FLUCTUATIONS AND DEFECTS IN THE SPIRAL STATE OF MAGNETIC SUPERCONDUCTORS

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We want to point out three properties of a magnetic superconductor: (i) The absence of true long-range order in the spiral state leads to the structure functions behaving like $(q_\parallel - q_0)^{\eta-2}$ and $(q_\perp^2)^{\eta-2}$ for $q_\perp = 0$ and $q_\parallel = 0$, respectively, where q_0 is the preferred momentum. The indices η are measured via Bragg-like neutron scattering. (ii) The state is perforated by line-like defects. (iii) Above some critical temperature the defect lines proliferate, thereby destroying the spiral quasi-order.

In a recent note [1] we argued that the condensate of long-range spiral order suggested by mean-field calculations in magnetic superconductors [2] could not properly exist due to catastrophic fluctuations. The Bragg-like reflex in neutron scattering would then really be due to these fluctuations. Experimentally, this theorists' subtle distinction requires high resolution of the line shape. Hoping that this will soon be available we calculate the singular behaviour of the structure factor $S(\boldsymbol{q})$ close to $q_\parallel \approx q_0, \boldsymbol{q}_\perp = 0$ and $\boldsymbol{q}_\perp \approx 0, q_\parallel = 0$. In addition we draw attention to the existence of line-like defects of the dislocation type (just as displayed on everyone's fingerprints), whose configurational fluctuations eventually destroy the pseudo-order and carry the system to the normal paramagnetic superconducting state.

Using the notation of ref. [1] we begin with the free energy of the relevant order parameter $M_\perp$ which is the magnetization orthogonal to the preferred momentum direction (say $\hat{z}$).

$$F = \frac{1}{2} \sum_{\boldsymbol{q}} [\tau + (|\boldsymbol{q}| - q_0)^2] |M_\perp(\boldsymbol{q})|^2 + \frac{\beta}{4} \int d^3x \, [M_\perp^2(\boldsymbol{x})]^2 \, . \tag{1}$$

Here τ contains the main temperature dependence and (1) is valid for $\tau \approx 0$ and $q \approx q_0$. Our analysis will start by assuming $\tau = \tau_0 < 0$ where the energy is minimized by the spiral solution

$$M_\perp \equiv M_1 + iM_2 = \sqrt{B/\alpha}\, e^{iq_0 z} \, . \tag{2}$$

We want to study soft long-wave fluctuations around this and multiply $M_\perp$ by a pure space dependent phase $e^{i\varphi(\boldsymbol{x})}$. By writing near q_0: $(|\boldsymbol{q}| - q_0)^2 \approx (1/4q_0^2)(q^2 - q_0^2)^2$ and working out the lowest derivatives, F reduces to the pure gradient energy (up to a constant)

$$F = \tfrac{1}{2} B \, [(\partial_3 \varphi)^2 + \lambda^2 (\boldsymbol{\partial}_\perp^2 \varphi)^2] \, , \tag{3}$$

where $\lambda^{-2} = 4q_0^2$. Since φ is a gaussian random field, the correlation function $\langle M_\perp(\boldsymbol{x}) M_\perp^\dagger(0)\rangle$ is easily calculated as [3]

$$\langle M_\perp(\boldsymbol{x}) M_\perp^\dagger(0)\rangle = (B/\alpha) \langle \exp\{\tfrac{1}{2} i[\varphi(\boldsymbol{x}) - \varphi(0)]\}\rangle = (B/\alpha) \exp\{-\tfrac{1}{2}\langle[\varphi(\boldsymbol{x}) - \varphi(0)]^2\rangle\} \, ,$$

where

$$-\tfrac{1}{2}\langle[\varphi(\boldsymbol{x})-\varphi(0)]^2\rangle = \langle\varphi(\boldsymbol{x})\varphi(0)\rangle - \langle\varphi(0)^2\rangle \equiv G_c(\boldsymbol{x}) = -\frac{T}{B}\int\frac{dq_3\,d^2q_\perp}{(2\pi)^3}(1-\cos\boldsymbol{q}\cdot\boldsymbol{x})\,(q_3^2+\lambda^2q_\perp^4)^{-1}$$

$$= -\frac{T\pi}{B\lambda(2\pi)^3}\int\frac{d^2q_\perp}{q_\perp^2}(1-\exp(-\lambda q_\perp^2|z|)\cos\boldsymbol{q}_\perp\cdot\boldsymbol{x}_\perp) = -\frac{T}{8\pi B\lambda}\int\frac{dq_\perp^2}{q_\perp^2}[1-\exp(-\lambda q_\perp^2|z|)J_0(q_\perp|\boldsymbol{x}_\perp|)]$$

$$= -\eta[\log(c^2x_\perp^2/d^2) + 2\gamma + E_1(x_\perp^2/4\lambda|z|)]\ . \tag{4}$$

