FATE OF FALSE VACUA - FIELD THEORY APPLIED TO SUPERFLOW

H.Kleinert

Institut für Theoretische Physik

Freie Universität Berlin, 1000 Berlin 33

ABSTRACT

The quantized states of superflow in a circular tube form a great number of metastable "false vacua". At the level of the field equations of motion there is absolute topological stability. Fluctuations cause decay with "critical bubbles" of large energies acting as a necessary trigger, just as in the evaporation process of a superheated liquid. The similarity and differences with respect to the infinite set of gauge field vacua are discussed.

I Introduction

During the last few years, two aspects of gauge theories have found special attention in elementary particle physics:

1) The existence of infinitely many vacua which differ by their topological properties.
2) The communication among the vacua via solutions of the field equations when continued to imaginary "time".

   The so obtained euclidean form of 3+1 dimensional field theory is formally equal to statistical mechanics in 4 spatial dimensions with the coupling constant g^2 playing the role of a temperature.

In this lecture I would like to show that a very similar situation is responsible for an important set of physical phenomena: The existence of superflow and its extremely slow decay in superconductors and superliquid ^{3}He. Certainly, since these systems are of a statistical nature as they are, no analytic continuation is necessary to imaginary time, the inverse temperature $\tau=1/T$ playing this rôle from the beginning. In superconductors, the theory has been developed a long time ago[1] but has recently found an essential improvement[2]. In ^{3}He, on the other hand, whose superliquid transition was discovered seven years ago[3], the more complicated interplay of dynamics[4] and topology has been understood only during the last year.

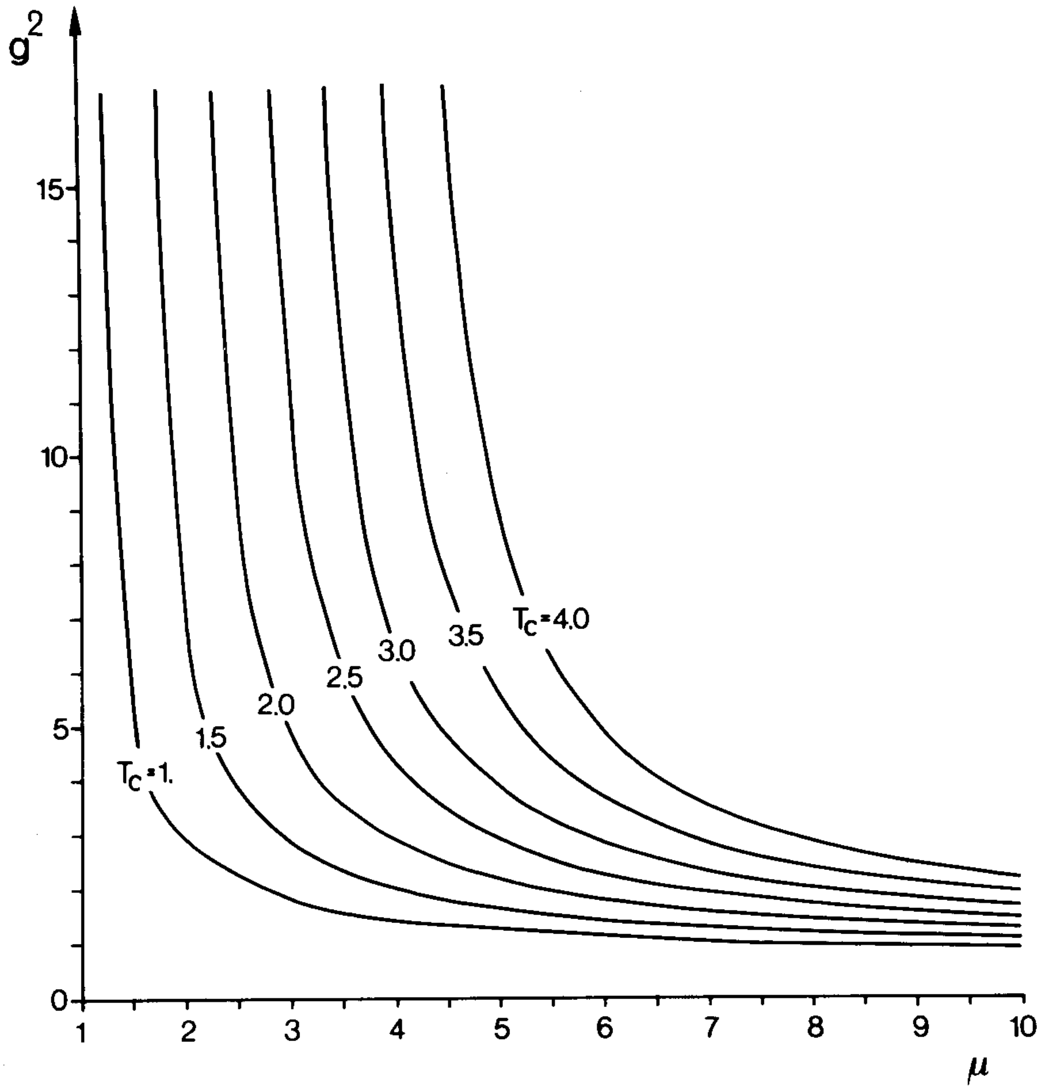
Common to both systems is the formation of a condensate of Cooper pairs: In a superconductor, these consist of two electrons (on the surface of the Fermi sea) in an s-wave. The attraction is caused by phonon exchange and is very weak. Physically, the exchange diagram accounts for the accumulation of positive ions along the path of the electron which acts as an attractive potential wake[+]. The binding energy determines the critical temperature at which the pairs break up, due to thermal collisions. It can be found as

$$T_c = \mu \, e^{-1/g^2} \approx 1 \, ^\circ K \tag{1}$$

Here μ is the ultraviolet cutoff for the phonon spectrum and g^2 is the strength of the potential wake. It turns out that all low-energy properties of the super-conductor can be described by using only this single energy parameter T_c (apart from the actual temperature, T employing natural units $\hbar = 1$, v_F = Fermi velocity = $1, 2m = 1$). Thus systems with many different μ and g are identical superconductors (see Fig.1). This is quite analogous to the existence of a dimensionally transmuted coupling constant in gauge theory. There an artificial mass parameter μ is needed to define a coupling strength of the mass-

---

+) I thank V.Weisskopf for an illuminating discussion of this process.



