SUPERFLOW IN ^{3}He-B IN THE PRESENCE OF A MAGNETIC FIELD AT ALL TEMPERATURES ☆

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We calculate gap functions, superfluid densities, critical currents, and susceptibilities for the B phase of ^{3}He in the presence of currents and a magnetic field. The distortion of the gap function is properly taken into account.

As laboratories begin exploring the flow properties of superfluid ^{3}He [1], external magnetic fields play a significant role in modifying experimental conditions. It is therefore desirable to understand theoretically the interplay of flow J and field H. For $T \lesssim T_c$, this has been done some time ago [2]. For arbitrary $T < T_c$ only the flow at $H = 0$ has been treated without [3] and now also including the distortion of the energy gap [4,5].

It is the purpose of this note to complete the picture by studying the situation with both $J \neq 0$ and $H \neq 0$.

Neglecting fluctuations in the order parameter A_{ai} of ^{3}He, the free energy density may be written as

$$f = -\tfrac{1}{2} T \sum_{\omega_n, \boldsymbol{p}} [\mathrm{tr}_{4\times 4} \log G^{-1}(\omega_n, \boldsymbol{p})] + (3g)^{-1} |A_{ai}|^2 + \text{const.}\,, \tag{1}$$

where

$$G(\omega_n, \boldsymbol{p}) \equiv \begin{pmatrix} \mathrm{i}\omega_n - \boldsymbol{p}^2/2m + \mu + \Omega_a \sigma_a/2 & A_{ai}\sigma_a \hat{p}_i \\ A^*_{ai}\sigma_a \hat{p}_i & \mathrm{i}\omega_n + \boldsymbol{p}^2/2m - \mu + \Omega_a\sigma_a/2 \end{pmatrix} \tag{2}$$

is the Green's function of the quasiparticles $(\psi, \mathrm{i}\sigma_2\psi^+)$ [6] in the presence of a magnetic field $H_a = \Omega_a/\gamma$ and a constant mean pair field A_{ai} [7,8]. [$\gamma \approx 2.04 \times 10^4\,(\mathrm{G\,s})^{-1}$ is the magnetic moment of ^{3}He atoms.] Diagonalizing G^{-1} we find $\mathrm{i}\omega_n \pm E^+$, $\mathrm{i}\omega_n \pm E^-$ along the diagonal with the quasiparticle energies ($\xi \equiv \boldsymbol{p}^2/2m - \mu$)

$$E^{\pm} = [\xi^2 + \tfrac{1}{4}\boldsymbol{\Omega}^2 + |A_{ai}\hat{p}_i|^2 \pm |\boldsymbol{\Omega}| (\xi^2 + |\hat{\Omega}_a A_{ai}\hat{p}_i|^2)^{1/2}]^{1/2}\,. \tag{3}$$

Flow is established by adding to the quasiparticle hamiltonian an external source term

$$-\boldsymbol{V} \cdot \tfrac{1}{2}\psi^+ \mathrm{i}\overleftrightarrow{\nabla}\psi \equiv -\boldsymbol{V}\boldsymbol{p}\,, \tag{4}$$

which enters into f with $-\boldsymbol{V}\boldsymbol{p}$ along the diagonal of G^{-1} [7–9], i.e. it simply changes ω_n to $\omega_n + \mathrm{i}\boldsymbol{V}\boldsymbol{p} \equiv \tilde{\omega}_n$ in all formulas. Because of the invariance of f under orbital and spin rotations we may choose $\boldsymbol{v}$ and $\boldsymbol{H}$ along the z-axis. (Notice, however, that fluctuations [10] *will* be sensitive to the relative angle between $\boldsymbol{v}$ and $\boldsymbol{H}$.) Then the constant

