RESONANCE DECAYS FROM O(3, 1) DYNAMICS. A REGULARITY IN THE PARTIAL DECAY WIDTHS*

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We have evaluated in closed form the scalar form factors in an irreducible representation of the noncompact group O(3, 1) containing antiparticles. In particular, we have calculated the decay of baryon resonances into the ground state and compared it with experiment.

In this note we report the calculation of the decay rates of baryon resonances of arbitrary spin into another baryon and a meson, and point out a remarkable regularity of the partial decay widths of baryons as a function of spin.

The basis of the calculation is the ordering of the observed baryon resonances into unitary irreducible representations of the dynamical group O(3, 1), a group isomorphic to the homogeneous Lorentz group, extended by parity, and containing antiparticles. The unitary irreducible representations are characterized by two numbers, a lowest spin j_0 that takes integer or half-integer values and a continuous imaginary number $j_1 = i\nu$. The state will be labeled by $|J, J_z\rangle$, where $J = j_0, j_0+1, j_0+2, \cdots$. After the extension by parity, the requirement of the existence of a four-vector current operator Γ_μ, which allows the particles to couple to the electromagnetic field, restricts the physically interesting representations of the group to the following types[1]:

Representation	States	Scalar, *ps*, vector, ···, vertex
$j_0=0, j_1=\frac{1}{2}$	no doubling of states	S, V
$j_0=\frac{1}{2}, j_1=0$	no doubling of states	S, V
$j_0=\frac{1}{2}, j_1=i\nu$	doubling	$S, P\ V, A$

The first two representations have been considered in a previous paper in the calculation of form factors and transition probabilities.[2] The doubling of states in the third representation will be associated with antiparticles because Γ_0 for these states has the opposite sign compared with the original states, just as in Dirac theory, and clearly occurs only for fermions. We denote the states of the two parts of the Hilbert spaces of the doubled representation by $|1\rangle$ and $|2\rangle$, and the parity of the lowest spin state by η. The parity of the other states are given by

$$P\left\{\begin{matrix}|1\rangle\\|2\rangle\end{matrix}\right\}=(-1)^{J-\frac{1}{2}}\sigma_1\left\{\begin{matrix}|1\rangle\\|2\rangle\end{matrix}\right\}. \qquad (1)$$

We then form the states with definite parity:

$$|\eta\pm\rangle=\frac{1}{\sqrt{2}}(|1\rangle\pm\eta|2\rangle), \qquad (2)$$

where the positive sign stands for particles and the negative sign for antiparticles, and obtain

$$P|\eta\pm\rangle=\pm(-1)^{J-\frac{1}{2}\eta}|\eta\pm\rangle. \qquad (3)$$

The ordering of the presently known baryon resonances according to this representation is shown in Fig. 1. Clearly we have several O(3, 1) towers for the same internal quantum numbers, but we remain in this work within the simplest possible dynamical group O(3, 1).

The general theory, the problem of mass spectrum, and the calculational techniques have been discussed before.[2,3] Here we shall consider the decay amplitudes of the type $N''\rightarrow N'$ + meson, and treat the meson as a scalar or a pseudoscalar vertex without assigning it, at the moment, to any O(3, 1) multiplet. We have then to evaluate the scalar or pseudoscalar amplitude

$$A=\left\langle J',J_z',\eta';p'\left|\begin{matrix}S\\P\end{matrix}\right|J,J_z,\eta;p\right\rangle, \qquad (4)$$

where $|J,J_z,\eta;p\rangle$ denotes the states boosted to a momentum p. In the rest frame of one of the baryons, taking the booster in the positive z direction, we have

$$A=\left\langle J',J_z',\eta'\left|\begin{matrix}S\\P\end{matrix}e^{-i\frac{1}{2}\xi M_3}\right|J,J_z,\eta\right\rangle \qquad (4')$$

with

$$\tanh\xi=p/E. \qquad (5)$$

If we denote the matrix elements of the finite group rotation in the original states $|1\rangle$ and

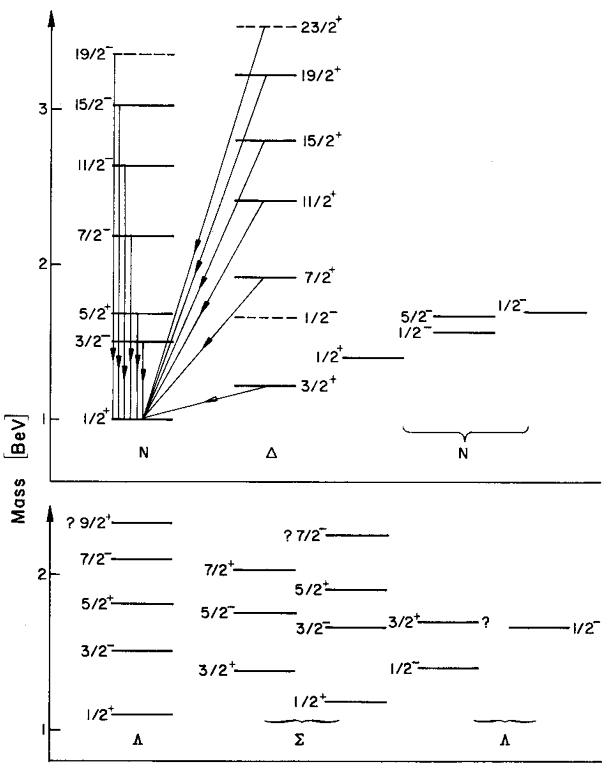

FIG. 1. The ordering of the presently known $Y=1$ and $Y=2$ baryon resonances according to the O(3, 1) representation $(\frac{1}{2},i\nu)$, and the calculated transitions.