1. The curves of identical theories in g^2,μ space. The renormalization group determines the reparametrization of any fixed theory along the corresponding curve.

less theory but all physical quantities depend only on the combination

$$\Lambda^2 = \mu^2 e^{-1/g^2(\mu^2)} \tag{2}$$

The critical temperature gives directly the size of Cooper pairs via+)

+) This relation holds only close to T_c. Trivial numerical factors are left out, for simplicity.

$$\xi(T) = \frac{1}{T_c}\left(1-\frac{T}{T_c}\right)^{-1/2} \approx 1000\ \mathring{A}\left(1-\frac{T}{T_c}\right)^{-1/2} \tag{3}$$

The presence of such large bound states causes the superconductor to be coherent over this distance $\xi(T)$. For this reason, $\xi(T)$ is called the coherence length.

In superliquid ^3He, the interatomic potential has a hard repulsive core for $r < 2.7\ \mathring{A}$. In the degenerate Fermi liquid, this gives rise to strong spin correlation with a preference of parallel spin configurations. Because of antisymmetry of the pair wave function, this amounts to a repulsion in even partial waves. Indeed, Cooper pairs are formed in the p-wave spin triplet state. The critical temperature is a thousand times lower than in superconductors: $T_c = .27^\circ$mk at p=35 bar . Since the masses of the ^3He atoms are larger than those of the electrons by about the same amount, the coherence length has the same order of magnitude in both systems.

The theoretical description of the behaviour of the condensate is greatly simplified by reexpressing the fundamental euclidean action directly in terms of the Cooper pair fields which are

$$\sigma(x) = \psi_e(x)\ \psi_e(x) \tag{4}$$

$$\sigma_{ai}(x) = \psi_{^3He}(x)\ \sigma_a\ \partial_i\ \psi_{^3He}(x) \tag{5}$$

respectively. Such a change of field variables can easily be performed in a path integral formulation[3] in which the partition function of the system reads

$$Z = \sum_{\psi_e} e^{-A[\psi_e]} \quad \text{or} \quad \sum_{\psi_{^3He}} e^{-A[\psi_{^3He}]} \tag{6}$$

By going from integration variables ψ to σ one can immediately find the alternative form[6]

$$Z = \sum_{\sigma} e^{-A[\sigma]} \tag{7}$$

where σ is the Cooper pair field (4) or (5).

The new action is very complicated. For temperatures close to T_c, however, it can be expanded in powers of the field σ and its derivatives. For static fields

$$-A[\sigma] = F/T = \frac{1}{T}\int d^3x\, f$$
$$= \frac{1}{T}\int d^3x \left[(-\log\frac{\mu}{T} + \frac{1}{g^2})|\sigma|^2 + \frac{1}{2T_c^2}|\sigma|^4 + \frac{1}{T_c^2}|\partial\sigma|^2 + \ldots \right] \tag{8}$$

where the dots denote the higher powers of σ and of their derivatives, each accompanied by an additional factor $1/T_c$.
In ^3He one has to take care of all different contractions among the spatial and spin indices i and a, respectively+). This generates 3 derivative terms

$$f_{der} = \frac{1}{T_c^2}\,(\partial_i\sigma^+_{aj}\,\partial_i\sigma_{aj} + \partial_i\sigma^+_{aj}\,\partial_j\sigma_{ai} + \partial_i\sigma^+_{ai}\,\partial_j\sigma_{aj}) \tag{9}$$

and five quartic potential terms

$$f_{pot} = \frac{1}{T_c^2}\{|tr(\sigma^T\sigma)|^2 + \beta_2[tr(\sigma^+\sigma)]^2$$
$$+\beta_3\, tr[(\sigma^+\sigma)(\sigma^+\sigma)^*] + \beta_4\, tr(\sigma^+\sigma)^2 + \beta_5\, tr[(\sigma\sigma^+)(\sigma\sigma^+)^*]\} \tag{10}$$

+) They contract separately, i.e. spin with spin, orbital with orbital indices, such that the theory is $SO(3)_{spin} \times SO(3)_{orbit}$ invariant.

It may be worth remarking that the transition from fields ψ_e or $\psi_{^3He}$ to that of the Cooper pair (4) or (5) is completely analogous to going from the fields ψ of the massive Thirring model to the scalar field ϕ defined by

$$\partial_\mu \phi \equiv \varepsilon_{\mu\nu} \bar{\psi}\gamma^\nu \psi \tag{11}$$

of the Sine-Gordon theory - the only difference being that in more than two dimensions the exact pair action is non-local and very complicated. However, if the system is studied close to the critical temperature and only with respect to its low-energy properties, the expansion of the free energy up to forth order in the field and up to second order in the derivatives contains all relevant information.

We shall now demonstrate how the field theory (8), and its extension (9),(10) in the case of ^{3}He, is capable of accounting for the properties of superflow.