Here $\gamma \approx 0.577$, η stands for $T/8\pi B\lambda$, $E_1(x) \equiv \int_x^\infty e^{-t}\,dt/t$ is the exponential integral, and $c^2 = q_{\rm cut}^2/q_0^2$ is a constant due to the transverse momentum cutoff. For $|z| \gg |x_\perp|$, $E_1(x) \to -\gamma - \log x$ and

$$G_c(\boldsymbol{x}) \sim c^{2\eta}\,e^{-\gamma\eta}\,(\pi^2/q_0\lambda|z|)^\eta\ . \tag{5}$$

For $|x_\perp| \gg |z|$, $E_1 \to 0$ and

$$G_c(\boldsymbol{x}) \sim c^{2\eta}\,e^{-2\gamma\eta}(4\pi^2/q_0^2\boldsymbol{x}_\perp^2)^\eta\ . \tag{6}$$

This gives for the structure factor

$$S(\boldsymbol{q}) \approx (q_\parallel - q_0)^{\eta-2} \quad \text{for } q_\perp = 0, \qquad S(\boldsymbol{q}) \approx (q_\perp^2)^{\eta-2} \quad \text{for } q_\parallel = 0\ . \tag{7}$$

Such a behaviour has been detected in Bragg-like reflexes of smectic liquid crystals [4] and we hope that neutron scattering on the spiral state will soon reveal the same deviations from true long-range order.

Given the simplicity of obtaining correlation functions we expect that also other fluctuation properties of the system can be understood by using this quasi-ordered state as a basis. For this it is essential to include macroscopic topological excitations [5]. Since the field φ is cyclic (φ and $\varphi + 2\pi n$ are indistinguishable) the system has singular vortex lines similar to He II. Recall that there the energy is $f = \frac{1}{2}B(\partial_i\varphi)^2$ such that $b_i \equiv B\,\partial_i\varphi$ is divergenceless, just as a magnetic field. Therefore one introduces a vector potential via $\boldsymbol{b} = \boldsymbol{\partial}\times\boldsymbol{a}$ and finds the forces between vortex lines from the gauge invariant coupling $\Sigma_n\, 2\pi n \oint dx_i^{(n)} a_i$ (yielding the Biot-Savart law) [6]. Here the field equation

$$\partial_3(\partial_3\varphi) - \lambda^2[\partial_1(\boldsymbol{\partial}_\perp^2\partial_1\varphi) + \partial_2(\boldsymbol{\partial}_\perp^2\partial_2\varphi)] = 0 \tag{8}$$

tells us that

$$\boldsymbol{b} \equiv B(\partial_3\varphi, -\lambda^2\boldsymbol{\partial}_\perp^2\partial_1\varphi, -\lambda^2\boldsymbol{\partial}_\perp^2\partial_2\varphi) \tag{9}$$

is divergenceless and may be written as $\boldsymbol{\partial}\times\boldsymbol{a}$. In terms of b_i the energy reads $(2B)^{-1}\{b_3^2 + [(\lambda\boldsymbol{\partial}_\perp^2)^{-1}b_1]b_1 + [(\lambda\boldsymbol{\partial}_\perp^2)^{-1}b_2]b_2\}$ such that in the long wave limit [7] and the gauge $\boldsymbol{\partial}_\perp \boldsymbol{a}_\perp = 0$

$$f = \frac{1}{2B}\sum_{\boldsymbol{k}}[\lambda^{-2}|a_3(\boldsymbol{k})|^2 + (k_\perp^2 + k_3^2/\lambda^2k_\perp^2)\,|\boldsymbol{a}_\perp(\boldsymbol{k})|^2]\ . \tag{10}$$

