NO PION CONDENSATE IN NUCLEAR MATTER DUE TO FLUCTUATIONS

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We show that if pion condensation occurs in a mean-field theory of infinite nuclear matter, fluctuations completely prevent the formation of a condensate as well as of the associated Goldstone mode. Thus if an increase of opalescence should ever be observed experimentally, it is these fluctuations which are measured rather than the scattering on the Goldstone modes. They preserve isotopic symmetry and increase very smoothly as the density passes the formerly critical density. There are no discontinuities in any thermodynamic quantity.

Since Migdal's pioneering work [1], much effort has been invested in deriving the dynamical details and possible experimental consequences of pion condensates in nuclei [2]. It seems to be an unanimous opinion that a condensate of pions of momentum $|q_c| \sim 2\mu$ ($\mu \equiv$ pion mass) should appear above some moderate critical density $\rho_c \approx 0.5\,\mu^3$. In the theory, the condensate is signalized by the continuous development of a non-vanishing expectation value of the pion field

$$\langle \boldsymbol{\pi} \rangle \sim (\rho/\rho_c - 1)^{1/2} \exp(i\boldsymbol{q}_c \boldsymbol{x})\, \boldsymbol{n} + \text{c.c.}$$

Experimentally, the transition is expected to announce itself by a marked critical opalescence due to the strong massless fluctuations transverse to the direction $\boldsymbol{n}$ accompanying the spontaneous breakdown of any continuous symmetry

Up to now, most discussions have been restricted to classical considerations as far as the pion field is concerned. Little is known [3] about the consequences of fluctuations in the immediate vicinity of and above ρ_c. It is the purpose of this note to introduce the appropriate theoretical techniques for their investigation and to derive a first result: We show that due to the many choices of the system as to the direction of $\boldsymbol{q}_c$ there are so large pretransitional fluctuations that the second-order phase transition, present at the mean-field level, is completely wiped out ‡1. A condensate can no longer form, i.e., all field components of the pion have zero expectation and isotopic symmetry is restored. There are certain remnants of the mean field transition ‡2. The expectation of the square of the pion field $\langle \boldsymbol{\pi}(x)^2 \rangle$ does approach the mean field behaviour $\rho/\rho_c - 1$ for $\rho \gg \rho_c$. Moreover, even though the susceptibilities are drastically modified by becoming isotropic in isospin, they do grow rapidly in $\rho/\rho_c - 1 > 0$ (exponentially for zero and quadratically for finite temperature) thus reflecting the infinite values of the transverse components at the mean field level.

Consider an action A describing matter in terms of some set of elementary particles, i.e., A contains local quark fields $\mathrm{q}(x)$ and others, say $\mathrm{G}(x)$, responsible for binding the quarks. The full quantum field theory is defined by a generating functional

$$Z[\eta, k] = \int D\mathrm{q}(x)\, D\mathrm{G}(x) \times \exp\left(\mathrm{i}A[\mathrm{q}, \mathrm{G}] + \mathrm{i}\int \mathrm{d}x(\bar{\eta}\mathrm{q} + \bar{\mathrm{q}}\eta + k\mathrm{G})\right), \quad (1)$$

‡1 Our result has nothing to do with smoothening of the transition due to finite-size effects which are small compared with what we discuss here if the nuclear radii R are very large, in particular $R \gg 1/q_c$.

‡2 Just as there is a supercurrent in a thin wire even though there is no condensate (fluctuation stabilized superconductivity).

where η, k are external sources. Only c-number fields occur and fluctuations are included explicitly rather than by field operators.

A pion field may be introduced carrying the properties of the composite $\bar{q}i\gamma_5\tau q$ field via an auxiliary functional integral

$$Z_{\mathrm{aux}} \equiv \int D\pi(x) \times \exp\left(i\int dx\,[\boldsymbol{\pi}(x) - \bar{q}(x)\, i\gamma_5 \boldsymbol{\tau} q(x)]^2\right), \tag{2}$$

which does not depend on the quark fields since the variables $\boldsymbol{\pi}(x)$ are integrated, at every space–time point, from $-\infty$ to $+\infty$. Thus Z can be multiplied with (2) without any change and we may describe the world just as well by $Z_{\mathrm{coll}} \equiv ZZ_{\mathrm{aux}}$. If we now integrate out all fields q(x), G(x), we remain with a new partition function [4] ‡3

$$Z_{\mathrm{coll}}[\eta, k] = \int D\pi(x) \exp(iA_{\mathrm{coll}}[\pi, \eta, k]). \tag{3}$$