II Superconductor

First we observe that with the critical temperature (1) the mass term in the action (8) can be written as

$$- \log \frac{T_c}{T} |\sigma|^2 \approx - (1- \frac{T}{T_c})|\sigma|^2 \tag{12}$$

It has the wrong sign for $T<T_c$ such that the field has no stable minimum at σ=o but oscillates around a new place

$$|\sigma|^2 = T_c^2 (1- \frac{T}{T_c}) + \sigma(1-\frac{T}{T_c}) \tag{13}$$

Thus if T lies sufficiently close to T_c, the higher power of contribute less and less and may be neglected. It is useful to take the factor $T_c(1- \frac{T}{T_c})^{1/2}$ out of the field σ and write the renormalized free energy as

$$f = - |\sigma|^2 + \frac{1}{2} |\sigma|^4 + |\partial\sigma|^2 \tag{14}$$

where we have made use of the coherence length (3) to introduce a dimensionless space variable and dropped an overall energy density factor proportional to $(1-\frac{T}{T_c})^2\, T_c^2$. The minimum of f lies now at $|\sigma|=1$ where it has the value

$$f = -\,1/2$$

This negative energy accounts for the binding of the Cooper pairs in the condensate and is therefore called condensation energy. In terms of (14), the partition function in equilibrium can be written as

$$Z = \sum_{\sigma} e^{-\frac{1}{T}\int d^3x\, f} \tag{15}$$

Let us now consider the flow properties of the system. Certainly, there is a divergenceless current

$$j(x) = \frac{1}{2}\ \psi^{\dagger}(x)\partial_x\ \psi(x) \tag{16}$$

associated with the transport of particle number. The important question to be understood is: How can this current become super?

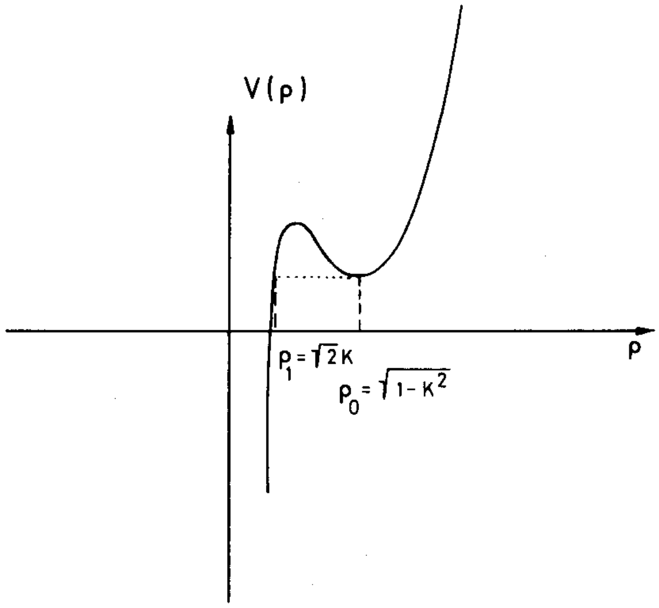
In order to see this let us set up a current in a long circular wire. If the thickness is chosen much smaller than the coherence length, transverse variations of σ are strongly suppressed with respect to longitudinal ones by the Boltzmann factor and the system depends only on the coordinate <u>along</u> the wire. If the cross section of the wire is absorbed in the definition of the temperature, we may simply study the partition function (15) for a one-dimensional problem along the z-axis. The field may be decomposed in polar coordinates

$$\psi(z) = \rho(z)e^{i\gamma(z)} \tag{17}$$

such that the free energy

$$f = -\,\rho^2 + \frac{1}{2}\,\rho^4 + \rho_z{}^2 + \rho^2\,\gamma_z{}^2 \tag{18}$$

leads to field equations



2. The potential $V(\rho)= -\rho^2+\rho^4/2-j^2/\rho^2$ showing the barrier to the left of ρ_0 to be penetrated if the supercurrent is to relax.

$$j = \rho^2 \gamma_z = \text{const.} \tag{19}$$

$$\rho_{zz} = -\rho + \rho^3 + \frac{j^2}{\rho^3} \tag{20}$$

The latter corresponds to the mechanical motion of a mass point in the potential, turned upsidedown (see Fig.2),

$$V(\rho) \equiv -\rho^2 + \frac{1}{2}\rho^4 - \frac{j^2}{\rho^2} \tag{21}$$

if z is considered as a "time". Obviously, there is a stationary solution

$$\gamma(z) = kz$$
$$\rho(z) \equiv \rho_0 = \sqrt{1-k^2} \tag{22}$$

Since the wire is closed, the phase $\gamma(z)$ has to be periodic over the length L and must be quantized according to

$$k_n = \frac{2\pi}{L} n \tag{23}$$

The corresponding energy density is (see Fig.3)

$$f(k_n) = -\frac{1}{2}(1-k_n^2)^2 \tag{24}$$

with a current

$$j(k_n) = \rho_o^2 (1-\rho_o^2) = k_n (1-k_n^2) \tag{25}$$

Notice that this current is bounded by

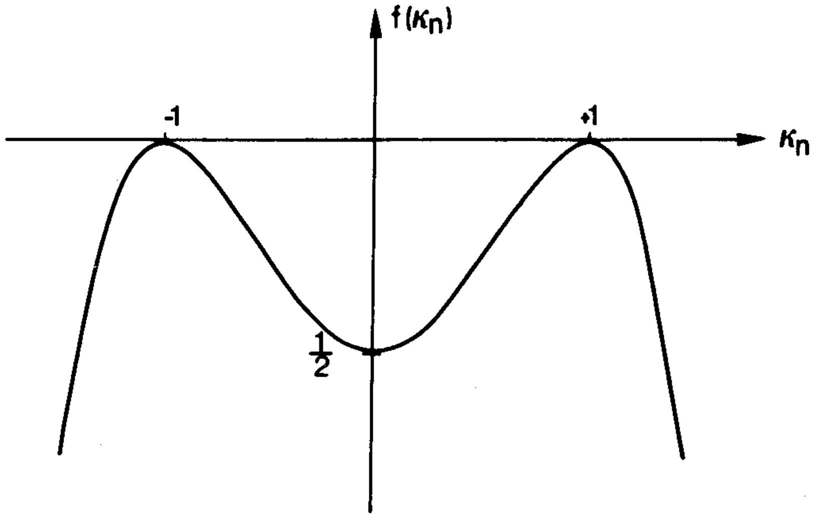
$$|j| < j_c \equiv \frac{2}{3\sqrt{3}} \tag{26}$$

No solution of the field equations can support a larger current than given by this critical value.