$|2\rangle$ by

$$\langle\tfrac{1}{2}J'J_z'|e^{-i\frac{1}{2}\xi M_3}|\tfrac{1}{2}JJ_z\rangle=\begin{pmatrix}V_{J_z}^{J'J}\\ \tilde{V}_{J_z}^{J'J}\end{pmatrix}\delta_{J_z'J_z}, \qquad (6)$$

we find for the scalar and pseudoscalar vertex amplitudes

$$A_{\text{scalar}}=(j_0^2-\nu^2-1)^{\frac{1}{2}}(V_{J_z}^{J'J}+\eta\eta'\tilde{V}_{J_z}^{J'J}),$$

$$A_{ps}=j_0\nu^{\frac{1}{2}}(V_{J_z}^{J'J}-\eta\eta'\tilde{V}_{J_z}^{J'J}),$$

$$\tilde{V}=V^*=V(-\nu). \qquad (7)$$

The matrix element in Eq. (6) involves those of finite Lorentz transformations,[4] and for $J_z=J$ and $J'>J$ one obtains

$$V_J^{J'J}(\xi,j_0,\nu)=N^{J'J}(\sinh\xi)^{J'-J}e^{i\nu\xi}e^{-\xi(j_0+J'+1)}F(J'+1-i\nu,J'+j_0+1,2J'+2;1-e^{-2\xi}),$$

$$N^{J'J}=(2)^{J'-J}\left[\frac{(J'+j_0)!(J'-j_0)!(J'+J)![(J+1)^2+\nu^2]\cdots(J'^2+\nu^2)(2J+1)!}{(J+j_0)!(J-j_0)!(J'-J)!(2J')!(2J'+1)!}\right]^{1/2}. \qquad (8)$$

For $\nu=0$ these functions reduce to the matrix elements of finite O(3, 1) transformations used in the previous paper.[2]

As an example we have evaluated Eq. (7b) for the decays $N^* \to N+\pi$, where N^* belongs to the $I=\frac{3}{2}$ and $I=\frac{1}{2}$ towers (Fig. 1), for different values of ν. The experimental points are from the latest tabulations.[5] The results are shown in Fig. 2, where the square of the amplitude

$$|M|^2 = \Gamma/\varphi, \tag{9}$$

$$\varphi = \text{phase space} = \frac{1}{2J+1}\,\frac{P_{\text{final}}}{M}E_{\text{nucleon}}E_{\text{pion}},$$

is plotted against the spin. The theory is expressed in terms of the parameter ξ; the experimental mass values enter via Eq. (5).

In a theory of this kind using directly the representations of noncompact dynamical groups, the only parameters that have to be determined are the values of the Casimir operators that specify the representation. In our case, j_0 is fixed to be $\frac{1}{2}$, essentially by the electromagnetic properties of the system. The value of the second Casimir operator ν must be related to the anomalous magnetic moment. We note that for mesons ν is 0. Therefore, the ν value giving the best fit for the decays should also give a better value for the magnetic moments and form factors. This is indeed the case and will be reported elsewhere.[6]

Because now a theory of the decay of resonances exists for all spin values, it is important to have more accurate experimental information on the partial decay widths and their errors, in particular for those resonances identified by a phase-shift analysis.

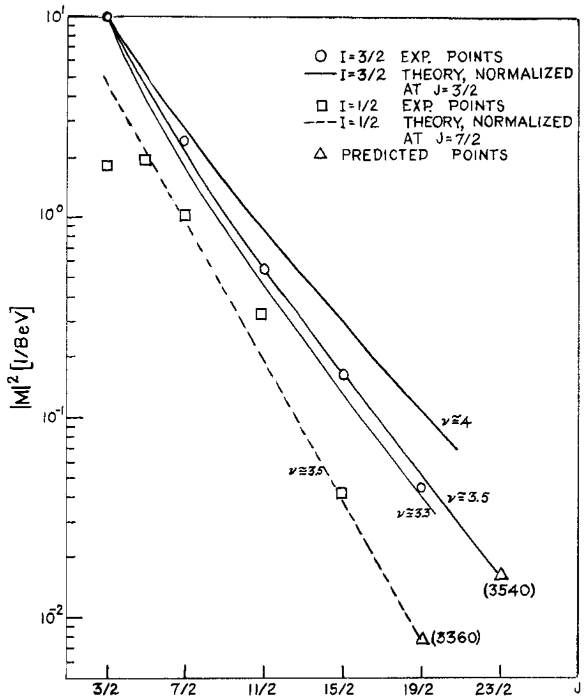

FIG. 2. Square of the decay amplitude as a function of spin for the transitions indicated in Fig. 1. Predicted partial widths for the $19/2^-$ and $23/2^+$ states, assuming the Regge-trajectory masses of 3360 and 3540 MeV, respectively, are ≈0.5 and ≈0.9 MeV.

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[1]There is still the possibility of mixing more than two O(3, 1) representations. Here we have studied the simplest representation allowing antiparticles of the simplest possible dynamical group.

[2]A. O. Barut and H. Kleinert, Phys. Rev. (to be published), and references therein.

[3]A. O. Barut and H. Kleinert, in Proceedings of the Fourth Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January, 1967 (to be published).

[4]S. Ström, Arkiv Fysik 29, 467 (1965).

[5]A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

[6]To be published.