Fuctional differentiation with respect to η (in multiples of 3) can now be used to generate any desired nuclear density. Setting $\eta = 0$ and $k = 0$ afterwards one obtains

$$^{\rho}Z_{\mathrm{coll}} = \int D\pi(x) \exp(i^{\rho}A_{\mathrm{coll}}[\pi]), \tag{4}$$

where $^{\rho}A_{\mathrm{coll}}[\pi]$ is the collective action of the pion field at a certain nuclear density ρ which we shall assume to be symmetric, $Z = N$.

To lowest approximation, the collective action $^{\rho}A_{\mathrm{coll}}[\pi]$ can be treated at the classical level at which one has equations of motion $\delta^{\rho}A_{\mathrm{coll}}/\delta\pi(x) = 0$ which are of the Hartree type [4,5]. In the long-wavelength limit, there are soft pion modes which can be described by an expansion in powers of the pion field

$$^{\rho}A^{\pi}_{\mathrm{coll}}[\pi] = \frac{1}{2} \sum_{|q_0| \sim \mu, |\boldsymbol{q}| \ll \mu} \boldsymbol{\pi}^{+}(q)(q_0^2 - \boldsymbol{q}^2 - \mu^2)\boldsymbol{\pi}(q) - \tfrac{1}{4}{}^{\rho}\beta(\pi_a^2)^2 + O(\pi^6), \tag{5}$$

where the quartic term is some non-local function of π.

The precise form of $\beta(\pi_a^2)^2$ depends on the model. For our qualitative discussion we shall only use [1,2,6] that for euclidean pion momenta this combination is positive definite. Second there are "pionic sound" waves with a kinetic action

$$^{\rho}A^{\mathrm{s}}_{\mathrm{coll}}[\pi] = \tfrac{1}{2}\mu^2 \sum_{|q_0| \ll \mu, |\boldsymbol{q}| \ll \mu} \pi^{+}(q)\left(1 - \frac{c_{\mathrm{s}}|\boldsymbol{q}|}{|q_0|}\right)\pi(q), \tag{6}$$

which may simply be added to (5).

It was the important observation of Migdal [1,2,6] that there is a third type of modes which have to be taken into account because of their low energy: If the dispersion curve of the "sound" wave (6), $q_0^2 = \omega^2(q)$, is continued to higher momenta there is a pronounced minimum at $q = q_{\mathrm{c}} \approx 2\mu$ ‡4. They have the crucial property that at a critical density $\rho = \rho_{\mathrm{c}} \approx 0.5\,\mu^3$, i.e. in the range of normal nuclear density, $\omega^2(q_{\mathrm{c}})$ vanishes and one can expand in the neighbourhood of ρ_{c} and q_{c}:

$$\omega^2(q) \approx \alpha[(1 - \rho/\rho_{\mathrm{c}}) + \xi_0^2(q - q_{\mathrm{c}})^2].$$

For $\rho > \rho_{\mathrm{c}}$, $\omega^2(q_{\mathrm{c}})$ changes sign signalizing the possible onset of a phase transition called pion condensation. In the following we shall investigate the fluctuation properties of this most relevant mode with the action close to ρ_{c} ‡5.

$$^{\rho}A^{\mathrm{r}}_{\mathrm{coll}}[\pi] = \frac{1}{2} \sum_{|q_0| \ll \mu, q_1 < |\boldsymbol{q}| < q_2} \boldsymbol{\pi}^{+}(q)[q_0^2 - \tau - (q - q_{\mathrm{c}})^2] \times \boldsymbol{\pi}(q) - \tfrac{1}{4}\beta(\pi_a^2)^2 \tag{7}$$

and the partition function

$$Z_{\mathrm{coll}} = \int D\pi(x) \exp(i^{\rho}A^{\mathrm{r}}_{\mathrm{coll}}[\pi]). \tag{8}$$

‡3 Some time ago, the same methods have been introduced into nuclear physics at an even more long distance level: The nucleon fields have been integrated out in favor of a nuclear density field thus obtaining Hartree equations plus their fluctuation corrections [5].

