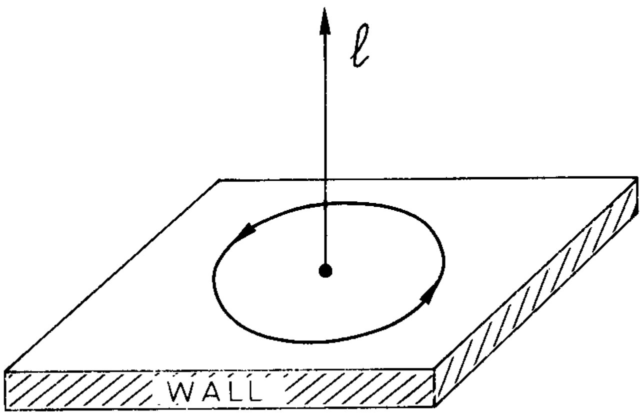

Fig. V

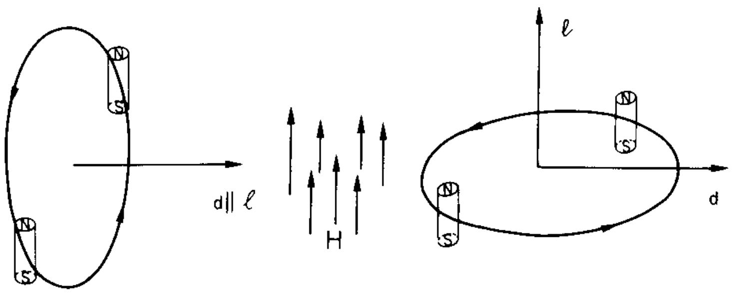

Fig. VI

With this preparation we can now present a few interesting classical field configurations.

1) Monopoles

Cooling a sphere smoothly through T_c plants the $\underset{\sim}{\ell}$ vectors orthogonal to the walls with $\underset{\sim}{\phi}^1, \underset{\sim}{\phi}^2$ covering them like orthonormal coordinates on a globe. With the liquid becoming "super" more and more inwards one may think that the $\underset{\sim}{\ell}$ lines grow radially until they hit a point singularity at the center. However, this is not true: Because of the mathematical theorem that a hedgehog cannot be combed without vortices, there must be singularities in the tangential $\underset{\sim}{\phi}^1, \underset{\sim}{\phi}^2$ fields either two of flux one (for example at north and south pole) or one of flux two (see Fig.VII). Thus ideally, one would have field lines

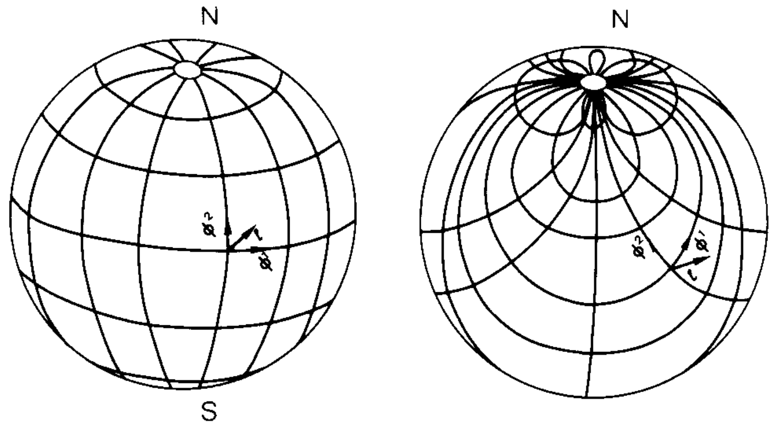

Fig. VII

growing as shown in Fig. VIIIa,b where the curly lines trace the singularities of the $\underset{\sim}{\phi}^1, \underset{\sim}{\phi}^2$ fields for the two possible extremes. But along these singular vortex lines the liquid must be normal since only if the field A_{ai} vanishes can the direction of $\underset{\sim}{\phi}^1, \underset{\sim}{\phi}^2$ be ill-defined. Thus the vortex lines form strings with a thickness equal to

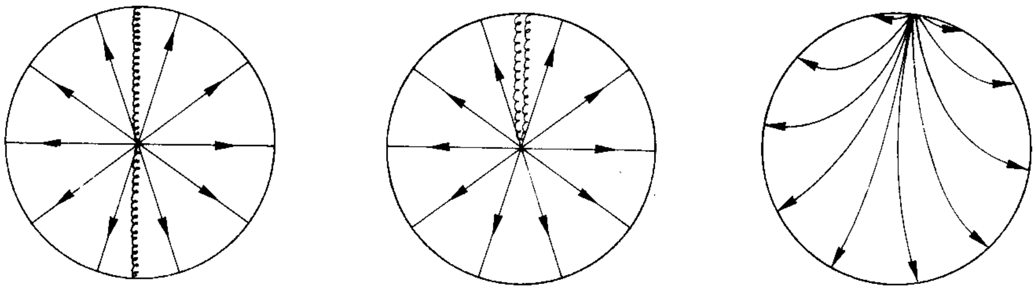
a) b) c)

Fig. VIII

the coherence length inside of which there is an accumulation of condensation energy (the difference between f(A=0) and f(A = equ.(18),(19)). Thus the liquid likes to keep these strings as short as possible and pulls the point singularity to the wall with an LlogL potential energy resulting in the flower-like field configuration VIIIc (called boojum). This object has an intrinsic angular momentum and and should be detectable by observing the rotation of little ^{3}He droplets as they are cooled through T_c into the superliquid phase.