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mean pair field A_{ai} in the B phase can be assumed to be of the form

$$A_{ai} = \mathrm{diag.}(\Delta_\perp, \Delta_\perp, \Delta_\parallel) \,, \tag{5}$$

allowing the gap parameter $\Delta_\parallel$ parallel to the flow to differ from the orthogonal one $\Delta_\perp$. Denoting the parameter of gap distortion by

$$r^2 \equiv 1 - \Delta_\parallel^2/\Delta_\perp^2 \,, \tag{6}$$

the quasiparticle energies are

$$E^\pm = \{\xi^2 + \tfrac{1}{4}\Omega^2 + \Delta_\perp^2(1 - r^2 z^2) \pm \Omega[\xi^2 + \Delta_\perp^2(1-r^2)z^2]^{1/2}\}^{1/2} \,. \tag{7}$$

The case $r = 1$ reduces to the planar phase $E^\pm = [(\xi \pm \frac{1}{2}\Omega)^2 + \Delta_\perp^2(1-z^2)]^{1/2}$, which at the BCS level under consideration is degenerate with the A phase [which is obtained from eq. (3) by inserting $A_{ai} = \hat{x}_a(\hat{x}_i + \mathrm{i}\hat{y}_i)$].

With eq. (7), the free energy density becomes simply

$$g \equiv f - VP = -\tfrac{1}{2}T \sum_{\omega_n, \boldsymbol{p}} \{[\log(\mathrm{i}\tilde{\omega}_n - E^+) + (E^+ \to -E^+)] + [\Omega \to -\Omega]\} + (3g)^{-1}(2\Delta_\perp^2 + \Delta_\parallel^2) + \mathrm{const.} \tag{8}$$

Derivatives with respect to $\Delta_\perp^2$ and $\Delta_\parallel^2$ lead to the orthogonal and longitudinal gap equations

$$\log\frac{T}{T_\mathrm{c}} = \int_{-1}^{1} \frac{\mathrm{d}z}{2} \begin{Bmatrix} \frac{3}{2}(1-z^2) \\ 3z^2 \end{Bmatrix} \gamma_{\substack{\perp \\ \parallel}}(\delta, \nu, \kappa) \,, \tag{9}$$

with the gap functions

$$\gamma_{\substack{\perp \\ \parallel}} \equiv \frac{2}{\pi\delta} \sum_{n=0}^{\infty} \mathrm{Re} \int_{-\infty}^{\infty} \mathrm{d}\xi \left(\frac{\xi^2 + c_n^2 \pm \kappa^2}{d_n} - \frac{1}{x_n^2 + \xi^2} \right) = \frac{2}{\delta} \sum_{n=0}^{\infty} \mathrm{Re}\left[\frac{\mathrm{i}}{\eta_n^+ + \eta_n^-} \left(1 - \frac{c_n^2 \pm \kappa^2}{\eta_n^+ \eta_n^-} \right) - \frac{1}{x_n} \right] , \tag{10}$$

where

$$c_n^2 \equiv (x_n - \mathrm{i}\nu z)^2 + 1 - r^2 z^2 \,, \qquad d_n = (\xi^2 + c_n^2 + \kappa^2)^2 - 4\kappa^2[\xi^2 + (1-r^2)z^2] \,,$$

$$\eta_n^\pm \equiv \{\kappa^2 - c_n^2 \pm 2\kappa[(1-r^2)z^2 - c_n^2]^{1/2}\}^{1/2} \,. \tag{11}$$

Here we have introduced the dimensionless variables

$$\delta = \Delta_\perp/\pi T \,, \quad \nu \equiv V p_\mathrm{F}/\Delta_\perp \,, \quad \kappa \equiv \gamma H/2\Delta_\perp = \Omega/2\Delta_\perp \,, \quad x_n \equiv \omega_n/\Delta_\perp = 2nH/\delta \,, \tag{12}$$

for convenience.