‡4 Because of the similarity with superfluid ^{4}He these may be called "pionic rotons".

‡5 We have gone to natural units $\xi_0 \equiv 1$, $\alpha \equiv 1$, abbreviated $\tau \equiv 1 - \rho/\rho_{\mathrm{c}}$ (in analogy with the conventional $-(1 - T/T_{\mathrm{c}})$ factor in statistical mechanics), and restricted the momentum summation to the interval $|\boldsymbol{q}| \in (q_1, q_2)$ with $q_1^2 \ll q_{\mathrm{c}}^2$, $q_2^2 \gg q_{\mathrm{c}}^2$ outside of which $\omega^2(q)$ is so large that fluctuations are irrelevant. We have omitted the terms of order π^6 since we want to work close to $\tau = 0$ where π remains small of order $\sqrt{-\tau}$.

For a first rough idea consider the pretransitional (i.e. $\tau \gtrsim 0$) behaviour of the pion propagator at zero separation ($q_0 = \mathrm{i}q_4$)

$$\langle \pi_a(x)\pi_b(x')\rangle|_{x=x'} = \delta_{ab} \int_{-\infty}^{\infty} \frac{\mathrm{d}q_4}{2\pi} \int_{q_1<|q|<q_2} \frac{\mathrm{d}^3q}{(2\pi)^3} \frac{1}{q_4^2 + \tau + (q - q_c)^2} . \tag{9}$$

The integral can be divided into three parts:

$$\frac{1}{2\pi^2} \int \frac{\mathrm{d}q_4}{2\pi} \int_{q_1}^{q_2} \mathrm{d}q \left(1 + \frac{2q_c(q - q_c) - q_4^2 - \tau}{q_4^2 + \tau + (q - q_c)^2} + q_c^2 \frac{1}{q_4^2 + \tau + (q - q_c)^2}\right), \tag{10}$$

of which the first is a constant, while the second is a smooth function of $\tau > 0$. The important contribution comes from the last part which diverges for $\tau \to 0$ as

$$\frac{1}{2\pi^2} \int \frac{\mathrm{d}q_4}{2\pi} \frac{\pi q_c^2}{(q_4^2 + \tau)^{1/2}} \sim -\frac{q_c^2}{4\pi^2} \log \tau . \tag{11}$$

The origin of this divergence lies in the gigantic directional fluctuations of $\boldsymbol{q}$ for $|\boldsymbol{q}|$ close to q_c which are degenerate on a whole spherical shell [‡6]. This is in contrast with standard phase transitions where the instability sets in at $q = 0$ and fluctuations remain small for $\tau \to +0$ ($\langle \pi^2 \rangle \sim c_1 + c_2 \sqrt{\tau}$). As usual in quantum field theory, the regular parts can be absorbed into τ and merely change $\tau \to a\tau + b$, i.e. the normalization of τ and the place where it vanishes. This amounts to a renormalization of $\alpha_1 \xi_0$ and the critical density ρ_c.

The singular piece, however, has dramatic consequences: If iterated in a Dyson equation it modifies the full propagator to the same form as (9) but with a fluctuation corrected value $\tau_f \approx \tau - K \log \tau_f$ where K is a positive constant and the regular piece already has been absorbed into τ. The sign of τ_f decides about the fate of the pion condensate including fluctuations. But here we have the following problem: As a function of τ the corrected value behaves, far above the critical point, in the same way as before, $\tau_f \sim \tau$. But as the previously critical point is approached, $\tau_f(\tau)$ flattens and remains positive for *all values of* τ. Thus, the instability is gone and there can no longer be a phase transition. The learned reader will object that the argument relies on perturbation theory and breaks down for $|\tau_f - \tau| \nless \tau$ such that in reality the system may just end up in a slight shift of the point of transition. The trouble is, however, of a more fundamental nature and is very much related to the old observation, in many-body physics, that one-dimensional systems have a unique ground state, which follows from Schrödinger theory, and that there are no Goldstone modes in two dimensions [7]. This might seem astonishing since here the system is 3 + 1-dimensional. The common point, however, is that the term (9) has exactly the same divergence as occurs in two- or one-dimensional systems depending on whether one looks at the quantum-mechanical or thermal case, and this gives rise to the fluctuation disaster. Instead of giving an abstract proof we hope to convince the reader by treating the full fluctuation problem (10) *exactly* in the somewhat academic limit that there are infinitely many pions, i.e. instead of an isotopic O(3) symmetry we take O(N) with $N \to \infty$. Simultaneously we have to let β tend to zero as β/N with fixed β. It is an experience in many-body theory that such results are not very far of reality if N is chosen to be 3 at the end.