2) Line Singularities

In a cylinder the $\hat{\ell}$ lines will develop inwards until they form a singular line at the axis (see the left of Fig.IX). But this again contains the large condensation energy and the field lines prefer avoiding it by flaring upwards like flames in a chimney [14] (see the right of Fig.IX).

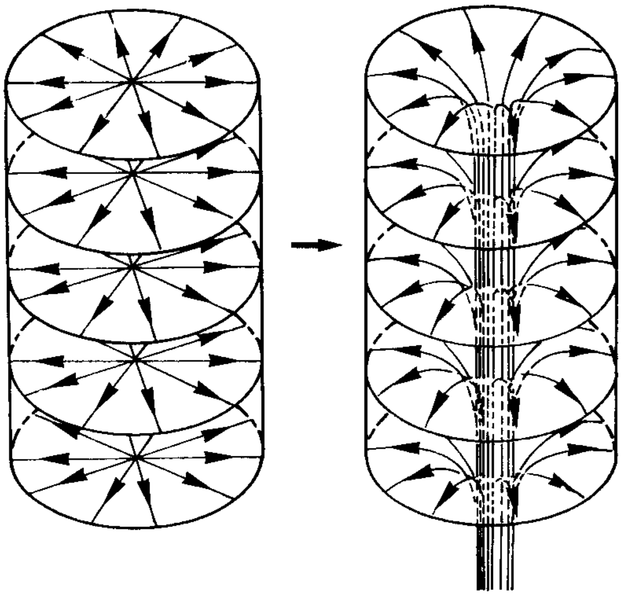

Fig. IX

3) Planar Singularities (Solitons)

In a magnetic field along the z axis the $\underset{\sim}{d}$ vectors will be forced in the xy plane, say

$$\underset{\sim}{d} = \sin\psi\,\hat{x} + \cos\psi\,\hat{y} \tag{23}$$

Due to the dipole force, also ℓ wants to have this direction. If the superliquid could develop smoothly, one would obtain a completely uniform $d \parallel \ell$ field. However, small perturbations will create defects with $d \parallel \ell$ in some direction and antiparallel in others. The size of such domain walls will be determined by the dipole length ξ_d. In order to study a particular (twist) domain wall assume $\underset{\sim}{\ell}$ to have the form

$$\underset{\sim}{\ell} = \sin\chi\,\hat{x} + \cos\chi\,\hat{y} \tag{24}$$

which is compatable with a Φ vector of the form

$$\Phi = e^{i\phi} (-\cos\chi\ \hat{x} + \sin\chi\ \hat{y} + iz) \tag{25}$$

The free energy reads

$$f = \{(\partial\psi)^2 + (\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2\} + \frac{1}{\xi_d^2}\sin^2(\chi-\psi) \tag{26}$$

This can be diagonalized by

$$v = \chi - \psi \tag{27}$$

$$u = \chi + 4\psi$$

as

$$f = \phi_z^2 + \frac{1}{20}u_z^2 + \frac{1}{5}v_z^2 + \frac{1}{\xi_d^2}\sin^2 v \tag{28}$$

The classical extrema are ϕ = const., u = const. with a soliton in the v variable [15] (see Fig. X)

