NO CRYPTOFERROMAGNETIC STATE DUE TO FLUCTUATIONS

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It is shown that the cryptoferromagnetic state which arises in a mean-field treatment of magnetic superconductors cannot form due to strong fluctuations. The Bragg-like neutron reflection is due to these fluctuations rather than a helical texture.

Field configurations with non-vanishing momentum $q_0 \neq 0$ occur, at the mean-field level, in several physical systems; examples are cholesteric liquid crystals [1], pion condensates [2], and magnetic superconductors [3] [‡1]. Fluctuations, however, drastically change the picture. In the first example, the phase transition is strongly shifted to lower temperatures and occurs only thanks to a cubic term in the free energy which changes the order from second to first [5]. Moreover, the transition does not proceed directly from the disordered to the chiral state but there is an intermediate blue phase in which the texture forms a lattice [5].

In pion condensates, where such a first-order mechanism is absent, the phase transition is apparently prevented completely [6].

It is the purpose of this note to show that in the magnetic superconductors the situation is similar to pion condensation: No long-range order can develop, all susceptibilities remain finite, and there are no Goldstone bosons. All phenomena attributed to the $q_0 \neq 0$ state [7] are really due to fluctuations and "pretransitional" in character.

Consider the free-energy density of the magnetic superconductor [8] which we write in natural units as

$$2f = \tau|\Delta|^2 + \tfrac{1}{2}|\Delta|^4 + |(-\mathrm{i}\boldsymbol{\partial} - 2eA)\Delta|^2 + \tau_M M^2 + \tfrac{1}{2}\beta M^4 + \xi_M^2(\boldsymbol{\partial}\cdot M)^2 + (\boldsymbol{\nabla}\times A - M)^2. \tag{1}$$

‡1 The helical texture in superfluid ^{3}He is not a good example since it forms only in a given external superflow, see ref. [4].

The partition function is given by

$$Z = \int \mathcal{D}\Delta\mathcal{D}\Delta^+\mathcal{D}M\mathcal{D}A \times \exp\left[-\int \mathrm{d}^3x \int_0^{1/T} \mathrm{d}t\,(\tfrac{1}{2}\dot{A}^2 + f)\right], \tag{2}$$

where we have included only static fluctuations except for the magnetic potential A where the time dependence is relevant [3] (t = imaginary time, T = temperature). Since A appears quadratically, it is integrated out and the exponent becomes

$$F[\Delta, M] = \frac{1}{2}\int \mathrm{d}^3x\,[\tau\Delta^2 + \tfrac{1}{2}\Delta^4 + (\partial\Delta)^2 + (\tau_M + 1)M^2 + \tfrac{1}{2}\beta M^4 + \xi_M^2(\boldsymbol{\partial}\cdot M)^2 - \tfrac{1}{2}(\boldsymbol{\nabla}\times M)G(\boldsymbol{\nabla}\times M) + \tfrac{1}{2}\,\mathrm{tr}\log\widetilde{G}^{-1}], \tag{3}$$

where

$$G_{ij}(x, x') = \int_0^{1/T} \mathrm{d}t\,\widetilde{G}_{ij}(x, t; x', t') \equiv \int_0^{1/T} \mathrm{d}t\,\langle A_i(x, t), A_j(x', t')\rangle \tag{4}$$

is the correlation function of the field A_i whose mass term μ depends on $\Delta(x)$ as

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$$\mu(x) = 2e\Delta(x)\,, \tag{5}$$

thereby accounting for the Meissner effect. Before integration we have chosen a gauge such that Δ becomes real. The trace of the logarithm collects the "black body" energy of the massive photons. It can be neglected except in the immediate vicinity of $\tau = 0$ where $\mu = 0$. Notice that if it is expanded in powers of Δ, it generates a cubic term. This changes the superconductive phase transition from second to first order [9], but since there is a factor e^3 the effect is so weak that it has never been seen [3].

The important feature of eq. (3) is the new bending energy for the magnetization. Assuming, for a moment, a constant order parameter Δ, we may invert

$$G_{ij}^{-1} = (\delta_{ij} - q_i q_j/\mu^2)/(q^2 + \mu^2)\,,$$

separate longitudinal and transverse components of $\boldsymbol{M}$, write the quadratic piece as

$$(\tau_M + 1 + \xi_M^2 q^2) M_\parallel^2$$
$$+ [\tau_M + 1 + \xi_M^2 q^2 + \mu^2/(\mu^2 + q^2)]\, M_\perp^2\,, \tag{6}$$

and realize that for

$$\gamma \equiv \xi_M \mu < 1 \tag{7}$$

the coefficient of the transverse part has a minimum at

$$q_0^2 = \mu^2 (1 - \gamma)/\gamma\,. \tag{8}$$

Close to it, the bending energy can be expanded as

$$[\tau_s + \alpha(q - q_0)^2]\, M_\perp^2\,, \tag{9}$$

where

$$\tau_s = \tau_M + 1 + (\gamma - 1)^2 \equiv \tau_s^0 (T/T_s - 1)\,,$$

$$\alpha = \gamma(4 - \gamma)\,. \tag{10}$$

The temperature T_s at which $\tau_s = 0$ marks the point below which a helical magnetic configuration $M_\perp = M_\perp^0 \exp(\mathrm{i} q_0 z)$ may form a stable ground state at the mean-field level.

We shall now show that this solution is an illusion. Fluctuations prevent the field from settling down.

In order to study this problem we may neglect $M_\parallel$ and the fluctuations in the gap parameter Δ since they remain hard close to $\tau_s = 0$. Only the fluctuations of $M_\perp$ are of a severe nature. In the critical regime, they are controlled by the free energy

$$2f_{M_\perp} = [\tau_s + \alpha(q - q_0)^2]\, M_\perp^2 + \tfrac{1}{2}\beta M_\perp^4$$
$$+ \text{irrelevant terms}\,. \tag{11}$$

From here on the conclusion follows precisely in the same way as in ref. [6]. It will suffice to repeat only the physics behind the formal argument: The bare correlation function

$$\langle M_\perp^2 \rangle = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, [\tau_s + \alpha(q - q_0)^2]^{-1}$$

has gigantic directional fluctuations of the wave vector $\boldsymbol{q}$ over a whole spherical shell and this leads to a divergence

$$\langle M_\perp^2 \rangle \sim 1/\sqrt{\tau_s} \tag{12}$$

for $\tau_s \to 0$. If this is inserted into Dyson's equation the renormalized τ_s^{ren} satisfies

$$\tau_s^{\text{ren}} \sim \tau_s + \text{const.}/\sqrt{\tau_s^{\text{ren}}}\,, \tag{13}$$

such that the transition can never take place. The expectation of $\langle M_\perp \rangle$ always remains zero.

The non-perturbative fluctuation effects are most easily accounted for by using higher effective actions as employed recently by the author [10].

The non-existence of the phase transition does not ruin many of the observational characteristics of the mean-field phase (see ref. [6]). The large fluctuations for $q \approx q_0$ can reflect neutrons very similarly to a stationary helical texture but the line width is increased and shows a typical "pretransitional" behaviour except that the renormalized τ_s^{ren} keeps decreasing as τ_s^{-2} for $\tau_s \to -\infty$, i.e. $\Delta q \sim (\tau_s^{\text{ren}}/\alpha)^{1/2} \to 1/(\sqrt{\alpha}\,\tau_s)$.

The idea to this note was conceived during an interesting lecture of Professor J. Keller on the problems of magnetic superconductors which I happened to attend just after finishing the related problem in pion condensates [6]. I am grateful to him for sending me his review with P. Fulde and for several critical discussions.

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