In order to simplify the algebra we shall replace the non-local interaction by a local $(\beta/N) \int \mathrm{d}^4x (\pi_a^2)^2$. The only difference lies in some angular average over the directions of $\boldsymbol{q}_c$ which can be accounted for by an effective change in the size of β. Then the full fluctuating theory of static fields close to $\tau = 0$ is defined by the generating functional [8]

$$Z_{\mathrm{coll}}[j] = \int D\pi(x) \exp\left(\mathrm{i}\, {}^{\rho}A^{\mathrm{r}}_{\mathrm{coll}}[\pi] + \mathrm{i} \int \mathrm{d}^4x\, j_a(x)\, \pi_a(x)\right) \equiv \exp(\mathrm{i}\, W[j]), \tag{12}$$

where j_a, $a = 1, ..., N$ are external sources for each pion field π_a. In order to perform the path integral we introduce a collective field [4,5] $\sigma \mathrel{\hat{=}} (\beta/N)\pi_a^2$ and rewrite $Z_{\mathrm{coll}}[j]$ as [‡7]

‡6 Notice the surface factor q_c^2.

‡7 This is an identical reformulation of (14) as can be verified by a quadratic completion and integration over the σ-field using the same argument as given after eq. (2).

$$Z_{\rm coll}[j] = \int D\pi(x)\, D\sigma(x)$$

$$\times \exp\left[i\left(\tfrac{1}{2}\pi_a^+ [q_0^2 - \omega^2(q) - \sigma]\, \pi_a + \frac{N}{4\beta}\int d^4x\, \sigma^2 + \int d^4x\, j_a(x)\pi_a(x) \right)\right]. \tag{13}$$

Notice that σ is, in general, non-diagonal in momentum space and $\pi^+ M\pi$ has to be read as in matrix algebra with q as indices. In the form (13) the $D\pi$ integral is gaussian and can be executed giving

$$Z_{\rm coll}[j] = \int D\sigma \exp\left\{ i\left[N\left(\frac{1}{4\beta}\int dx\, \sigma^2(x) + \tfrac{1}{2} i\, {\rm tr}\log iG_\sigma^{-1} \right) + \tfrac{1}{2} i\, j_a G_\sigma j_a \right]\right\} \equiv \exp(iW[j]). \tag{14}$$

Here G_σ is the propagator of a pion in an external σ-field, i.e. in momentum space: $iG_\sigma^{-1} = q_0^2 - \omega^2(q) - \sigma$. The important point is the factor N in front of the brackets. As $N \to \infty$, the σ-field is squeezed into the extremum of the exponent, say $\sigma = \Sigma[j]$, and can no longer fluctuate (saddle point theorem). Thus

$$W[j] = N\left(\frac{1}{4\beta}\int dx\, \Sigma^2_{(x)} + \tfrac{1}{2} i\, {\rm tr}\log iG_\Sigma^{-1} \right) + \tfrac{1}{2} i\, j_a G_\Sigma j_a, \tag{15}$$

is the *exact* generating functional of all connected pion Green's functions. It is now easy to study the question of spontaneous symmetry breakdown. For this we introduce the effective action [8] via a Legendre transform $\Gamma[\Phi] + j_a\Phi_a = W[j]$ with $\Phi_a \equiv \delta W[j]/\delta j_a(x) \equiv iG_\Sigma j_a$ being the ground-state expectation value of the pion field. Inserting this into (15) we find the result, exact to leading order N,

$$\Gamma[\Phi, \Sigma] = N\left(\frac{1}{4\beta}\int dx\, \Sigma^2_{(x)} + \tfrac{1}{2} i\, {\rm tr}\log iG_\Sigma^{-1} \right) + \tfrac{1}{2}\Phi_a [q_0^2 - \tau - (q - q_c)^2 - \Sigma]\, \Phi_a. \tag{16}$$