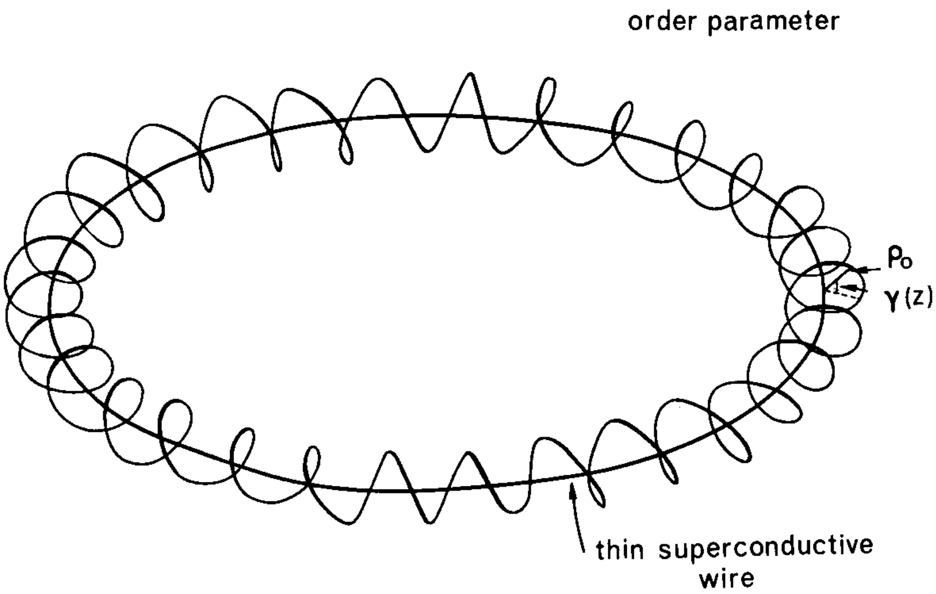
We shall now demonstrate that all states of current j_n smaller than j_c are infact "super" in the sense of having an extremely long lifetime (in practice ranging from hours to years). In the sense of field theory each state k_n can be considered as a "false vacuum" which eventually will decay to the true vacuum, the state of no current[7].

In order to understand this enormous stability of the states we notice that the temperature is very small such that the temperature fluctuations leave ρ very close to ρ_o. We can thus picture the field configuration as a spiral of radius ρ_o wound around the wire with the azimuthal angle representing the phase $\gamma(z) = k_n z$ (see Fig.4).
If the temperature is zero, ρ is frozen at ρ_o and the winding number is absolutely stable on topological grounds. The current runs on forever. The "false vacuum" has an infinite lifetime.

In order that the current may relax by one unit it is necessary that at some place thermal fluctuations carry $\rho(z)$ to zero. There the phase becomes undefined and may slip by 2π.



3. The condensation energy as a function of the velocity parameter $k_n = \frac{2\pi}{L} n$.



4. If ρ is frozen at ρ_0, the field configuration may be pictures as a spiral of radius ρ_0 with pitch $\frac{\partial\gamma(z)}{\partial z} = \frac{2\pi}{L} n$. The supercurrent is absolutely stable since the winding number n is locked topologically.

Now, since $T \ll 1$, such phase slips are extremely rare. In order that an excursion of $\rho(z)$ to $\rho = o$ has an appreciable measure in the functional sum (7) one must first look for solutions of the equations of motion+) which carry $\rho(z)$ as closly as possible to zero. From our experience with mechanics we are used to imagining the motion of a mass point in the potential $-V(\rho)$. It is easily realized that there is, in fact, a solution which carries $\rho(z)$ from ρ_o at $z = -\infty$ up the curve $V(\rho)$ to $\rho = 2K^2$ and back to ρ_o at $z = \infty$ (see Fig.5). Explicitly :

$$\rho_b(z) = 1-k^2 - \frac{\omega^2}{2} / ch^2 \ \frac{\omega}{2}(z-z_o) \tag{27}$$

with energy

$$F_b = \int dz \ f(\rho_b) = \frac{4}{3}\omega = \frac{4}{3}\sqrt{2(1-3k^2)} \tag{28}$$

where ω is the curvature of $V(\rho)$ close to ρ_o

$$V(\rho) \approx \omega^2 (\rho-\rho_o)^2 + \ldots \tag{29}$$

The solution reaches the point of smallest ρ at z_o :

$$\rho(z_o) = 2k^2 \tag{30}$$

This is still non-zero and does not permit a phase slip. We shall now see, however, that quadratic fluctuations around this solution are sufficient to reduce the current. Let us insert a small deviation