For $T \lesssim T_\mathrm{c}$, x_n becomes very large and we can pick up the leading $1/x_n^3$ terms in the gap equations (9), obtaining

$$1 - \frac{T}{T_\mathrm{c}} \approx \delta^2 \left[1 + \begin{Bmatrix} \frac{1}{5}(2\nu^2 - r^2) \\ \frac{3}{5}(2\nu^2 - r^2) + 2\kappa^2 \end{Bmatrix} \right] \tfrac{7}{8}\zeta(3) + \ldots \,, \tag{13}$$

which reduces to [with $\Delta_\mathrm{B}^2 \equiv \pi^2 T_\mathrm{c}^2 [8/7\zeta(3)](1 - T/T_\mathrm{c})$],

$$\Delta_\parallel^2/\Delta_\mathrm{B}^2 \approx 1 - 3v^2 - 6h^2 \,, \qquad \Delta_\perp^2/\Delta_\mathrm{B}^2 \approx 1 + \tfrac{3}{2}h^2 \,, \tag{14}$$

in agreement with the Ginzburg–Landau calculations of refs. [2,4]. In eqs. (14) we have used $v^2 = V^2/V_0^2(1 - T/T_\mathrm{c})$ and $h^2 \equiv H^2/H_0^2(1 - T/T_\mathrm{c})$, where $V_0 = (2m^*\xi_0)^{-1} \approx 6.3$ cm/s and $H_0 = p_\mathrm{F}/m^*\xi_0\gamma \approx 16.4$ kG are natural units for the velocity and the magnetic field with $\xi_0 \equiv [7\zeta(3)/48\pi^2]^{1/2} p_\mathrm{F}/m^* T_\mathrm{c} \approx 559$ Å being the coherence length, and the numbers holding for zero pressure [11] ($m^* \approx 3m_{^3\mathrm{He}}$). We can now obtain the superfluid density $\rho_\mathrm{s}^\parallel$ parallel to flow and field by forming the derivative with respect to V:

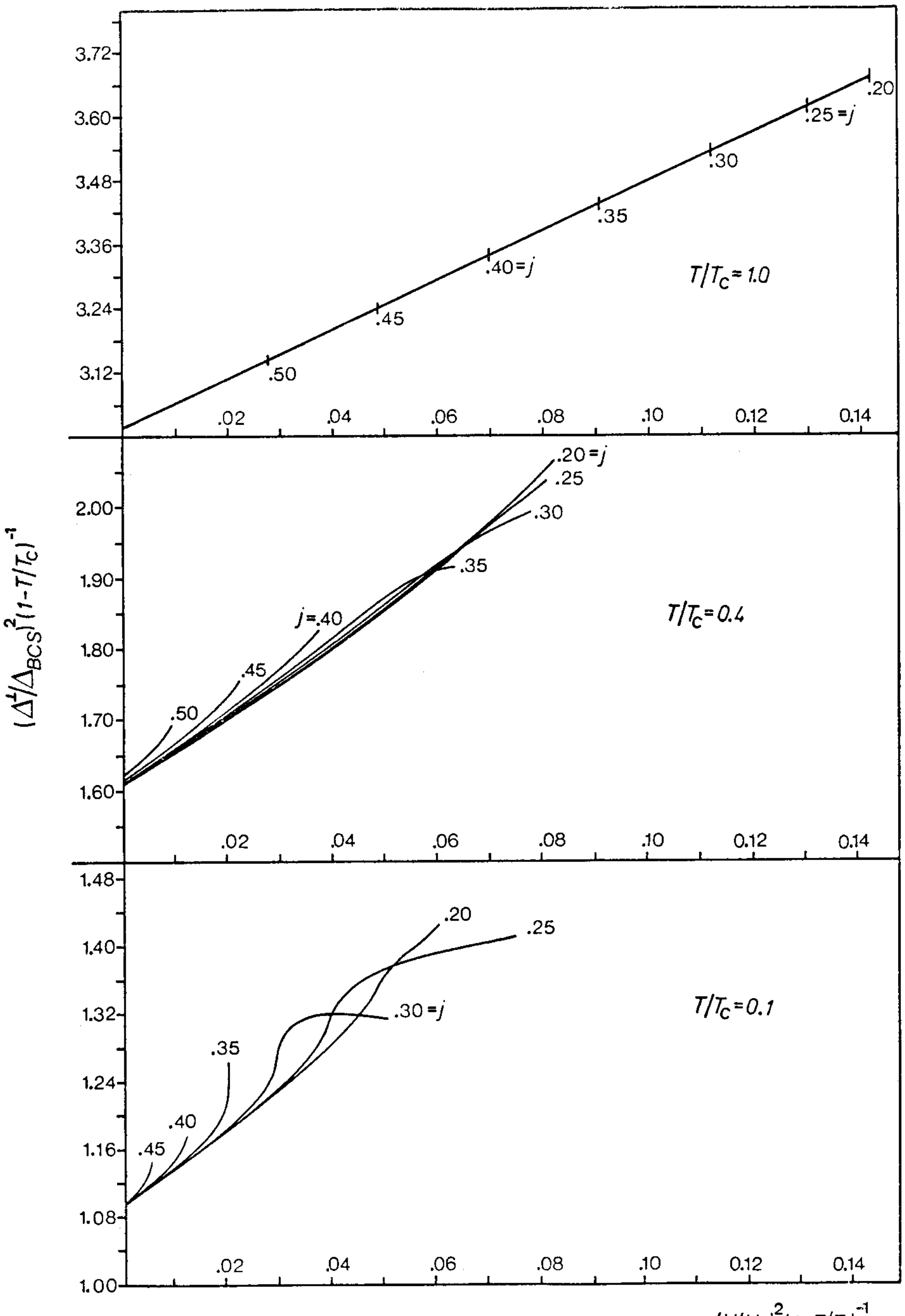

Fig. 1. The gap function $\Delta_\perp$ orthogonal to flow and field is displayed in the form $(\Delta_\perp/\Delta_{BCS})^2(1-T/T_c)^{-1}$, where $\Delta_{BCS} \approx 1.76$ T_c, as a function of the reduced magnetic field (Fermi-liquid uncorrected) $h^2 \equiv (H/H_0)^2(1-T/T_c)^{-1}$ at different fixed supercurrents $j \equiv (J/J_0)(1-T/T_c)^{-3/2}$. The natural units are $H_0 \approx 16.4$ kG and $J_0 \equiv \rho V_0 \approx \rho \times 6.3$ cm/s. For $T \lesssim T_c$, the curves are straight lines lying on the top of each other, as they start out at (0, 3.02), but ending at different values of H where the fixed current becomes critical. For lower temperatures, the curves become quite different.