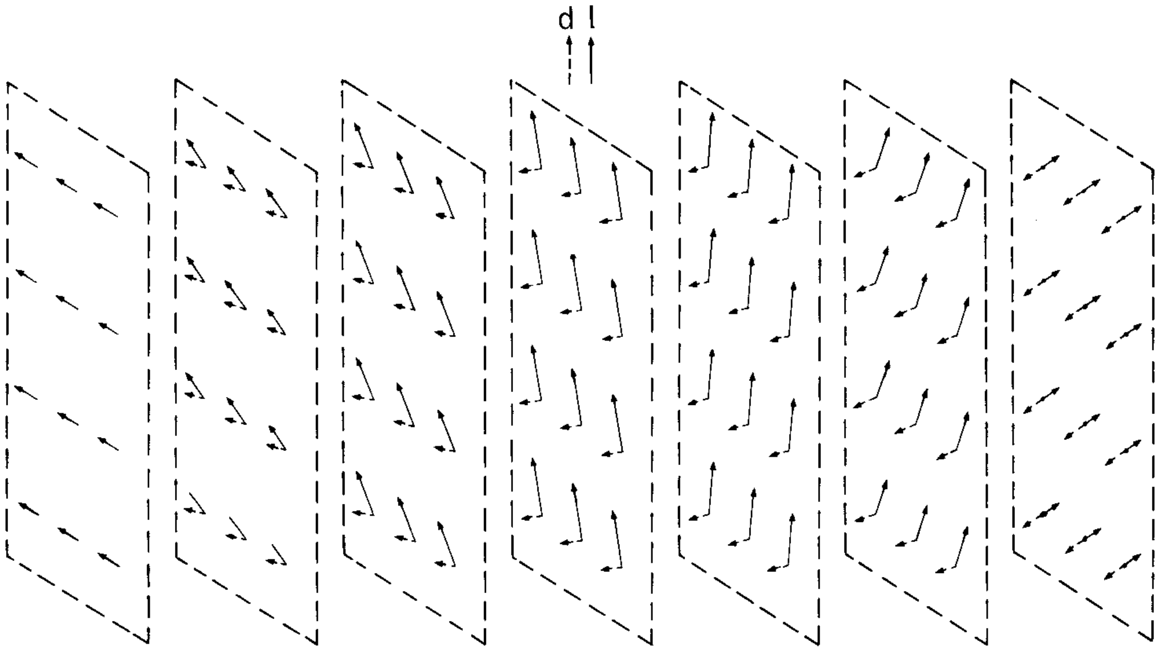

Fig. X

$$\mathrm{tg}\,\frac{v_{sol}}{2} = e^{\pm z/\xi_{sol}} \quad , \quad \sin v_{sol} = \mathrm{ch}^{-1}(z/\xi_{sol}) \tag{29}$$

of size

$$\xi_{sol} = \frac{1}{\sqrt{5}}\,\xi_d$$

The detection of all these classical field configurations proceeds most simply via nuclear magnetic resonance experiments. A vibrating $\underset{\sim}{H}_\omega$ field causes the $\underset{\sim}{d}$ vector to oscillate around the equilibrium position generating spin waves in the system. For these any spatial inhomogeneity acts as a potential wall (or mountain) which can trap (or repell) them. For example, the soliton just found will catch spin waves in a bound state. Consider small deviations

$$\psi = \psi_{sol} + S \tag{30}$$

Then the energy fluctuates as

$$\delta^2 f = \delta_z^{\ 2} + \frac{1}{\xi_d^{\ 2}}\left[1 - \frac{2}{\mathrm{ch}^2(z/\xi_{sol})}\right]\delta^2 \tag{31}$$

This is extremized by the solution of the Schrödinger equation

$$\{-\partial_z^{\ 2} + \frac{1}{\xi_d^{\ 2}}\left[1 - \frac{2}{\mathrm{ch}^2(z/\xi_{sol})}\right]\}\ \delta(z) = \omega^2\ \delta(z) \tag{32}$$

which has a single bound state

$$\delta(z) \propto \frac{1}{[\mathrm{ch}(z/\xi_{sol})]^s} \tag{33}$$

with

$$s = \frac{1}{2}\left[-1+\sqrt{1+4\frac{2}{\xi_d^2}\xi_{sol}^2}\right] = \frac{1}{2}\left[-1 + \sqrt{\frac{13}{5}}\right] \approx .306 \qquad (34)$$

The energy is

$$\omega^2 = \frac{1}{2}\,(\sqrt{65} - 7) = \frac{1}{\xi_d^2} \qquad (35)$$

while the continuum has a spectrum

$$\omega^2 = k^2 + \frac{1}{\xi_d^2} \qquad (36)$$

This bound state can be seen as a satellite frequency in NMR experiments shifted down by a factor of $\sqrt{\frac{1}{2}(\sqrt{65}-7)} \approx .728$ with respect to the normal line in complete agreement with the data [16].

Finally, let us give an example of the use of topology. Since the dipole force aligns $\underset{\sim}{d}$ and $\underset{\sim}{\ell}$, the parameter space of ^{3}He-A is for extended objects ($\gg\xi_d$) SO_3 only. Its homotopy groups are

$$\pi_1 = Z_2 \quad , \quad \pi_2 = 0. \qquad (37)$$

Notice that the first statement implies that ^{3}He-A is not really a superliquid at all since it is not able to pile up a large number of flux quanta in a torus (for this π_1 has to be equal to Z).[17]

In the B phase, θ is pinched to $\theta_d = \arccos\left(-\frac{1}{4}\right) \approx 104^\circ$ such that the remaining parameter space is $S^2 \times S^1$ (S^1 from the phase and S^2 from the direction vector $\underset{\sim}{n}$). Now the homotopy groups are

$$\pi_1 = Z \quad , \quad \pi_2 = Z \qquad (38)$$

such that there are infinitely many line and point singularities. ^{3}He-B is a superliquid just as He-II.

For a more complete discussion of the non-trivial field configurations you are referred to Ref. 18).

Apart from these applications of recently popular methods of particle physics and field theory, ^{3}He may also serve directly high-energy experiments whenever a coherent accumulation of small effects must be detected. An example could be the electric dipole moments of the Cooper pairs caused by P violating (T conserving) neutral currents. These would line up with LxS and could pile up to a macroscopic effect in the condensate [19).]

With ^{3}He physics being a rapidly expanding field of research, I have been able to present only a glimpse of the many interesting phenomena to be explained by theory. Hopefully, the similarity of the problems as well as the methods of their solution may provide particle physicists with the same degree of inspiration as has been derived in the past from superconductors and superliquid ^{4}He.

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