$$\rho(z) = \rho_b(z) + \delta\rho(z) \tag{31}$$

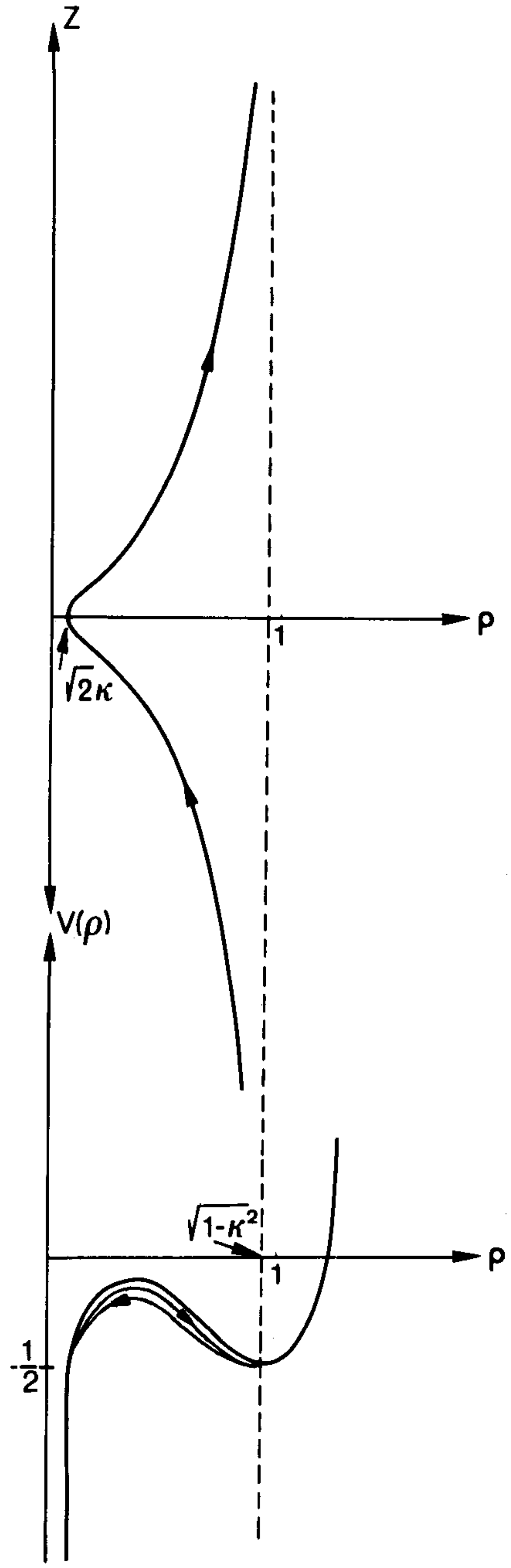
into the free energy. Respecting the equation of motion for ρ_b, the lowest variation of F is of second order

$$\delta^2 F = \int dz \ \delta\rho \ (-\partial_z^{\ 2} + V''(\rho))\delta\rho \tag{32}$$

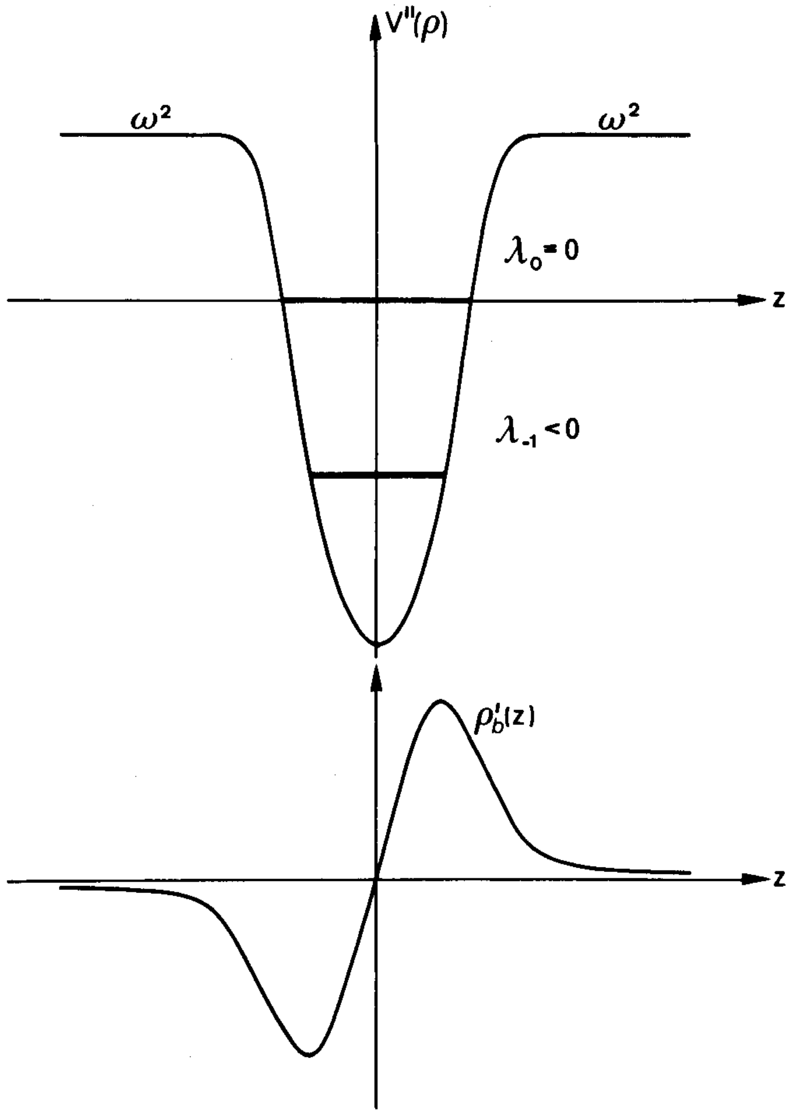
But this expression is nôt positive definite. This can be seen

---

+) This follows from the method of steepest descent generalized from integrals to path integrals.