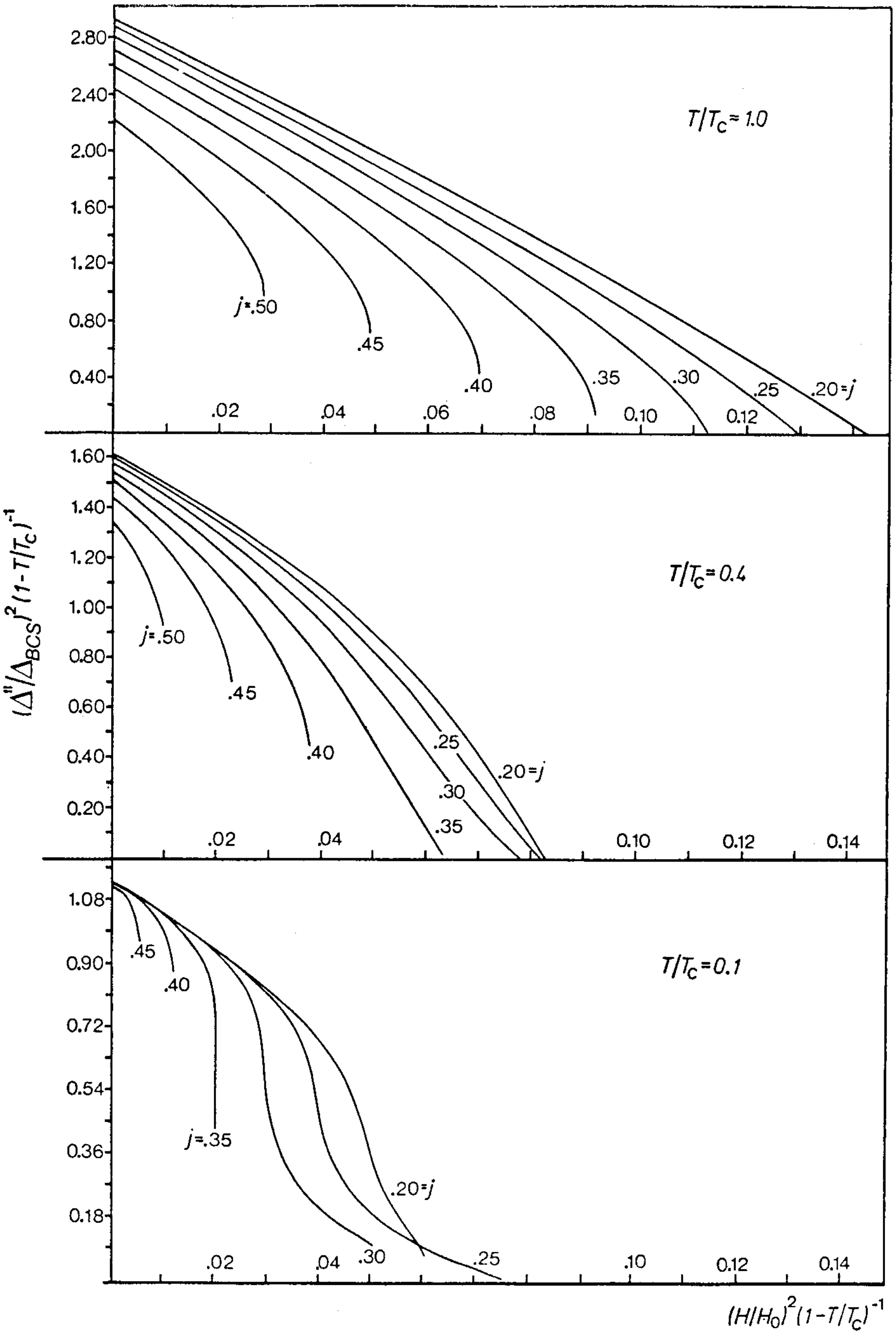

Fig. 2. The gap function $\Delta_{\parallel}$ parallel to flow and magnetic field is displayed in the form $(\Delta_{\parallel}/\Delta_{BCS})^2(1 - T/T_c)^{-1}$ similarly to fig. 1. For larger values of j, $\Delta_{\parallel}$ touches the abscissa. This amounts to a smooth transition into the A-phase before reaching the critical current.