Let us now ask whether there can be a static condensate of momentum q_c, i.e. $\Phi_a = \Phi_a^c \delta_{q,q_c}\delta_{q_c,0}$ i $\Sigma(x) = \Sigma^c$. For this the effective potential

$$V(\Phi^c, \Sigma^c) = -\Gamma[\Phi^c, \Sigma^c]/VT = -N\left(\frac{1}{4\beta}\Sigma^{c2} - \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} \log(q_4^2 + \tau + (q - q_c)^2 + \Sigma^c) \right) + \tfrac{1}{2}(\tau + \Sigma^c)\Phi_a^{c2}, \tag{17}$$

has to be minimal at $\Phi_a^c = \langle 0|\pi_a|0\rangle \neq 0$. We see that this requires $\tau_f \equiv \tau + \Sigma^c = \partial^2 V/\partial\Phi_a^{c2} = 0$. But differentiating (17) with respect to Σ^c we find the following equation for τ_f

$$\tau = \tau_f + (\beta/4\pi^2)[(q_c^2 - \tau_f/2)\log(\tau_f/q_c^2) + 4q_c\sqrt{\tau_f}] + (\beta/N)\Phi_a^{c2}, \tag{18}$$

where we have used (10) and absorbed the divergencies in a redefinition of coupling constant g and τ. Eq. (18) leads to precisely the same problem as before only that now the result is exact (in the limit of large N). There is no value of τ (i.e. density) for which τ_f can be forced to vanish. Thus $\Phi_a^c = \langle 0|\pi_a|0\rangle = 0$ and there can be no condensate nor Goldstone modes.

Notice that eq. (18) makes an exact statement about the susceptibility of pion fluctuations which is defined as

$$\begin{aligned}\delta_{q,q'}\chi(q)_{ab} &= \langle 0|\pi_a(q)\pi_b(q')|0\rangle \\ &= -[\delta^2\Gamma[\Phi]/\delta\Phi_a(q)\delta\Phi_b(q')]^{-1}_{\Phi^c} \\ &= \delta_{q,q'}\delta_{ab}/[q_4^2 + \tau_f + (q - q_c)^2].\end{aligned} \tag{19}$$

This is a smooth function in τ, isotropic in isospin. For $\rho \gg \rho_c$ and $q_4 = 0$, $q = q_c$, χ grows exponentially in $-\tau = \rho/\rho_c - 1$ as a washed-out remnant of the two infinite transverse susceptibilities at the mean-field level.

For high nuclear temperature the integration over $dq_4/2\pi$ in (10), (17) has to be replaced by a simple temperature factor T while setting $q_4 = 0$. Then the effect is even more drastic as we can see from (11): instead of the fluctuation logarithm there is even a square-root singularity at $\tau_f = 0$ and eq. (18) becomes

$$\tau = \tau_f + (\beta/2\pi^2)T\{2q_c\log(\tau_f/q_c^2) - [(q_c^2 - \tau_f)/\sqrt{\tau_f}]\,\pi\} + (\beta/N)\Phi_a^{c2}. \tag{20}$$

For arbitrary temperature one has to sum over Matsubara frequencies $T\Sigma_{(2\pi/T)q_4=0,\pm1,\pm2,...}$ rather than just $q_4 = 0$. Again the divergence is of the square-root type.

The behaviour of the local square of the pion field can be calculated from $V(\Phi)$ as follows

$$\langle\pi^2(x)\rangle = 2\partial V(\Phi)/\partial\tau|_{\Phi^c} = N\Sigma^c/\beta = N(\tau_f - \tau)/\beta. \quad (21)$$

It is a smooth function of τ. For large $\pm\tau$ it may be approximated by the mean-field behaviour 0, $-N\tau/\beta$, respectively ‡8. It will be interesting to see what modifications arise to this argument from the possibility of a condensate with lattice texture ‡9.

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‡8 It should be mentioned that eq. (18) becomes physically inconsistent for very large $\tau_f \approx q_c \exp(8\pi^3/\beta)$ indicating an instability of the limit $N \to \infty$ of the field theory. This presents no problem since the initial action (7) may only be used for $\tau \approx 0$.

‡9 In cholesteric liquid crystals where fluctuations are similar the phase transition is saved by a lattice texture (blue phase) [9].

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