5. An extremal excursion ("critical bubble") corresponds to a mass point sitting at ρ_0, rolling under the influence of negative gravity up the hill unto the point $\rho=2k^2$, and returning back to ρ_0, the variable z playing the role of a time variable.



6. The infinitesimal translation of the "critical bubble, ρ_b' is an antisymmetric wave function of zero energy. Hence there must be a negative-energy bound state.

either by explicitly solving the eigenvalue problem

$$(-\partial_z{}^2+V''(\rho_b))\psi_n(z)= (-\partial_z{}^2-1+3\rho_b{}^2-3\frac{j^2}{\rho_b^4})\psi_n(z)=\lambda_n\psi(z) \qquad (33)$$

or by the following much simpler reasoning : Equ.(33) is a Schrödinger equation in a potential which asymptotically is a constant ($=\omega^2$). For z approaching z_o , the potential becomes smaller due to the curvature of V(ρ) decreasing (see Fig.6). At $z=z_o$ it has a minimum at a negative value. The small vibrations are unstable if there is a bound state with a negative eigenvalue $\lambda_{-1}<o$. Its existence can be argued by noticing first that there is certainly a zero eigenvalue due to translational invariance: Since the wire is very long, the solution $\rho_b(z)$ which has its closest approach to the origin at z_o exists for any finite z_o ($<<L$) . Thus a small translation of z_o is certainly a fluctuation which does not change the energy such that

$$\psi_o = [\rho_b(z+\delta z_o)-\rho_b(z)] /\delta z_o = \rho_b'(z) \tag{34}$$

must solve the fluctuation problem (33) with $\lambda_o=o$. This can indeed be verified by an explicit calculation: Since

$$-\rho_{b\ zz} + V'(\rho_b) = o \tag{35}$$

one has

$$-\partial_z^2 \ \rho_b' + V''(\rho)\ \rho_b' = o \tag{36}$$

such that (33) is fulfilled. Now this zero frequency solution has an important property: Since $\rho_b(z)$ is an even function in $z - z_o$, $\rho_b'(z)$ is odd and has a node at z_o . Therefore it cannot be the ground state of the Schrödinger problem and there must be another lower lying state, i.e. with $\lambda_{-1}<\lambda_o= o$. The vibration problem is therefore unstable [7].

The whole process is quite analogous to the nucleation of the vapour phase in a superheated liquid [7] . There bubbles have to form containing vapour inside. The extremal size (critical bubble) is determined by the balance of surface tension and pressure (solution of equations of motion of surface). The bubbles are unstable against radial fluctuations: expansion leads to the transformation of the whole liquid into vapour (the volume energy wins), contraction leads to the return to the liquid phase (the surface energy wins). Associated with this is a negative eigenvalue of the corresponding differential equation [7]. Because of the complete analogy one may call the distorted solution $\rho_b(z)$ a "critical bubble" whose decay mediates the phase slip.

For the precise calculation of the decay rate we refer to Ref.2).

Let us summarize the physical situation: Thermal fluctuations lead to the presence of "critical bubbles" along the uniform spiral. Their presence is very rare due to the small Boltzmann factor+)

$$e^{-\frac{1}{T}\frac{4}{3}\omega} << 1 \tag{37}$$

The bubbles cause distortions of the spiral bringing the size of the field close to zero somewhere along the wire. The quadratic fluctuations around the bubbles do reach the zero field point and allow the phase to slip. The current relaxes by one unit of 2π and the spiral returns to the uniform configuration with one winding number less (and the bubble having disappeared)

What are the parallels with gauge theories? The quantum number $k = \partial_z\gamma$ characterizing the superliquid velocity of the "false vacua" can be written as

$$k = e^{i\gamma(z)}\, i\partial_z\, e^{-i\gamma(z)} = U^{-1}(z)\, i\partial_z\, U(z) \tag{38}$$

Thus the circular superliquid velocity corresponds to a vector potential for a pure gauge transformation. In the superliquid, the energy depends on k like

$$f = -\frac{1}{2} + k^2 + O(k^2) \tag{39}$$

In a gauge theory it does not, due to gauge invariance. Notice that if the gauge field had a mass term

$$f = A_z^2$$

then also its energy would have a k^2 dependence and the infinitely many vacua of different winding number would no longer be degenerate.

+) A cross section factor of the wire is absorbed into T, for simplicity, together with the previous factors $(1-\frac{T}{T_c})^2\, T_c^{\,2}$ when going from (8) to (14).

III Superliquid ^{3}He

Consider now the topologically more interesting system of superliquid ^{3}He. The discussion of the minima of the energy (10) is not as trivial as in the case of the superconductor (12). Here at least 11 local extrema are known with the two lowest ones being present in the laboratory, depending on pressure and temperature. They are called B and A phase(for a phase diagram see Ref.3 or 6). If the size of the field σ_{ai} is frozen, they may be parametrized as

$$B: \quad \sigma_{ai} = \rho_o R_{ai}(z) e^{i\phi(z)} \qquad (40)$$

$$A: \quad \sigma_{ai} = \rho_o \ell_a(z)(\phi_i^{(1)} + i\phi_i^{(2)})(z) \qquad (41)$$

where $R_{ai}(z)$ is an arbitrary rotation matrix. $\underset{\sim}{\phi}^{(1)} \perp \underset{\sim}{\phi}^{(2)}$ are unit vectors characterizing the plane in which the Cooper pairs move and

$$\underset{\sim}{\ell} = \underset{\sim}{\phi}^{(1)} \times \underset{\sim}{\phi}^{(2)} \qquad (42)$$

points in the direction of the arbital angular momentum of the Cooper pairs.+)