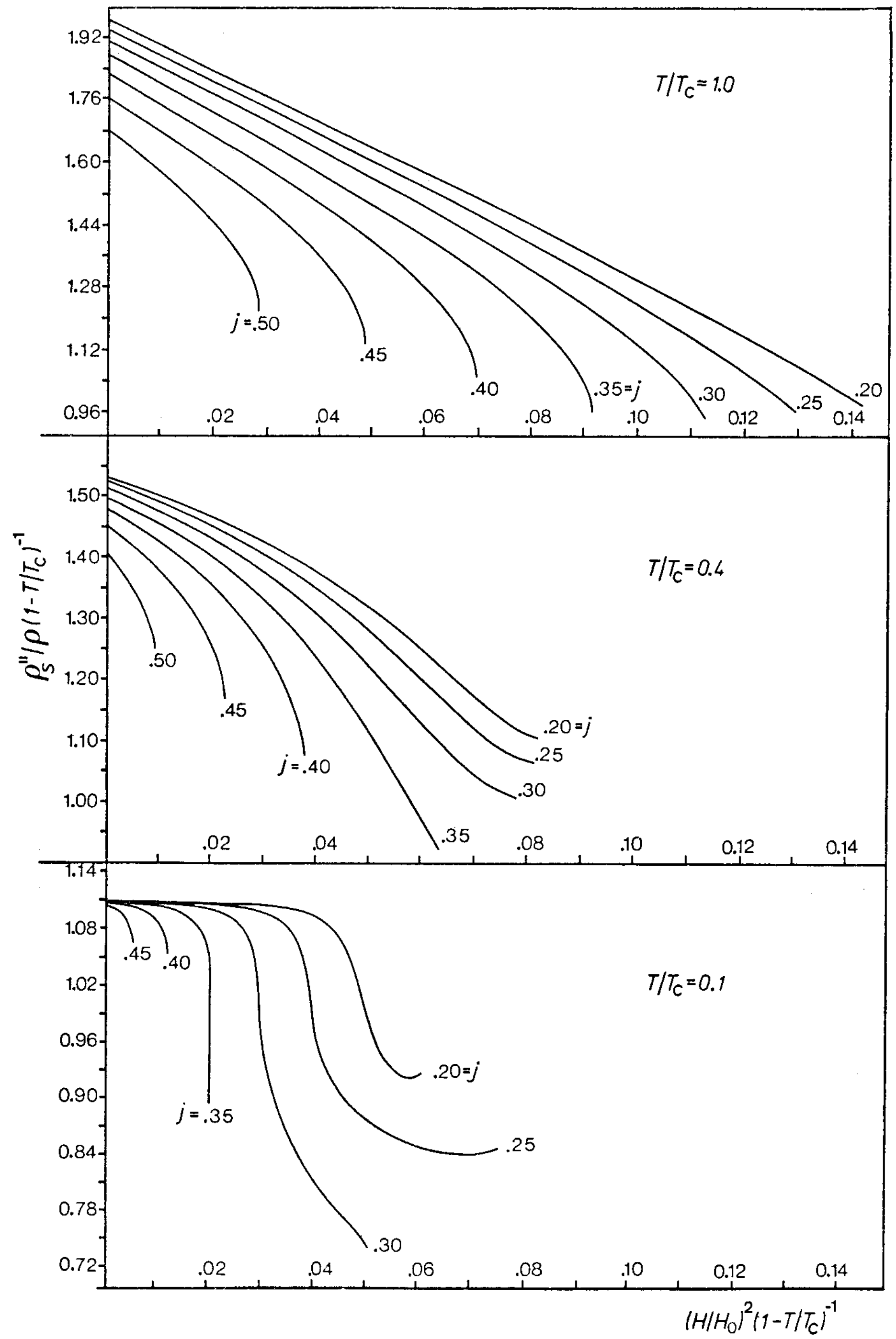

Fig. 3. The superfluid density $(\rho_s^{\|}/\rho)(1 - T/T_c)^{-1}$ is shown, with the same conventions as in fig. 1.