The coupling of a defect loop L is obtained as follows: Let n be the vortex strength of L such that for any circuit around L, φ changes by $2\pi n$, i.e. $\oint dk_i\,\partial_i\varphi = 2\pi n$. Then φ must have a jump by $2\pi n$ on some surface spanned by L. Suppose now that this loop is inserted in a given smooth field configuration φ thereby changing it to $\varphi + \delta\varphi$. The field energy (3) changes by

$$\delta f = B\int d^3x\,(\partial_3\varphi\,\partial_3\delta\varphi - \lambda^2\boldsymbol{\partial}_\perp^2\partial_1\varphi\,\partial_1\delta\varphi - \lambda^2\boldsymbol{\partial}_\perp^2\partial_2\varphi\,\partial_2\delta\varphi) = \int d^3x\,b_i\,\partial_i\delta\varphi = -\int d^3x\,\partial_i(b_i\varphi) = -\int dS_i\,b_i\varphi, \tag{11}$$

where the surface integral runs over a thin ellipsoid enclosing the surface S. This integral can be evaluated via the discontinuity $\Delta\varphi|_S = 2\pi n$ as

$$\delta f = 2\pi n\int_S dS_i\,b_i\ . \tag{12}$$

Now $\boldsymbol{b} = \boldsymbol{\partial} \times \boldsymbol{a}$ can be used to transform this into the gauge-invariant local coupling

$$\delta f = 2\pi n \oint_{L} \mathrm{d}x_i \, a_i(\boldsymbol{x}) \,. \tag{13}$$

The grand-canonical ensemble of interacting lines can now be described in complete analogy with defect theories of melting [8]: We introduce a disorder field $\psi(\boldsymbol{x})$ whose free correlation function

$$\langle \psi(\boldsymbol{x}') \, \psi^+(\boldsymbol{x}) \rangle = \int_0^\infty \mathrm{d}L \, \mathrm{e}^{-(\epsilon/T - w)L} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \, \mathrm{e}^{\mathrm{i}\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x}) - a^2 \boldsymbol{k}^2 L/2D} = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \, \mathrm{e}^{\mathrm{i}\boldsymbol{k}(\boldsymbol{x}'-\boldsymbol{x})} \, [a^2 \boldsymbol{k}^2/2D + (\epsilon/T - w)]^{-1}, \tag{14}$$

gives the probability for a line of any length to run from $\boldsymbol{x}$ to $\boldsymbol{x}'$. Here D is the space dimension, $w \approx \log 2D$ the entropy per link, a the thickness of the line, and ϵ the core energy per length a. The grand-canonical ensemble of free random loops is then given by the partition function [8]

$$Z = \int \mathcal{D}\psi \, \mathcal{D}\psi^+ \exp\left(- \int \mathrm{d}x \, [(a^2/2D) \, |\partial\psi|^2 + (\epsilon/T - w) \, |\psi|^2] \right) . \tag{15}$$

The long-range interaction due to the gauge field is incorporated by the usual minimal replacement

$$\boldsymbol{\partial} \to \boldsymbol{\partial} - (2\pi \mathrm{i}/T) \boldsymbol{a} \,, \tag{16}$$

where we have restricted ourselves to only the basic lines with unit winding number.

Certainly, there will be short range steric interactions which we may parametrize by a term $\frac{1}{2} q |\psi|^4$. Thus the total partition function of the system as seen from the spiral state becomes

$$Z = \int \mathcal{D}A \, \mathcal{D}\psi \, \mathcal{D}\psi^+ \exp\left(\frac{1}{2} \sum_k \, [\lambda^{-2} |A_3(\boldsymbol{k})|^2 + (k_\perp^2 + k_3^2/\lambda^2 k_\perp^2) |A_\perp(\boldsymbol{k})|^2] \right)$$

$$\times \exp \int \mathrm{d}^3 x \, [|(\partial_i - 2\pi \mathrm{i} \sqrt{B} A_i/T) \, \psi|^2 + (\epsilon/T - w) \, |\psi|^2 + \tfrac{1}{2} q |\psi|^4] \,. \tag{17}$$

This will be the most economic tool for studying the critical behaviour at the transition temperature which is given by $T_c = \epsilon/w$. Above T_c the disorder field destabilizes leading to $\langle \psi \rangle \neq 0$ which signalizes the proliferation of defect lines and the destruction of the spiral state.

It goes without saying that our partition function (17) can be used for studying the completely analogous phenomena in smectics where the importance of defect lines has been stressed before [9] and, of course, in pion condensates [7].

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