We can now immediately see that the B phase has, in a long circular tube, superflow properties very similar to those of the thin superconductor. For ρ_o frozen, a uniform flow is given by

$$\phi(z) = k_n z \ , \ k_n = \frac{2\pi}{L} n \qquad (42)$$

and relaxation can occur only by fluctuations of ρ_o to the origin. For this the formation of energetic "critical ρ bubbles" is needed causing a high stability of the superflow states k_n (the false vacua). The situation is quite different in the A phase. Let ρ_o

---

+) In writing this form in the A phase we have taken into account the hyperfine interaction which forces the spin direction into the orbital plane (see Refs. 3 and 6).

be frozen and consider a uniform superflow along the z axis with $\underset{\sim}{\ell} || z$. A Galileian factor e^{ikz} is equivalent to rotating $\underset{\sim}{\phi}^{(1)} \perp \underset{\sim}{\phi}^{(2)}$ around $\underset{\sim}{\ell}$ as one proceeds along the tube. The configuration many again be visualized by the end point of $\underset{\sim}{\phi}^{(1)}$ (say) forming a spiral of unit radius around the tube.

So far everything looks the same as in the previous two cases. The important difference, however, is that we have <u>chosen</u> $\underset{\sim}{\ell}$ to be parallel to the z axis. If the direction of $\underset{\sim}{\ell}$ is allowed to vary, it is quite easy to see that the spiral can be deformed until it is a straight line for n = even, or only one winding is left for n = odd. The proof for this is identical to the standard way of showing the doubly connectedness of the rotation group SO(3). The positions of the dreibein $\underset{\sim}{\phi}^{(1)}, \underset{\sim}{\phi}^{(2)}, \underset{\sim}{\ell}$ may be parametrized by a rotation matrix which is necessary to transform it to a certain fixed configuration (say $\underset{\sim}{\phi}^{(1)}, \underset{\sim}{\phi}^{(2)}, \underset{\sim}{\ell}$ in x,y,z direction). This in turn may be written as

$$e^{i\theta \underset{\sim}{n} \underset{\sim}{L}} \tag{43}$$

where $\underset{\sim}{n}$ is the axis and θ the angle of rotation. The vectors $\theta\, \underset{\sim}{n}$ are lying in a sphere of radius π with diametrially opposite surface points identified.

The position of the dreibein along the circular tube may be drawn as a closed path in this parameter space. It is easily seen that there are two topologically inequivalent classes depending on whether there is an even or odd number of jumps between diametrially opposite points (see Fig.7). But the state of superflow which was set up in the beginning ($\underset{\sim}{\ell} || z$ and $\underset{\sim}{\phi}^{(1)}$ describing a spiral) corresponds exactly to a straight line path from the south to the north pole, jumping to the south and continuing again to the north pole and so on n-times. The continuous deformability of the path to either of the two fundamental ones (which is the point at the origin or a single straight line from south to north pole) is equivalent to the continuous reduction of superflow to no or one unit of flux. In either case the flow would not be "super" at all. Thus, contrary to superconductors