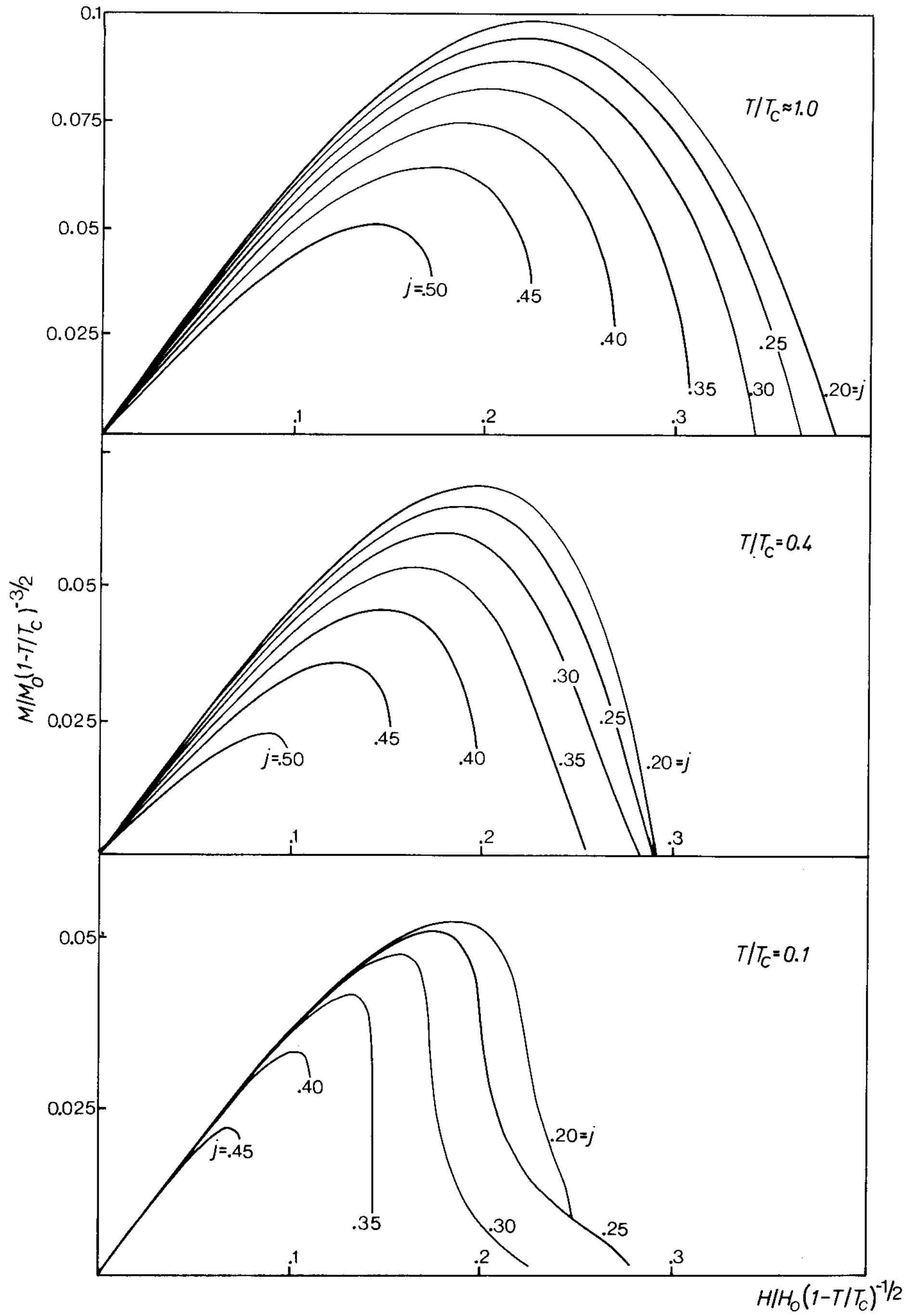

Fig. 4. The magnetization $m \equiv (M/M_0)(1 - T/T_c)^{-3/2} = (\chi_s/\chi_0)h(1 - T/T_c)^{-1}$ as a function of the *uncorrected* reduced magnetic field h. For a geometric Fermi-liquid correction see the text.

$$\frac{\rho_s^{\|}}{\rho} \equiv \frac{\partial(f-VP)}{\partial V}\frac{1}{\rho V}+1=\frac{6}{\delta v}\mathrm{Re}\int_{-1}^{1}\frac{dz}{2}z\sum_{n=0}^{\infty}\mathrm{i}(x_n-\mathrm{i}vz)\int_{-\infty}^{\infty}\frac{d\xi}{\pi}\frac{\xi^2+c_n^2+\kappa^2}{d_n}$$

$$=-\frac{6}{\delta v}\mathrm{Re}\int_{-1}^{1}\frac{dz}{2}z\sum_{n=0}^{\infty}\frac{x_n-\mathrm{i}vz}{\eta_n^+ + \eta_n^-}\left(1-\frac{c_n^2+\kappa^2}{\eta_n^+\eta_n^-}\right). \tag{15}$$

In the Ginzburg–Landau domain $T \lesssim T_c$ the expansion in $1/x_n$ leads to

$$\rho_s^{\|}/\rho \approx 2(1-\tfrac{9}{5}v^2-3h^2)\,, \tag{16}$$

again in agreement with refs. [2,4].