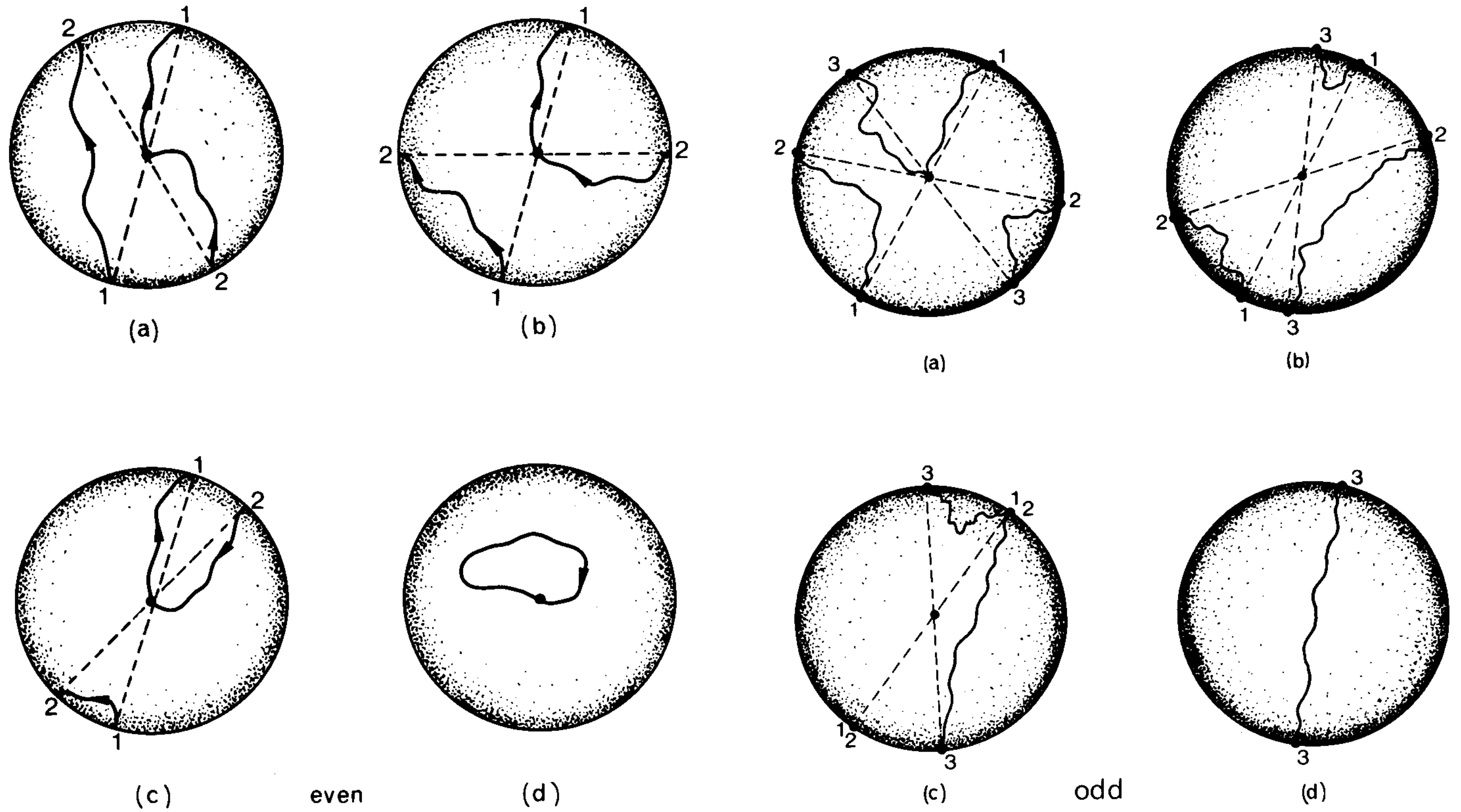


Fig. 7. States of superflow correspond to a closed contour in the SO(3) parameter sphere. For an even number of flux quanta the contour can be deformed continuously into a point which is a state of no current. States with an odd quantum number can be contracted until there is only one diametral line from south to north pole (say) corresponding to a single unit of flux.

and ^{3}He B, the freezing of ρ_0 does not topologically stabilize the flow. There is, however, another energy barrier, which still allows superflow to occur: It lies in the space of directions of the dreibein. By looking at the derivative terms of the free energy one can see that a superflow $\partial\sigma_{ai} \propto v_s \sigma_{ai}$ causes an attraction to the ℓ vector of the form

$$f_{der} \propto -(\ell \cdot v_s)^2$$

Thus there is, in fact, a potential barrier against the deformation described before: In the idealized case that the vector ℓ is tightly locked to the forward direction, the spiral $\phi^{(1)}$ around the tube becomes topologically stable. Decay can occur by the formation of a "bubble" in directional space (say the angle between ℓ and v_s). Again, a very small Boltzmann factor causes a high stability of the different current states. Notice that now the superflow is related to

$$A_z = R\partial_z R^{-1} \qquad (45)$$

where R is the rotation matrix (43). The different "false vacua" correspond, just as before, to states of pure gauges, now in a non-abelian version. Again, the bending energies are equivalent to inserting a mass term in a gauge theory destroying the degeneracy of the infinitly many vacua of different winding number.

Finally, let us mention one more interesting aspect of the topology of superflow in ^{3}He-A : If a magnetic field is turned on parallel to the z axis it attracts the spins of the Cooper pairs. Since these lie in the orbital plane there is a force pulling ℓ out of the direction of flow. It turns out that for H larger than a certain critical value there is a new equilibrium position in which ℓ forms a fixed non-zero angle with the current. As a consequence, there is an additional topologically stable quantum number: The number by which ℓ winds around the z axis. Since the other number, by which $\phi^{(1)}$ winds around the direction of flow, is completely independent of this, there is a doubly infinite set of false vacua characterized by two macroscopic quantum numbers n_1, n_2. ^{3}He-A has become a double superliquid [5]. What is the physical nature of this second superflow? Since the Cooper pairs are formed in a p-wave there are two mechanisms of matter flow: The first consist of the Cooper pairs themselves moving. The second is observed even if all Cooper pairs are at rest. If their orbital planes are not all parallel but have $\nabla \times \ell = o$, the circular orbits of neighbouring pairs do no longer cancel each other and a transport of neighbouring pairs is observed. This is quite analogous to the source of the magnetic field in the presence of matter

$$\nabla \times B = j + \nabla \times M$$

Even with no current flowing a magnetic field is generated if the magnetization has a curl due to the non-cancellation of the microscopic current loops.

IV Conclusion

At very low temperatures, many-body systems such as superconductors and superliquid ^{3}He can be described by a simple field theory. Many of the properties of the vacuum which have recently come under investigation in the field theory of elementary particles are present in these systems and carry responsibility for the phenomenon of superflow.

REFERENCES

1. J.S.Langer and V.Ambegaokar, Phys.Rev. 164,498 (1967)
   D.E. Mc Cumber and B.I.Halperin, Phys.Rev. 131, 1054 (1970)

2. H.Duru, H.Kleinert and N.Ünal, Berlin preprint, Oct.1979

3. For a review and references see
   H.Kleinert, Fortschr.Physik 26,565-671 (1978) and
   Collective Field Theory of Superliquid ^{3}He, Berlin Preprint, Sept.1978.

4. H.Kleinert, Y.R.Lin-Liu and K.Maki, Suppl.J.de Phys. 6, C6-59 (1978) (paper presented at the Low Temperature Conference LT15) Phys.Letters 70A,27 (1979)
A.Fetter, Suppl.J.de Phys. 6, C6-46 (1978) Phys.Rev.Lett.40, 1656 (1978).

5. H.Kleinert, Phys.Letters 71A, 66 (1979).

6. This is discussed in my 1978 Erice lectures to appear in "The New Aspects of Subnuclear Physics, ed. by A.Zichichi, Plenum Press 1978.

7. J.S.Langer, Am.Phys.41,108 (1967)
P.Frampton, Phys.Rev.Letters 37, 1378 (1976) and Phys.Rev.D15, 2924 (1977).
S.Coleman, Phys.Rev. D15, 2929 (1977) and Erice Lectures 1977.