The susceptibility $\chi_s^{\|}$ is obtained in complete analogy as

$$\frac{\chi_s^{\|}}{\chi_0}=\frac{\partial(f-VP)}{\partial H}\frac{1}{\chi_0 H}+1=\frac{2}{\delta}\mathrm{Re}\int_{-1}^{1}\frac{dz}{2}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{d\xi}{\pi}\frac{\xi^2-c_n^2-\kappa^2+2(1-r^2)z^2}{d_n}$$

$$=\frac{2}{\delta}\mathrm{Re}\int_{-1}^{1}\frac{dz}{2}\sum_{n=0}^{\infty}\frac{\mathrm{i}}{\eta_n^+ + \eta_n^-}\left[1+\frac{c_n^2+\kappa^2-2(1-r^2)z^2}{\eta_n^+\eta_n^-}\right], \tag{17}$$

where $\chi_0 = 2N(0)(\gamma/2)^2 = \frac{3}{4}\gamma^2\rho/p_F^2$ is the value for the degenerate electron gas. In the Ginzburg–Landau limit $T \to T_c$ this becomes

$$\chi_s^{\|}/\chi_0 \approx \tfrac{2}{3}\Delta_{\|}^2/\Delta_B^2 = \tfrac{2}{3}(1-3v^2-6h^2)\,. \tag{18}$$

The results are displayed in figs. 1–4. We have plotted gaps and superfluid density for fixed currents against the magnetic field. Instead of the susceptibility, however, we have preferred to show the reduced magnetization.

$$m \equiv M/M_0(1-T/T_c)^{-3/2} = (\chi_s/\chi_0)(H/H_0)(1-T/T_c)^{-3/2} = (\chi_s/\chi_0)h(1-T/T_c)^{-1}\,.$$

This has an advantage when it comes to including Fermi-liquid corrections: for these we have to read the velocities and magnetic fields in all our formulas as the local quantities V^*, H^* which are related to the physical V, H by an additional molecular field:

$$\{1+\tfrac{1}{3}F_1^s[1-\rho_s^{\|}(V^*,H^*)/\rho]\}V^* = V\,, \tag{19}$$

$$\{1+F_0^a[1-\chi_s(V^*,H^*)/\chi_0]\}H^* = H\,. \tag{20}$$

Under this replacement, currents [3,4] and magnetizations remain invariant. Thus the plots for $\Delta_\perp$, $\Delta_{\|}$ can be used directly the way they are in order to extract the *corrected* functions of current and magnetization. The super fluid density $\rho_s^{\|}$, on the other hand, has to be divided by a factor $[1+\frac{1}{3}F_1^s(1-\rho_s^{\|}/\rho)]$. Experimentally, it is usually the magnetic field which is given. Then we may find H^* and $M(H^*)$ by writing eq. (20) as

$$h^*(1+F_0^a)/F_0^a - h/F_0^a = m(h^*)\,, \tag{21}$$

which amounts to the following geometric construction: In fig. 4, draw a straight line of slope $(1+F_0^a)/F_0^a$ through the point $-h/F_0^a$ on the ordinate. The intersection of this line with our curves gives the reduced magnetization $m(h^*)$ together with the uncorrected magnetic field h^* to be used in figs. 1–3 for reading off gaps and superfluid densities.

Notice that the Fermi-liquid corrected velocities are simply $V = J[1+\frac{1}{3}F_1^s(1-\rho_s^{\|}/\rho)/\rho_s^{\|}]$. By plotting curves of constant j we have eliminated the local velocity v^*.

Our results agree with previous calculations at zero H [12] and zero V [13].

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